THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 UNIVERSITY MATHEMATICS 2025-2026 Term 1
Suggested Solutions of WeBWork Coursework 5

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1. Differentiate the following function:

$$f(t) = \frac{\sqrt[6]{t-1}}{\sqrt[6]{t}}$$

Solution: Write $f(t) = \big((t-1)/t\big)^{1/6}$. Then by chain rule,

$$\begin{aligned} f'(t) &= \frac{1}{6}\left(\frac{t-1}{t}\right)^{-5/6} \cdot \frac{d}{dt}\left(\frac{t-1}{t}\right) \\ &= \frac{1}{6}\left(\frac{t-1}{t}\right)^{-5/6} \cdot \frac{d}{dt}\left(1-\frac{1}{t}\right) \\ &= \frac{1}{6}\left(\frac{t-1}{t}\right)^{-5/6} \cdot \frac{1}{t^2} \\ &= \frac{1}{6t^2}\left(\frac{t-1}{t}\right)^{-5/6}. \end{aligned}$$

2. Calculate the derivative of the following function.

$$f(x) = \frac{e^x}{(e^x+3)(x+2)}$$

Solution: Use the quotient rule with numerator $u = e^x$ and denominator $v = (e^x+3)(x+2)$. Then

$$\begin{aligned} f'(x) = \frac{u'v - uv'}{v^2} &= \frac{e^x(e^x+3)(x+2) - e^x\big[(e^x+3)\cdot 1 + (x+2)\cdot e^x\big]}{(e^x+3)^2(x+2)^2} \\ &= \frac{e^x\big[(e^x+3)(x+2) - (e^x+3) - (x+2)e^x\big]}{(e^x+3)^2(x+2)^2} \\ &= \frac{e^x\big[(e^x+3)(x+1) - (x+2)e^x\big]}{(e^x+3)^2(x+2)^2} \\ &= \frac{e^x\big[(e^x x + 3x + e^x + 3) - (e^x x + 2e^x)\big]}{(e^x+3)^2(x+2)^2} \\ &= \frac{e^x(3x+3-e^x)}{(e^x+3)^2(x+2)^2}. \end{aligned}$$

3. Differentiate
$$g(x) = \ln\left(\frac{2-x}{2+x}\right).$$

Solution: Let $h(x) = \frac{2-x}{2+x}$. Then $h'(x) = \frac{(2+x)(-1)-(2-x)(1)}{(2+x)^2} = \frac{-4}{(2+x)^2}$. Thus by chain rule,
$$g'(x) = \frac{h'(x)}{h(x)} = \frac{-4}{(2+x)^2} \cdot \frac{2+x}{2-x} = \frac{-4}{(4-x^2)}.$$

4. Let
$$f(x) = |x| \ln(2-x).$$
Find $f'(x)$.

Solution: $d = 2$, because $2 - x > 0$. $c = 0$, because $|x| = \begin{cases} -x, & x < 0, \\ 0, & x = 0, \\ x, & x > 0. \end{cases}$

For $x < 0$:
$$f(x) = -x\ln(2-x), \qquad f'(x) = -\ln(2-x) + \frac{x}{2-x}.$$
For $x > 0$:
$$f(x) = x\ln(2-x), \qquad f'(x) = \ln(2-x) - \frac{x}{2-x}.$$
At $x = 0$ the left-hand derivative is $-\ln 2$ and the right-hand derivative is $\ln 2$, so the derivative does not exist at $x = 0$.

Thus with $c = 0$ and $d = 2$,
$$f'(x) = \begin{cases} -\ln(2-x) + \frac{x}{2-x}, & x < 0, \\ \text{DNE}, & x = 0, \\ \ln(2-x) - \frac{x}{2-x}, & 0 < x < 2. \end{cases}$$

5. Find $\frac{dy}{dx}$ if
$$5x^3y^2 - 3x^2y = 2.$$

Solution: Differentiate both sides with respect to x:
$$15x^2y^2 + 10x^3y\frac{dy}{dx} - 6xy - 3x^2\frac{dy}{dx} = 0.$$
Collect $\frac{dy}{dx}$ terms:
$$(10x^3y - 3x^2)\frac{dy}{dx} = 6xy - 15x^2y^2.$$
Therefore
$$\frac{dy}{dx} = \frac{6xy - 15x^2y^2}{10x^3y - 3x^2} = \frac{3y(2-5xy)}{x(10xy-3)}.$$

6. Let

$$f(x) = \frac{5x^2}{(4-5x)^5}.$$

Find the equation of the tangent line to the graph of f at $x = 2$.
Solution: Compute $f(2)$ and $f'(2)$. We have

$$f(x) = 5x^2(4-5x)^{-5}, \quad f'(x) = 10x(4-5x)^{-5} + 5x^2(-5)(4-5x)^{-6}(-5).$$

