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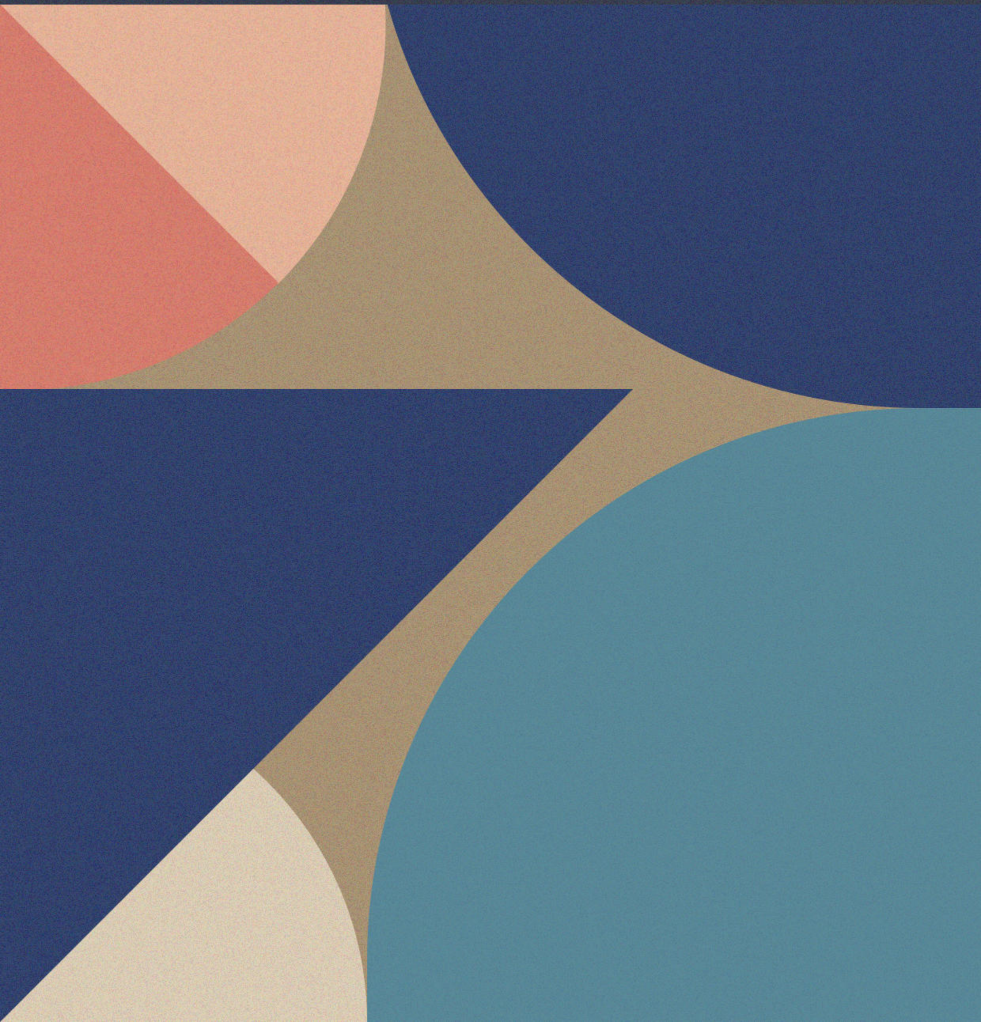


Math1010F

Nov. 13. 2025

Ex1.1# Let $f, g: [-\pi, \pi] \to \mathbb{R}$ be the functions defined by

$$f(x) = 2\cos(2x) - 4\sin x + 1,$$

and $g(x) = |f(x)|$, for any $x \in [-\pi, \pi]$.

1a) Consider the function $f(x)$.

