THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530A: Games & Strategic Thinking 2025-2026 Term 1
Homework Assignment 2
Due Date: November 28, 2025 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature ____________________ Date ____________________

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. homework assignments via 2025R1 Games and Strategic Thinking (UGEB2530A)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

The deadline for HW2 is November 28, 2025 (Friday) before 11:59 PM. Please plan ahead and submit your HW2 on time.

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.

- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Find all the pure Nash equilibria of the games with the following game bimatrices and state whether they are Pareto optimal.

   (a) $\begin{pmatrix} (1,3) & (4,6) \\ (2,5) & (1,3) \end{pmatrix}$

   (b) $\begin{pmatrix} (-1,2) & (3,6) & (1,-3) \\ (3,1) & (5,-1) & (4,2) \\ (6,3) & (-2,2) & (3,0) \end{pmatrix}$

2. Consider the 2-person game with the following bimatrix

$$\begin{pmatrix} (1,4) & (6,1) \\ (4,3) & (2,5) \end{pmatrix}$$

   (a) Find the prudential strategy of each player and the security levels of the players.

   (b) Find the Nash equilibrium of the game and the corresponding payoffs to the players.

3. Let us consider the 2-person game used by a non-profit organisation (social hut) that wishes to aid a needy man if he looks for work but not if he does not try. The payoffs are $5, -2$ (for organisation, needy man) if the organisation aids and the needy man tries to work; $1, 3$ if the organisation does not aid and the needy man tries to work; $4, 2$ if the organisation aids and the needy man does not try to work; and $-3, 0$ in the remaining case.

   (a) Write down the game bimatrix. (Use organisation aid as the row player)

   (b) Find the prudential strategy of each player and the security levels of the players.

   (c) Find the Nash equilibrium of the game and the corresponding payoffs to the players.

4. The battle between a batsman and a bowler in a cricket game just reached the head-to-head competition stage. The bowler is either taking an inswing or an outswing action, while the batsman is either using an offside or a legside action. If the bowler uses inswing and the batsman uses offside, the profits for the bowler and the batsman will be 5 and 5 respectively. If the bowler uses inswing and the batsman uses legside, the profits for the bowler and the batsman will be 9 and 12 respectively. If the bowler uses outswing and the batsman uses offside, the profits for the bowler and the batsman will be 13 and 11 respectively. If the bowler uses outswing and the batsman uses legside, the profits for the bowler and the batsman will be 7 and 8 respectively.

   (a) Write down the game bimatrix. (Use the bowler as the row player)

   (b) Find the prudential strategies and the security levels of the two competitors.

   (c) Find all the Nash equilibria and the corresponding payoffs to two competitors.

5. Answer the following questions:

(a) ABC Corporation plans to undertake three subsidiary projects with minimum investments: Project A with an investment of \$4000, Project B with \$6000, and Project C with \$5000 respectively. The projects are trial projects and have chances to be successful.

	Project A	Project B	Project C
Probability of success	0.8	0.4	0.5
Profit if successful	20000	25000	24000
Loss if fails	2000	2000	1000

Draw a decision tree and make a decision.

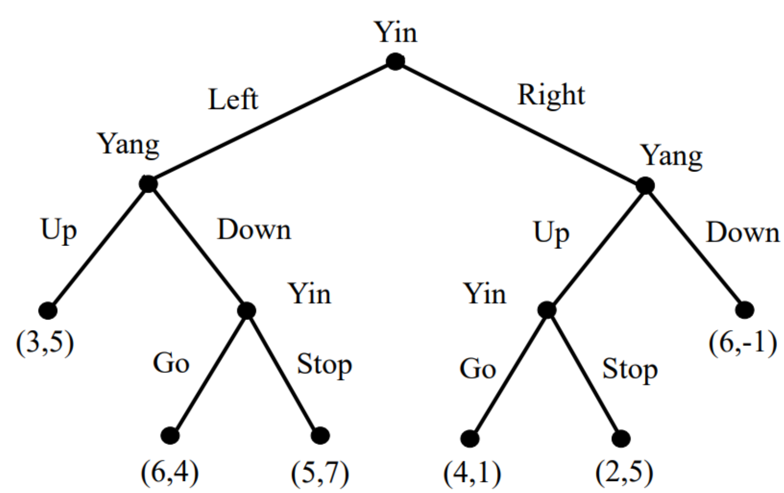
(b) A man considers investing an initial amount in either Project A or Project B. For Project A, he invests \$8000, and for Project B, he invests \$7000.

- The production outcomes upon completion of **Project A** can result in three possible states of nature:
  - High demand with a probability of 0.5, yielding a profit of \$12,000
  - Average demand with a probability of 0.3, yielding a profit of \$8,000
  - Low demand with a probability of 0.2, yielding a profit of \$5,000
- The production outcomes upon completion of **Project B** also have three possible states of nature:
  - High demand with a probability of 0.6, yielding a profit of \$10,000
  - Average demand with a probability of 0.3, yielding a profit of \$9,000
  - Low demand with a probability of 0.1, yielding a profit of \$5,000

