THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010 UNIVERSITY MATHEMATICS 2025-2026 Term 1
Suggested Solutions of WeBWork Coursework 2

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Q1. Find the domain of the function

$$f(x) = \frac{\sqrt{7-2x}}{x^2-25}.$$

Solution. Variable x should satisfy

$$\begin{cases} 7-2x \geq 0 \\ x^2 \neq 25 \end{cases} \implies \begin{cases} x \leq 3.5 \\ x \neq \pm 5 \end{cases}$$

Therefore, domain of f is $(-\infty, -5) \cup (-5, 3.5]$.

Q2. The domain of the function $g(x) = \log_a(x^2-16)$ is $(-\infty,$ ____) and (____, ∞).

Solution. Variable x should satisfy

$$x^2 - 16 > 0 \implies x < -4 \text{ or } x > 4,$$

Therefore, domain of f is $(-\infty, -4) \cup (4, \infty)$.

Q3. Given that $f(x) = \dfrac{1}{x}$ and $g(x) = 2x+4$, calculate
(a) $(f \circ g)(x)$= ______, its domain is all real numbers except ______ .
(b) $(g \circ f)(x)$= ______, its domain is all real numbers except ______ .
(c) $(f \circ f)(x)$= ______, its domain is all real numbers except ______ .
(d) $(g \circ g)(x)$= ______, its domain is (______, ______) .

Solution.

(a)

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= \frac{1}{2x+4}. \end{aligned}$$

Variable x should satisfy $2x + 4 \neq 0$ which is $x \neq -2$. Therefore, its domain is all real numbers except -2.

(b)

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= \frac{2}{x} + 4.\end{aligned}$$

Variable x should satisfy $x \neq 0$. Therefore, its domain is all real numbers except 0.

(c)

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= \frac{1}{\frac{1}{x}} \\ &= x.\end{aligned}$$

Variable x should satisfy $x \neq 0$. Therefore, its domain is all real numbers except 0.

(d)

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= 2(2x+4)+4 \\ &= 4x+12.\end{aligned}$$

Its domain is $\mathbb{R} = (-\infty, \infty)$.

Q4. Given the functions, $f(x) = \dfrac{x-6}{x-3}$ and $g(x) = \sqrt{x+4}$, find the following domains. Use interval notation.

Domain of f	________
Domain of g	________
Domain of $f+g$	________
Domain of $\dfrac{f}{g}$	________
Domain of $\dfrac{g}{f}$	________
Domain of $f(g(x))$	________
Domain of $g(f(x))$	________

Solution.

f: Variable x should satisfy $x - 3 \neq 0$, then its domain $\{x : x \neq 3\} = (-\infty, 3) \cup (3, \infty)$.

g: Variable x should satisfy $x + 4 \geq 0$, then its domain $\{x : x \geq -4\} = [-4, \infty)$.

$f+g$: Its domain is $[-4, 3) \cup (3, \infty)$.

$\dfrac{f}{g} = \dfrac{\frac{x-6}{x-3}}{\sqrt{x+4}}$: Its domain is $(-4, 3) \cup (3, \infty)$.

$\dfrac{g}{f} = \dfrac{\sqrt{x+4}}{\frac{x-6}{x-3}}$: Its domain is $[-4, 6) \cup (6, \infty)$.

$f(g(x)) = \dfrac{\sqrt{x+4}-6}{\sqrt{x+4}-3}$: Variable x should satisfy $x+4 \geq 0$ and $\sqrt{x+4}-3 \neq 0$. Its domain is $[-4,5) \cup (5,\infty)$.

$g(f(x)) = \sqrt{\dfrac{x-6}{x-3}+4}$: Variable x should satisfy $x \neq 3$ and $\dfrac{x-6}{x-3}+4 \geq 0$, which implies $x < 3$ or $x \geq \dfrac{18}{5}$. Its domain is $(-\infty, 3) \cup \left[\dfrac{18}{5}, \infty\right)$.

Q5. Suppose $f(x) = 6x - 7$ and $g(y) = \dfrac{y}{6} + \dfrac{7}{6}$.

(a) Find the composition $g(f(x))$.

(b) Find the composition $f(g(y))$.

(c) Are the functions f and g inverse to each other?

Solution.

(a) The composition

$$g(f(x)) = \frac{6x-7}{6} + \frac{7}{6} = x.$$

(b) The composition

$$f(g(y)) = 6\left(\frac{y}{6} + \frac{7}{6}\right) - 7 = y.$$

(c) Yes. Since $g(f(x)) = x$ and $f(g(y)) = y$.

Q6. Find the inverse function to $y = f(x) = \dfrac{4-2x}{7-6x}$.