A. Find the x-intercepts and y-intercepts of the curve $y = f(x)$.

B. Compute $f'(x)$ and $f''(x)$.

C. Determine the critical points of $f(x)$ in $(-\pi, \pi)$.

D. Determine the relative extrema of $f(x)$ in $(-\pi, \pi)$.

E. Does $f(x)$ attain any absolute extrema in $[-\pi, \pi]$? If yes, where?

F. Sketch the curve $y = f(x)$ for $x \in [-\pi, \pi]$.

Proof. 1a) A. Let $f(x) = 2\cos(2x) - 4\sin x + 1$. Then, $f(x)$ can be rewritten as $f(x) = -4\sin^2 x - 4\sin x + 3 = -(2\sin x + 1)^2 + 4$. Set $f(x) = 0$, then $2\sin x + 1 = \pm 2$, that is, $x = \frac{\pi}{6}$ and $x = \frac{5}{6}\pi$. $(x \in [-\pi, \pi])$

Besides, letting $x = 0$, we have $f(0) = 3$. Hence, x-intercepts: $(\frac{\pi}{6}, 0)$ and $(\frac{5}{6}\pi, 0)$; and y-intercept: $(0, 3)$.

B. We have $f'(x) = -4\sin(2x) - 4\cos x$; $f''(x) = -8\cos(2x) + 4\sin x$.

C. Noting that $f'(x) = -8\sin x\cos x - 4\cos x = -4\cos x(2\sin x + 1)$ for $x \in [-\pi, \pi]$. Set $f'(x) = 0$, that is, $\cos x = 0$ or $2\sin x + 1 = 0$. Hence, the critical points of f is $x = -\frac{\pi}{2}, \frac{\pi}{2}, -\frac{\pi}{6}, -\frac{5}{6}\pi$.

D. We use First derivative test to find the relative extrema. We consider the following intervals. which are divided by critical point.

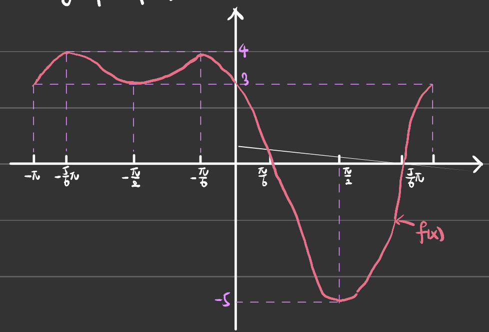
• Intervals	$(-\pi, -\frac{5}{6}\pi)$	$(-\frac{5}{6}\pi, -\frac{\pi}{2})$	$(-\frac{\pi}{2}, -\frac{\pi}{6})$	$(-\frac{\pi}{6}, \frac{\pi}{2})$	$(\frac{\pi}{2}, \pi)$
• f' sign	+	−	+	−	+
• Monotonicity	$f\uparrow$	$f\downarrow$	$f\uparrow$	$f\downarrow$	$f\uparrow$

Hence, f attains relative maximum at $x=-\frac{5}{6}\pi$ and $x=-\frac{\pi}{6}$, with $f(-\frac{5}{6}\pi)=4$ and $f(-\frac{\pi}{6})=4$; f attains relative minimum at $x=-\frac{\pi}{2}$ and $x=\frac{\pi}{2}$, with $f(-\frac{\pi}{2})=3$ and $f(\frac{\pi}{2})=-5$.

E. Since f is continuous function on a closed and bounded interval. then f attains respectively the absolute maximum and absolute minimum. at an endpoint of its domain, or at a critical point in the interior of its domain.

Noting that $f(\pi)=3$ and $f(-\pi)=3$. Hence, f attains the absolute maximum at $x=-\frac{5}{6}\pi$ and $x=-\frac{\pi}{6}$, with $\max f(x)=4$. f attains the absolute minimum at $x=\frac{\pi}{2}$ with $\min f(x)=-5$.

F. This is the graph of f:



□

Ex 1.1# Let $f, g : [-\pi, \pi] \to \mathbb{R}$ be the functions defined by

$$f(x) = 2\cos(2x) - 4\sin x + 1,$$

and $g(x) = |f(x)|$, for any $x \in [-\pi, \pi]$.

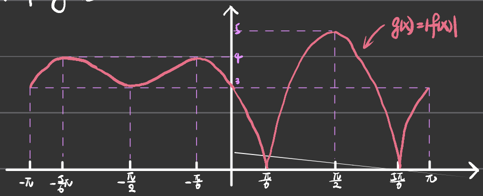
(b) Consider the function $g(x)$.