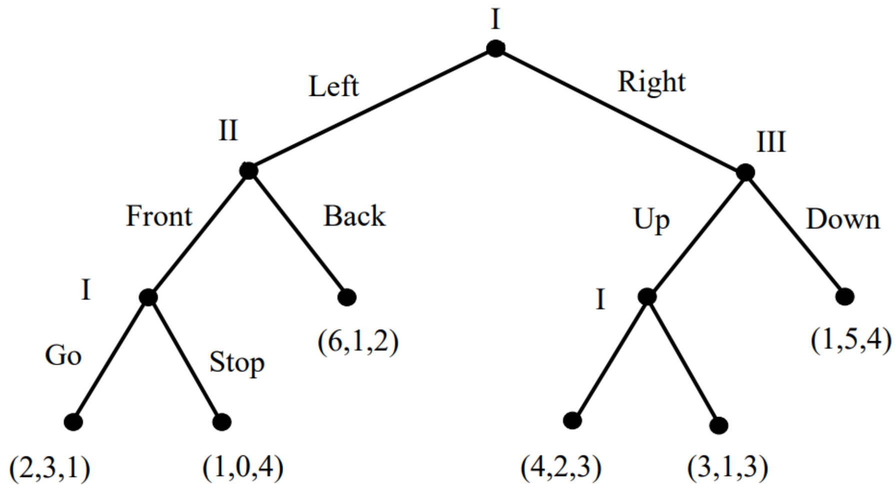
Draw a decision tree and choose the project with the maximum return.

6. Solve the following game trees by backward induction and write down the payoff pair in the solution.

a)



b)

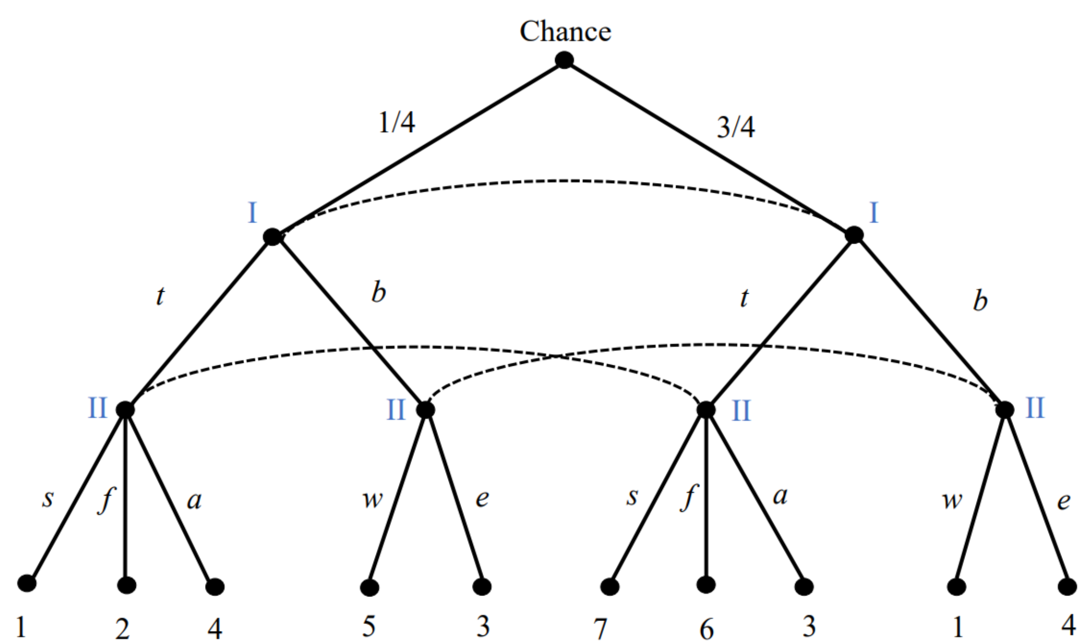


7. Mary and Abe start with \$10 in each of their piles. They take turns choosing one of two actions, continue or stop, with Mary choosing first. Each time a player says continue, \$10 will be removed from her pile, and \$20 will be added to the other player's pile. The game automatically stops when the total amount in their piles reaches \$60.

   (a) Draw the game tree of the game.

   (b) Solve the game by backward induction and write down the payoffs of the players in the solutions.

8. Consider a zero sum game between Player I and Player II with game tree



Here s denotes spirit, f denotes fire, a denotes air, w denotes water, e denotes earth, t denotes top, and b denotes bottom.

The numbers assigned to the terminal nodes are the payoffs to Player I.

(a) Write down all strategies of Player I and Player II.

(b) Find the strategic form (game matrix) of the zero sum game.

9. Mickey chooses a number from 5, 6 and 7. Minnie, who knows whether the chosen number is odd or even but does not know the exact value, must choose between 2 or 3. Then Minnie pays Mickey with an amount equal to the (absolute) difference of the numbers.

   (a) Draw the game tree of the game.

   (b) Write down all strategies of Mickey and Minnie.

   (c) Find the maximin strategy of Mickey, the minimax strategy of Minnie and the value of the game.

10. Players I and II play the following bluffing game. Each player bets \$1. Player I is given a card which is high or low; each is equally likely. Player I sees the card, player II doesn't. Player I can raise the bet to \$2 or fold. If player I folds, player I loses \$1 to player II. If player I raises, player II can call or fold. If player II folds, he loses \$1 to player I no matter what the card is. If player II calls, then player I wins \$2 from player II if his card is high and loses \$2 to player II if the card is low.

    (a) Draw the game tree of the game.

    (b) Write down all strategies of players I and II.

    (c) Write down the strategic form (game matrix) of the game.

    (d) Solve the game.

11. Anna has two coins. One is fair (probability 1/2 of heads and 1/2 of tails) and the other is biased with probability 1/4 of heads and 3/4 of tails. Anna knows which coin is fair and which is biased. She selects one of the coins and tosses it. The outcome of the toss is announced to Elsa. Then Elsa must guess whether Anna chose the fair or biased coin. If Elsa is incorrect, she pays \$2 to Anna, and if she is correct, she receives \$2 from Anna.

    (a) Draw the game tree.

    (b) Write down all strategies of Anna and Elsa.

    (c) Write down the strategic form of the game.

    (d) Solve the game.