THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530A: Games & Strategic Thinking 2025-2026 Term 1
Homework Assignment 1
Due Date: October 17, 2025 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

____________________ ____________________

Signature Date

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2025R1 Games and Strategic Thinking (UGEB2530A)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Write your solutions on A4 white paper or use an iPad or other similar device to present your answers and submit a digital form via Gradescope. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Please be aware that you can only use a ball-point pen to write your answers for any exams.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

Please attempt to solve all the problems. Your solutions for problems 1 - 10 are to be submitted.

1. Evaluate the following matrix products.

(a) $\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

(c) $\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix}$

2. Suppose a dice and 2 coins are tossed together. Let x be the number obtained from the dice and y be the number of heads shown among the coins.

(a) Fill in the blanks in the following tables:

x	1	2	3	4	5	6
$P(\text{getting } x)$						

y	0	1	2
$P(\text{getting } y)$			

(b) Using part (a), fill in the blanks in the following table:

z	1	2	3	4	5	6	7	8
$P(x + y = z)$								

(c) Now, evaluate the expected value of $x + y$.

3. In a game, two players call out one of the numbers 1, 2, or 3 simultaneously. Let S be the sum of the two numbers. If S is even, then player 2 pays S dollars to player 1. If S is odd, then player 1 pays S dollars to player 2.

(a) Write down the game matrix (with player 1's payoffs).

(b) Find the expected payoff of player 1 if player 1 calls out the numbers 1, 2, 3 with probabilities 0.5, 0.3, 0.2 respectively, and player 2 calls out the numbers 1, 2, 3 with probabilities 0.1, 0.4, 0.5 respectively.

(c) Suppose player 2 calls out the numbers 1,2,3 with probabilities 0.1, 0.4, 0.5 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?

4. In a modified rock-paper-scissors game, the loser pays the total number of fingers among the two gestures to the winner. The payoff is 0 if there is a draw.

(a) Write down the game matrix (with player 1's payoffs).
(Use rock, paper, scissors, as the order of strategies.)

(b) Suppose player 1 uses $(0.2, 0.4, 0.4)$ and player 2 uses $(0.3, 0.5, 0.2)$. Find the expected payoff of player 1.

(c) If player 1 uses $(0.2, 0.4, 0.4)$, what is player 2's best strategy?

(d) If player 2 uses $(0.3, 0.5, 0.2)$, what is player 1's best strategy?

(e) By considering equalizing strategies, find a Nash equilibrium and the value of the game.

5. For each of the following payoffs matrices (2-person, zero-sum, simultaneous games), circle all the saddle point(s) (if any).

(a) $$\begin{pmatrix} -1 & -4 & 4 & -2 \\ -4 & 4 & -1 & 0 \\ 2 & 3 & -1 & 1 \end{pmatrix}$$

(b) $$\begin{pmatrix} -3 & 5 & -2 & 0 \\ 0 & -5 & -1 & -3 \\ 1 & 3 & 7 & 4 \\ -2 & 2 & 3 & 1 \end{pmatrix}$$

(c) $$\begin{pmatrix} 4 & 3 & 5 & 3 \\ 2 & 1 & -1 & -20 \\ 3 & 3 & 4 & 3 \\ -16 & 0 & 16 & 1 \end{pmatrix}$$

6. By eliminating dominated strategies, find the optimal strategies for both players and determine the value of the game.

		Player B				
		I	II	III	IV	V
Player A	I	2	4	3	8	4
	II	5	6	3	7	8
	III	6	7	9	8	7
	IV	4	2	8	4	3

7. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

(a) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}$

8. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

   (a) $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \end{pmatrix}$

   (b) $\begin{pmatrix} -1 & 6 \\ 0 & 4 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$

9. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

   (a) $\begin{pmatrix} 1 & 0 & 4 \\ 0 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

   (c) $\begin{pmatrix} 2 & -4 & -4 \\ -4 & 6 & -4 \\ -4 & -4 & 16 \end{pmatrix}$

   (d) $\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{pmatrix}$

10. We are considering one of three alternatives A, B, or C under uncertain conditions. The payoff matrix is below:

Alternative	Conditions		
	#1	#2	#3
A	3,000	4,500	6,000
B	1,000	9,000	2,000
C	4,500	4,000	3,500

Determine the best plan by each of the following criteria and show work:

   (a) Laplace

   (b) Maximin

   (c) Maximax

   (d) Hurwicz (assume that $\alpha = .65$)

   (e) Regret (minimax)

   (f) We have a choice of two investment strategies: stocks and bonds. The returns for each under two possible economic conditions are as follows:

States of Nature: $p1 = 0.75$, $p2 = 0.25$

Alternative	Condition 1	Condition 2
Stocks	$10,000	-$4,000
Bonds	$7,000	$2,000

i Compute the expected value and select the best alternative.

ii What probabilities for condition 1 and condition 2 would have to exist to be indifferent toward stocks and bonds.