

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 UNIVERSITY MATHEMATICS 2025-2026 Term 1
Suggested Solutions of WeBWork Coursework 3

If you find any errors or typos, please email us at
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1.

$$\lim_{x \rightarrow 3} \left(\sqrt{x^2 + 6} - \frac{x^2 + 3x}{x} \right) = \lim_{x \rightarrow 3} (\sqrt{x^2 + 6} - (x + 3)).$$

Substituting $x = 3$ gives

$$\sqrt{15} - 6 \approx -2.1270166538.$$

Answer: -2.1270166538 .

2.

$$f(x) = \begin{cases} \sqrt{-4-x} + 3, & x < -5, \\ 3, & x = -5, \\ 2x + 14, & x > -5. \end{cases}$$

- Left-hand limit: $\lim_{x \rightarrow -5^-} f(x) = 4$.
- Right-hand limit: $\lim_{x \rightarrow -5^+} f(x) = 4$.
- Two-sided limit: 4.

Answer: Left limit 4; Right limit 4; Two-sided limit 4.

3.

$$g(x) = \begin{cases} 3x + 4, & x < 8, \\ 24, & x = 8, \\ 3x - 4, & x > 8. \end{cases}$$

- (a) $\lim_{x \rightarrow 8^-} g(x) = 28$.
- (b) $\lim_{x \rightarrow 8^+} g(x) = 20$.
- (c) Limit does not exist (DNE).
- (d) $g(8) = 24$.

Answer: (a) 28; (b) 20; (c) DNE; (d) 24.

4.

$$\lim_{x \rightarrow 7^-} \left(\frac{1}{x-7} - \frac{1}{|x-7|} \right).$$

As $x \rightarrow 7^-$, we have $x - 7 < 0$ and thus $|x - 7| = -(x - 7)$. Then

$$\frac{1}{x-7} - \frac{1}{|x-7|} = \frac{1}{x-7} - \frac{1}{-(x-7)} = \frac{2}{x-7} \rightarrow -\text{INF}.$$

Answer: -INF.

5.

$$\left| \sin x \cos\left(\frac{1}{x^2}\right) \right| \leq |\sin x| \rightarrow 0, \quad x \rightarrow 0.$$

By the squeeze theorem,

$$\lim_{x \rightarrow 0} \sin x \cos\left(\frac{1}{x^2}\right) = 0.$$

Answer: 0.

6.

$$f(x) = \frac{x^2 + 4}{x^2 - 9}.$$

Vertical asymptotes: $x = \pm 3$. Horizontal asymptote: $y = 1$.

- $\lim_{x \rightarrow -3^-} f(x) = \infty$.
- $\lim_{x \rightarrow -3^+} f(x) = -\infty$.

- $\lim_{x \rightarrow 3^-} f(x) = -\infty$.
- $\lim_{x \rightarrow 3^+} f(x) = \infty$.
- $\lim_{x \rightarrow -\infty} f(x) = 1$.
- $\lim_{x \rightarrow \infty} f(x) = 1$.

Answer:

$$\lim_{x \rightarrow -3^-} f = \text{INF}, \quad \lim_{x \rightarrow -3^+} f = -\text{INF},$$

$$\lim_{x \rightarrow 3^-} f = -\text{INF}, \quad \lim_{x \rightarrow 3^+} f = \text{INF}, \quad \lim_{x \rightarrow -\infty} f = 1, \quad \lim_{x \rightarrow \infty} f = 1.$$

7.

- (a) If $\lim_{x \rightarrow 1^-} f(x) = 5$ then $\lim_{x \rightarrow 1} f(x) = 5$.

False.

- (b)

$$\text{If } \lim_{x \rightarrow 1^-} f(x) = 5 \text{ then } \lim_{x \rightarrow 1^+} f(x) = 5.$$

False.

- (c)

$$\text{If } \lim_{x \rightarrow 1} f(x) = 5 \text{ then } \lim_{x \rightarrow 1^-} f(x) = 5.$$

True.

- (d)

$$\text{If } \lim_{x \rightarrow 1} f(x) = 5 \text{ then } \lim_{x \rightarrow 1^+} f(x) = 5.$$

True.

- (e)

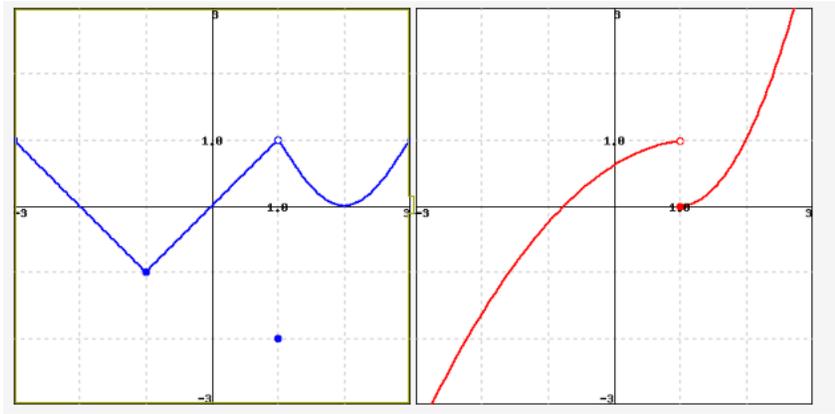
A. If the left-hand limit exists, then the two-sided limit exists.

B. If the two-sided limit exists, then the left- and right-hand limits both exist and are equal.

C. If the left- and right-hand limits both exist and are equal, then the two-sided limit exists.

D. If the right-hand limit exists, then the two-sided limit exists.

Correct choices: B, C.



8.

Use the given graphs of the function f (left, in blue) and g (right, in red) to find the following limits

- 1) $\lim_{x \rightarrow 1} [f(x) + g(x)] = \text{DNE}$
- 2) $\lim_{x \rightarrow 2} [f(x) + g(x)] = 0 + 1 = 1$
- 3) $\lim_{x \rightarrow 0} f(x)g(x) = 0$
- 4) $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 0$
- 5) $\lim_{x \rightarrow -1} \sqrt{3 + f(x)} = \sqrt{3 - 1} = \sqrt{2}$

9.

Consider

$$\lim_{x \rightarrow 0^+} \sqrt{x} \sin\left(\frac{\pi}{x}\right).$$

Since

$$|\sqrt{x} \sin(\pi/x)| \leq \sqrt{x} \rightarrow 0,$$

the limit is 0.

Answer: 0.

10.

$$\lim_{x \rightarrow 0} \frac{x^2}{\sin^2(7x)} = \lim_{x \rightarrow 0} \frac{1}{49} \cdot \left(\frac{7x}{\sin(7x)}\right)^2 = \frac{1}{49}.$$

Answer: 1/49.

11.

$$\lim_{x \rightarrow 0} \frac{\tan(5x)}{\tan(4x)} = \lim_{x \rightarrow 0} \frac{\sin(5x) \cos(4x)}{\sin(4x) \cos(5x)} = \frac{5}{4} \lim_{x \rightarrow 0} \frac{\sin(5x)}{5x} \lim_{x \rightarrow 0} \frac{4x}{\sin(4x)} \lim_{x \rightarrow 0} \frac{\cos(4x)}{\cos(5x)} = \frac{5}{4}.$$

Answer: $\frac{5}{4}$.