A. Sketch the curve $y = g(x)$ for $x \in [-\pi, \pi]$.

B. Determine the relative extrema of $g(x)$ in $(-\pi, \pi)$.

C. Does $g(x)$ attain any absolute extrema in $[-\pi, \pi]$? If yes, where?

Proof. (b) A. The graph of g is



B. By the statement of part (a) D and the graph of g, we can conclude that f attains relative maximum at $x = -\frac{5}{6}\pi, -\frac{\pi}{6}$ and $\frac{\pi}{2}$, with $f(-\frac{5}{6}\pi) = 4$, $f(-\frac{\pi}{6}) = 4$ and $f(\frac{\pi}{2}) = 5$; f attains relative minimum at $x = -\frac{\pi}{2}, \frac{\pi}{6}$ and $\frac{5}{6}\pi$, with $f(-\frac{\pi}{2}) = 3$, $f(\frac{\pi}{6}) = 0$ and $f(\frac{5}{6}\pi) = 0$.

C. By the discussion in part (a) E, we can conclude that f attains the absolute maximum at $x = \frac{\pi}{2}$ with $\max f(x) = 4$; f attains the absolute minimum at $x = \frac{\pi}{6}$ and $x = \frac{5}{6}\pi$, with $\min f(x) = 0$. $\square$

Ex1.2# Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \sqrt[3]{x^3 - x^2 - x + 1},$$

for any $x \in \mathbb{R}$.

1a) Find the x-intercepts and y-intercepts of the curve $y = f(x)$.

1b) What are the asymptotes of the curve $y = f(x)$.

1c) Is $f(x)$ continuous on $\mathbb{R}$?

Proof. 1a) Noting that $x^3 - x^2 - x + 1 = x^2(x-1) - (x-1) = (x-1)^2(x+1)$. Setting $f(x) = 0$, we have $x = \pm 1$. Besides, noting that $f(0) = 1$. Hence, x-intercepts: $(1, 0)$ and $(-1, 0)$; y-intercept: $(0, 1)$.

1b) It is clear that $f(x)$ has no vertical asymptote. Since $\lim_{x\to+\infty} f(x) = +\infty$ and $\lim_{x\to-\infty} f(x) = -\infty$, then $f(x)$ has no horizontal asymptote.

Next, we assume $f(x)$ has oblique asymptote $y = ax + b$. Then

$$a = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{1}{x}\left[\sqrt[3]{x^3 - x^2 - x + 1}\right]$$
$$= \lim_{x\to+\infty} \sqrt[3]{1 - \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}} = 1;$$

$$b = \lim_{x\to+\infty} [f(x) - ax] = \lim_{x\to+\infty} \left[\sqrt[3]{x^3 - x^2 - x + 1} - x\right]$$
$$= \lim_{x\to+\infty} \left\{ x\left[1 - \left(\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}\right)\right]^{\frac{1}{3}} - x \right\} \quad \text{Taylor's}$$
$$= \lim_{x\to+\infty} \left\{ x\left[-\frac{1}{3}\left(\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^3}\right) + o\left(\frac{1}{x^2}\right)\right] - x \right\} = -\frac{1}{3}.$$

If we let $x \to -\infty$, we can get the same result. Hence, $f(x)$ only has one asymptotes, i.e., $y = x - \frac{1}{3}$.

1c) Let $g(u) = \sqrt[3]{u}$ and $h(x) = x^3 - x^2 - x + 1$. Since g and h are continuous on $\mathbb{R}$, then $f(x) = g[h(x)]$ is also continuous.

Ex1.2# Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \sqrt[3]{x^3 - x^2 - x + 1},$$

for any $x \in \mathbb{R}$.

(d) Show that $f'(x) = \frac{3x^2-2x-1}{3[f(x)]^2}$, for all $x \in \mathbb{R} \setminus \{-1, 1\}$. At which points is $f(x)$ not differentiable?

(e) Determine the relative extrema of $f(x)$?

(f) Does $f(x)$ attain any absolute extrema? If yes, where?

Proof. (d) Noting that $[f(x)]^3 = x^3 - x^2 - x + 1$ $(*)$, taking the derivative of $(*)$ and we get $3[f(x)]^2 f'(x) = 3x^2 - 2x - 1$, that is, $f'(x) = \frac{(x-1)(3x+1)}{[f(x)]^2}$, for $x \in \mathbb{R} \setminus \{-1, 1\}$. Hence, f is not differentiable at $x = \pm 1$. (when $f(x) = 0$)

(e) By $f'(x)$, we know that the critical points of $f(x)$ is $x = \pm 1, -\frac{1}{3}$. Then, we consider the following intervals, which are divided by critical points. We have table.