Solution. Express x in terms of y:

$$\begin{aligned} y &= \frac{4-2x}{7-6x} \\ 7y - 6xy &= 4 - 2x \\ (2-6y)x &= 4 - 7y \\ x &= \frac{4-7y}{2-6y} \end{aligned}$$

Therefore, the inverse function of f is:

$$x = g(y) = \frac{4-7y}{2-6y}.$$

Q7. Part 1. Evaluate the following limit by simplifying the expression and then evaluating the limit.

$$\lim_{x \to 4} \frac{x^2 + x - 20}{x - 4}.$$

Part 2.

$$\lim_{x\to 2}\frac{x^4+x^2-20}{x-2}$$

Solution.

Part 1. The limit

$$\begin{aligned}\lim_{x\to 4}\frac{x^2+x-20}{x-4} &= \lim_{x\to 4}\frac{(x-4)(x+5)}{x-4}\\ &= \lim_{x\to 4} x+5\\ &= 9.\end{aligned}$$

Part 2. The limit

$$\begin{aligned}\lim_{x\to 2}\frac{x^4+x^2-20}{x-2} &= \lim_{x\to 2}\frac{(x^2+5)(x^2-4)}{x-2}\\ &= \lim_{x\to 2}\frac{(x^2+5)(x+2)(x-2)}{x-2}\\ &= \lim_{x\to 2}(x^2+5)(x+2)\\ &= 36.\end{aligned}$$

Q8. Evaluate the limit

$$\lim_{b\to 8}\frac{\frac{1}{b}-\frac{1}{8}}{b-8}.$$

Solution. The limit

$$\begin{aligned}\lim_{b\to 8}\frac{\frac{1}{b}-\frac{1}{8}}{b-8} &= \lim_{b\to 8}\frac{\frac{8-b}{8b}}{b-8}\\ &= \lim_{b\to 8}\frac{-1}{8b}\\ &= -\frac{1}{64}.\end{aligned}$$

Q9. Let a be a positive real number. Evaluate the limit

$$\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{5(x-a)}.$$

Solution. The limit

$$\begin{aligned}\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{5(x-a)} &= \lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{5(\sqrt{x}+\sqrt{a})(\sqrt{x}-\sqrt{a})}\\ &= \lim_{x\to a}\frac{1}{5(\sqrt{x}+\sqrt{a})}\\ &= \frac{1}{10\sqrt{a}}.\end{aligned}$$

Q10. Determine whether the sequences are increasing, decreasing, or not monotonic.

(a) $a_n = \dfrac{1}{4n+6}$.

(b) $a_n = \dfrac{n-4}{n+4}$.

(c) $a_n = \dfrac{\sqrt{n+4}}{6n+4}$.

(d) $a_n = \dfrac{\cos n}{4^n}$.

Solution.

(a) For all $n \in \mathbb{N}$,

$$\begin{aligned}a_{n+1}-a_n &= \frac{1}{4n+10}-\frac{1}{4n+6}\\ &= \frac{-4}{(4n+10)(4n+6)} < 0.\end{aligned}$$

Therefore, a_n is decreasing.

(b) For all $n \in \mathbb{N}$,

$$\begin{aligned}a_{n+1}-a_n &= \frac{n-3}{n+5}-\frac{n-4}{n+4}\\ &= \frac{8}{(n+4)(n+5)} > 0.\end{aligned}$$

Therefore, a_n is increasing.

(c) For all $n \in \mathbb{N}$,

$$\begin{aligned}a_{n+1}-a_n &= \frac{\sqrt{n+5}}{6n+10}-\frac{\sqrt{n+4}}{6n+4}\\ &= \frac{\sqrt{(6n+4)^2(n+5)}-\sqrt{(6n+10)^2(n+4)}}{(6n+10)(6n+4)}\\ &= \frac{\sqrt{36n^3+228n^2+256n+80}-\sqrt{36n^3+264n^2+580n+400}}{(6n+10)(6n+4)}\\ &< 0.\end{aligned}$$

Therefore, a_n is decreasing.

(d) a_n is not monotonic, since

$$a_1 = \frac{\cos 1}{4} > 0,$$
$$a_2 = \frac{\cos 2}{4^2} < 0,$$
$$a_5 = \frac{\cos 5}{4^5} > 0.$$

Q11. Determine whether the sequence

$$a_n = \frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{n}{n^2}$$

converges or diverges. If it converges, find the limit. Note that

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}.$$

Solution. The sequence

$$\begin{aligned} a_n &= \frac{1}{n^2}\frac{n(n+1)}{2} \\ &= \frac{n+1}{2n} \\ &= \frac{1}{2} + \frac{1}{2n}. \end{aligned}$$

The sequence a_n is decreasing since $a_{n+1} - a_n = \frac{1}{2n+2} - \frac{1}{2n} < 0$. It is bounded by $\left[\frac{1}{2}, 1\right]$. By *Monotone Convergence Theorem*, a_n convergences. The limit

$$\begin{aligned} \lim_{n\to\infty} a_n &= \lim_{n\to\infty} \frac{n+1}{2n} \\ &= \frac{1}{2} + \lim_{n\to\infty} \frac{1}{2n} \\ &= \frac{1}{2}. \end{aligned}$$