Simplify the derivative:

$$f'(x) = 10x(4-5x)^{-5} + 125x^2(4-5x)^{-6}.$$

Evaluate at $x = 2$:

$$\begin{aligned} f(2) &= \frac{5 \cdot 4}{(-6)^5} = \frac{20}{-7776} = -\frac{5}{1944}, \\ f'(2) &= 10 \cdot 2(-6)^{-5} + 125 \cdot 4(-6)^{-6} = 20(-6)^{-5} + 500(-6)^{-6} \\ &= 20(-6)^{-6}\big(-6\big) + 500(-6)^{-6} = 20(-6)^{-6}(-6+25) \\ &= 20 \cdot 19 \cdot (-6)^{-6} = 380(-6)^{-6} = \frac{380}{6^6}. \end{aligned}$$

Thus the tangent line at $x = 2$ is

$$y = f'(2)(x-2) + f(2) = \frac{380}{6^6}(x-2) - \frac{5}{1944}.$$

7. If the equation of motion of a particle is given by $s(t) = A\cos(wt + d)$, the particle is said to undergo simple harmonic motion . Assume $0 \leq d < \pi$.
(a) Find the velocity $v(t)$ of the particle at time t.
(b) What is the smallest positive value of t for which the velocity is 0? Assume that w and d are positive.
Solution:

   (a) Velocity: $v(t) = s'(t) = -A\omega \sin(\omega t + d)$.

   (b) Smallest positive t where $v(t) = 0$: solve $\sin(\omega t + d) = 0$, i.e. $\omega t + d = n\pi$. The smallest positive solution is for $n = 1$:

$$t = \frac{\pi - d}{\omega}.$$

8. A parabola is defined by the equation

$$x^2 - 2xy + y^2 + 6x - 10y + 25 = 0.$$

Find points with horizontal and vertical tangents.
Solution: Differentiate implicitly with respect to x:

$$2x - 2y - 2xy' + 2yy' + 6 - 10y' = 0.$$

Collect y':

$$y'(-2x + 2y - 10) + 2x - 2y + 6 = 0, \quad \Rightarrow \quad y' = \frac{2x - 2y + 6}{2x - 2y + 10}.$$

Horizontal tangents occur when numerator equals to 0, i.e. $2x - 2y + 6 = 0 \implies y = x + 3$. Substitute into the curve:

$$x^2 - 2x(x+3) + (x+3)^2 + 6x - 10(x+3) + 25 = 0.$$

Solving the above equation, we have

$$\text{horizontal tangent points } (1, 4).$$

Vertical tangents occur when denominator equals to 0, i.e. $2x - 2y + 10 = 0 \implies y = x + 5$. Substitute into the curve and solve to get

$$\text{vertical tangent points } (0, 5).$$

9. $f(x) = x^2 \arctan(9x)$. Differentiate:

$$f'(x) = 2x \arctan(9x) + x^2 \cdot \frac{9}{1 + (9x)^2} = 2x \arctan(9x) + \frac{9x^2}{1 + 81x^2}.$$

10. $f(x) = \dfrac{3x^2}{\sqrt{3x^2 + 2}}$. Find $f'(x)$ and evaluate at $x = 1$ and $x = -2$.
Solution: Write $f(x) = 3x^2(3x^2 + 2)^{-1/2}$. Then

$$\begin{aligned} f'(x) &= 6x(3x^2+2)^{-1/2} + 3x^2\left(-\tfrac{1}{2}\right)(3x^2+2)^{-3/2} \cdot 6x \\ &= \frac{6x}{\sqrt{3x^2+2}} - \frac{9x^3}{(3x^2+2)^{3/2}}. \end{aligned}$$

Evaluate:

$$\begin{aligned} f'(1) &= \frac{6}{\sqrt{5}} - \frac{9}{5\sqrt{5}} = \frac{30-9}{5\sqrt{5}} = \frac{21}{5\sqrt{5}}, \\ f'(-2) &= \frac{6(-2)}{\sqrt{3 \cdot 4 + 2}} - \frac{9(-8)}{(14)^{3/2}} = -\frac{12}{\sqrt{14}} + \frac{72}{14\sqrt{14}} = \frac{-168+72}{14\sqrt{14}} = \frac{-96}{14\sqrt{14}} = -\frac{48}{7\sqrt{14}}. \end{aligned}$$

11. The equation of the tangent line to the graph of $y = x\cos(3x)$ at $x = \pi$ is given by $y = mx + b$. Find m, b.
Solution: By product rule, $y' = \cos(3x) - 3x\sin(3x)$. At $x = \pi$ we have $\cos(3\pi) = -1$, $\sin(3\pi) = 0$. Hence $m = -1$. The point on the curve is $(\pi, \pi\cos(3\pi)) = (\pi, -\pi)$, so the tangent line is $y = -x$ and thus $b = 0$.