Intervals	$(-\infty, -1)$	$(-1, -\frac{1}{3})$	$(-\frac{1}{3}, 1)$	$(1, +\infty)$
$f'(x)$ sign	$f' > 0$	$f' > 0$	$f' < 0$	$f' > 0$
Monotonicity	$f \uparrow$	$f \uparrow$	$f \downarrow$	$f \uparrow$
Extrema		local max	local min	

Hence, by the First Derivative Test, f attains a relative maximum at $x = -\frac{1}{3}$ with $f(-\frac{1}{3}) = \frac{\sqrt[3]{32}}{3}$. f attains a relative minimum at $x = 1$ with $f(1) = 0$.

(f) Since $\lim_{x \to +\infty} f(x) = +\infty$ and $\lim_{x \to -\infty} f(x) = -\infty$, so f does not attain any absolute extrema.

Ex1.2# Let $f: \mathbb{R} \to \mathbb{R}$ be the function defined by

$$f(x) = \sqrt[3]{x^3 - x^2 - x + 1},$$

for any $x \in \mathbb{R}$.

(g) Show that $f''(x) = \dfrac{-8(x-1)^2}{9[f(x)]^5}$ for all $x \in \mathbb{R} \setminus \{-1, 1\}$.

(h) Does $f(x)$ have any point of inflection? If yes, where?

(i) Sketch the curve $y = f(x)$ for $x \in [-2, 2]$.

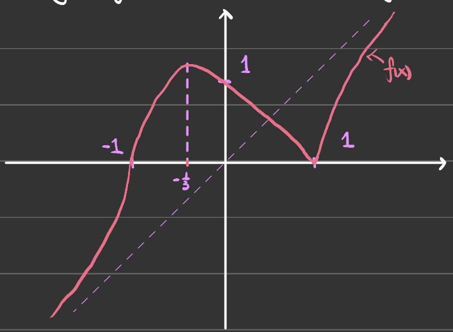
Proof. (g) By (d), we have $3[f(x)]^2 f'(x) = 3x^2 - 2x - 1$. Taking derivative again, it holds that $3[f(x)]^2 f''(x) + 6f(x)[f'(x)]^2 = 6x - 2$, that is, $f''(x) = \dfrac{-8(x-1)^2}{9[f(x)]^5}$.

(h) Next, we consider the following intervals, which are divided by critical point of $f'(x)$ and $f(x)$. We have table.

◦ Intervals	$(-\infty, -1)$	$(-1, -\frac{1}{3})$	$(-\frac{1}{3}, 1)$	$(1, +\infty)$
◦ Monotonicity	$f \uparrow$	$f \uparrow$	$f \downarrow$	$f \uparrow$
◦ $f''(x)$ sign	$f'' > 0$	$f'' < 0$	$f'' < 0$	$f'' < 0$
◦ Concavity	up	down	down	down

Hence, f'' changes sign at $x = -1$, that is, inflection point: $(1, f(1))$.

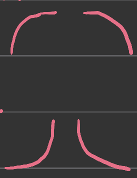
(i)



Remark:

down.

up.



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Ex 1.3# 1(a) Show that for any $\alpha>0$, $\alpha+\alpha^{-1}\geqslant 2$.

1(b) Hence, or otherwise, show that $\cosh x\geqslant 1$ for any $x\in\mathbb{R}$.

1(c) Apply the Mean Value Theorem to show that

$$|\sinh a-\sinh b|\geqslant |a-b|.$$

whenever $a\neq b$.

Proof. 1(a) Let $\alpha>0$. We have $\alpha+\alpha^{-1}-2=\alpha(\alpha^2-2\alpha+1)=\alpha(\alpha-1)^2\geqslant 0$. Then $\alpha+\alpha^{-1}\geqslant 2$.

1(b) By 1(a), letting $\alpha=e^t$, then $\cosh x=\frac{e^x+e^{-x}}{2}\geqslant\frac{2}{2}=1$.

1(c) Analysis: Lebesgue's Mean Value Theorem: $f(b)-f(a)=f'(\xi)\cdot(b-a)$.

Hence, it is nature to set $f(x)=\sinh x$.

Let $f(x)=\sinh x$. Then $f(x)$ is continuous on $[a,b]$; $f(x)$ is differentiable on (a,b); and $f'(x)=\cosh x$. By the Mean Value Theorem, there exists some $\xi\in(a,b)$ such that

$$(\sinh a-\sinh b)=\cosh\xi\cdot(a-b).$$

Then, noting that $\cosh\xi\geqslant 1$, it holds that

$$|\sinh a-\sinh b|\geqslant|\cosh|\cdot|a-b|\geqslant|a-b|. \qquad \square$$

Ex 1.3# 1d) Apply the Mean Value Theorem to show that

$$|\sinh(a-b)| \leqslant |a-b| \cosh a \cosh b. \qquad (*)$$

whenever $a \neq b$.

Proof. 1d) Analysis: Lebesgue's Mean Value Theorem: $f(b)-f(a) = f'(\xi)\cdot(b-a)$.

In order to create $f(x)$, we need to rewrite $(*)$ first. Noting that $\sinh(a-b) = \sinh a \cosh b - \cosh a \sinh b$, then $(*)$ can be rewritten as

$$|\sinh a \cosh b - \cosh a \sinh b| \leqslant |a-b| \cosh a \cosh b. \qquad (**)$$

By part 1b), we know that $\cosh a \cosh b \geqslant 1$. then letting $(**) \times [\cosh a \cosh a]^{-1}$, we get

$$\left|\frac{\sinh a}{\cosh a} - \frac{\sinh b}{\cosh b}\right| \leqslant |a-b|. \qquad (***)$$

It is sufficient to prove that $(***)$ is correct.

Let $f(x) = \tanh x$. Then $f(x)$ is continuous on $[a,b]$; $f(x)$ is differentiable on (a,b); and $f'(x) = [\cosh x]^{-2}$. By the Mean Value Theorem, there exists some $\xi \in (a,b)$ such that

$$(\tanh a - \tanh b) = [\cosh \xi]^{-2} \cdot (a-b).$$

Then, noting that $\cosh \xi \geqslant 1$, it holds that

$$|\tanh a - \tanh b| \leqslant |a-b|.$$

which implies $(***)$ is correct. Hence,

$$|\sinh(a-b)| \leqslant |a-b| \cosh a \cosh b. \qquad \square$$

Ex 1.4# Let $a, b \in \mathbb{R}$ with $1 < a < b$.

1(a) Show that there exists some $\xi \in (a, b)$ such that

$$\log(\log b) - \log(\log a) = \frac{b-a}{\xi \log \xi}.$$

1(c) Hence, or otherwise, show that

$$\frac{1}{b \log b} < \frac{\log(\log b) - \log(\log a)}{b-a} < \frac{1}{a \log a}.$$

Proof. 1(a) Let $f(x) = \log(\log x)$. Then $f(x)$ is continuous on $[a, b]$; $f(x)$ is differentiable on (a, b); and $f'(x) = \frac{1}{x \log x}$. By the Mean Value Theorem, there exists some $\xi \in (a, b)$ such that

$$\log(\log b) - \log(\log a) = \frac{b-a}{\xi \log \xi}.$$

1(b) By part 1(a), we have

$$\frac{\log(\log b) - \log(\log a)}{b-a} = \frac{1}{\xi \log \xi}. \qquad (*)$$

Let $g(x) = x \log x$, for $x \in (1, +\infty)$. Since $g'(x) = \log x + 1 > 0$, $g(x)$ is strictly increasing. Noting that $1 > a > \xi > b$, then we have

$$a \log a < \xi \log \xi < b \log b,$$

that is,

$$\frac{1}{b \log b} < \frac{1}{\xi \log \xi} < \frac{1}{a \log a}.$$

This, combining with $(*)$, we can conclude that

$$\frac{1}{b \log b} < \frac{\log(\log b) - \log(\log a)}{b-a} < \frac{1}{a \log a}. \qquad \square$$

Ex 1.5# Let $0<\alpha<\beta$.

(a) Let $g: \mathbb{R}_+ \to \mathbb{R}$ be the function defined by $g(x) = x^{-\alpha} + \alpha \log x - 1$ for any $x \in \mathbb{R}_+$.

A. Compute the derivative of g.

B. Show that $g(x) > 0$ for any $x \in (0,1)$.

(b) Apply (a), or otherwise, to show that the limit

$$\lim_{t \to 0^+} t^\beta \log t.$$

exists. What is the value of this limit?

Proof. (a) A. For any $x \in \mathbb{R}_+$, $g'(x) = -\alpha x^{-\alpha-1} + \alpha x^{-1} = \alpha x^{-1}(1 - x^{-\alpha})$.

B. When $x \in (0,1)$, since $\alpha > 0$, we have $0 < x^\alpha < 1$, that is, $x^{-\alpha} > 1$.

Hence, $g'(x) = \alpha x^{-1}(1 - x^{-\alpha}) < 0$. It follows that g is strictly decreasing on $(0,1]$. Hence, for any $x \in (0,1)$, we have

$$g(x) > g(1) = 1^\alpha + \alpha \log(1) - 1 = 0.$$

(b) By part (a), we have $t^{-\alpha} + \alpha \log t - 1 > 0$ for $t \in (0,1)$ (*). In order to create $t^\beta \log t$, we let $\alpha^{-1}\beta^t \times$ (*), then it holds that $\alpha^{-1} t^{\beta-\alpha} + t^\beta \log t - \alpha^{-1} t^\beta > 0$. That is to say, for $t \in (0,1)$,

$$t^\beta \log t > \alpha^{-1} t^\beta - \alpha^{-1} t^{\beta - \alpha}.$$

Noting that $0 < \alpha < \beta$, so

$$\lim_{t \to 0^+} t^\beta \log t \geq \lim_{t \to 0^+} \alpha^{-1} t^\beta - \lim_{t \to 0^+} \alpha^{-1} t^{\beta-\alpha} = 0.$$

Besides, since $t^\beta > 0$ and $\log t < 0$ when $t \in (0,1)$, $\lim_{t \to 0^+} t^\beta \log t \leq 0$.

Hence, $\lim_{t \to 0^+} t^\beta \log t = 0$. □

Ex 1.6# Let $a>b>0$ and define

$$f(x)=\begin{cases}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}, & \text{for } x>0;\\ \sqrt{ab}, & \text{for } x=0.\end{cases}$$

1a) Show that f is right continuous at $x=0$.

1b) Show that $\lim_{x\to+\infty} f(x)=a$.

Proof. 1a) If f is right continuous at $x=0$. then $\lim_{x\to0^+} f(x)=f(0)$. By L'Hospital law.

$$\begin{aligned}\lim_{x\to0^+} f(x) &= \lim_{x\to0^+}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}\\ &= \exp\left[\lim_{x\to0^+}\frac{1}{x}\log\left(\frac{a^x+b^x}{2}\right)\right] \qquad \frac{0}{0}\\ &= \exp\left[\lim_{x\to0^+}\frac{2}{a^x+b^x}\cdot\frac{a^x\log a+b^x\log b}{2}\right]\\ &= \exp\left(\frac{\log a+\log b}{2}\right)=\sqrt{ab}=f(0).\end{aligned}$$

Hence, f is right continuous at $x=0$.

1b) Since $a>b>0$, then $\lim_{x\to+\infty}\left(\frac{b}{a}\right)^x=0$. By L'Hospital rule.

$$\begin{aligned}\lim_{x\to+\infty}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}} &= \exp\left[\lim_{x\to+\infty}\frac{1}{x}\log\left(\frac{a^x+b^x}{2}\right)\right] \qquad \frac{\infty}{\infty}\\ &= \exp\left[\lim_{x\to+\infty}\frac{2}{a^x+b^x}\cdot\frac{a^x\log a+b^x\log b}{2}\right]\\ &= \exp\left[\lim_{x\to+\infty}\frac{\log a+\left(\frac{b}{a}\right)^x\log b}{1+\left(\frac{b}{a}\right)^x}\right]\\ &= \exp(\log a)=a.\end{aligned}$$

$\square$

Ex 1.6# Let $a>b>0$ and define

$$f(x)=\begin{cases}\left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}, & \text{for } x>0;\\ \sqrt{ab}, & \text{for } x=0.\end{cases}$$

1c) Let $h(t)=(1+t)\log(1+t)+(1-t)\log(1-t)$ for $0\leq t<1$. and $g(x)=\log f(x)$ for $x\in\mathbb{R}_+$.

A. Show that $h(t)>h(0)$ for $0<t<1$.

B. For $x>0$. letting $t=\frac{a^x-b^x}{a^x+b^x}$. Show that $0<t<1$ and

$$h(t)=2\left[\frac{a^x\log a^x+b^x\log b^x}{a^x+b^x}+\log\left(\frac{2}{a^x+b^x}\right)\right].$$

C. Show that for $x>0$.

$$x^2g'(x)=\frac{a^x\log a^x+b^x\log b^x}{a^x+b^x}+\log\left(\frac{2}{a^x+b^x}\right)$$

Hence. deduce that $f(x)$ is strictly increasing on $[0,+\infty)$.

Proof. 1c) A. Let $h(t)=(1+t)\log(1+t)+(1-t)\log(1-t)$ for $0\leq t<1$. We have $h'(t)=\log\left(\frac{1+t}{1-t}\right)$. Since $\frac{1+t}{1-t}>1$ if $t\in(0,1)$, then $h'(t)>0$ if $t\in(0,1)$. Hence. $h(t)$ is strictly increasing on $t\in(0,1)$. Then. let's select $s\in(0,t)$. then $f(t)>f(s)$. Since $h(t)$ is increasing on $t\in[0,1)$ and $s>0$, so we have $f(s)\geq f(0)$. Then $f(t)>f(0)$. □

Remark: the difference of strictly increasing and increasing.

B. Let $t=\frac{a^x-b^x}{a^x+b^x}$. Then we have $1+t=\frac{2a^x}{a^x+b^x}$ and $1-t=\frac{2b^x}{a^x+b^x}$. Besides, we also have $\log(1+t)=\log a^x+\log\frac{2}{a^x+b^x}$ and $\log(1-t)=$ $\log(1-t)=\log b^x+\log\frac{2}{a^x+b^x}$. Hence, by the definition of $h(t)$, it holds that

$$h(t) = (1+t)\log(1+t) + (1-t)\log(1-t)$$
$$= \frac{2a^x}{a^x+b^x}\left(\log a^x + \log\frac{2}{a^x+b^x}\right) + \frac{2b^x}{a^x+b^x}\left(\log b^x + \log\frac{2}{a^x+b^x}\right)$$
$$= 2\left[\frac{a^x\log a^x + b^x\log b^x}{a^x+b^x} + \log\left(\frac{2}{a^x+b^x}\right)\right].$$

□

C. Let $g(x) = \log f(x)$. Then. $g(x) = \frac{1}{x}\log\left(\frac{a^x+b^x}{2}\right)$ for $x \in \mathbb{R}_+$. By the quotient rule. we get

$$g'(x) = \frac{\frac{2}{a^x+b^x}\cdot\frac{a^x\log a + b^x\log b}{2}x - \log\left(\frac{a^x+b^x}{2}\right)}{x^2}.$$

that is. $x^2 g'(x) = \frac{a^x\log a^x + b^x\log b^x}{a^x+b^x} + \log\left(\frac{2}{a^x+b^x}\right).$

Since $g(x) = \log f(x)$. then $f(x) = \exp[g(x)]$ and $f'(x) = \exp[g(x)]g'(x)$.

Hence. if we want to prove $f'(x) > 0$. it suffies to prove $g'(x) > 0$.

Combining part B and part C we have $x^2 g'(x) = \frac{1}{2}h(t)$. Since $h(t) > h(0)$ (By part A), then $x^2 g'(x) > \frac{1}{2}h(0) = 0$, that is, $g'(x) > 0$ if $x \in \mathbb{R}_+$. Hence. $f'(x) > 0$ if $x \in \mathbb{R}_+$, that is. $f(x)$ is strictly increasing on $x \in (0, +\infty)$. (no include $x=0$)

Since we need to prove f is strictly increasing on $[0, +\infty)$, then we need to prove $f(t) > f(0)$ when $t > 0$. Let's select $0 < z < s < t$, then we have $f(t) > f(s) > f(z)$. Besides. by part 1(a). f is right continuous, that is, $f(s) \geq \lim_{z\to 0^+} f(z) = f(0)$. Hence. $f(t) > f(0)$.

Therefore. f is strictly increasing on $[0, +\infty)$.

□

Remark: