For a function $f: A \to B$

(1) f is said to be an injective function if

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

(2) f is said to be a surjective function if

$$\forall y \in B,\ \exists x \in A \text{ s.t. } f(x) = y$$

Every element in B, we can find $x \in A$, such that $f(x) = y$

(3) f is said to be a bijective if and only if

f is both injective and surjective

Example 1.3.9

Show that $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x+3$ is a bijective function

Real number

Proof: (1) Injective:

$f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow x_1 = x_2$, f is injective

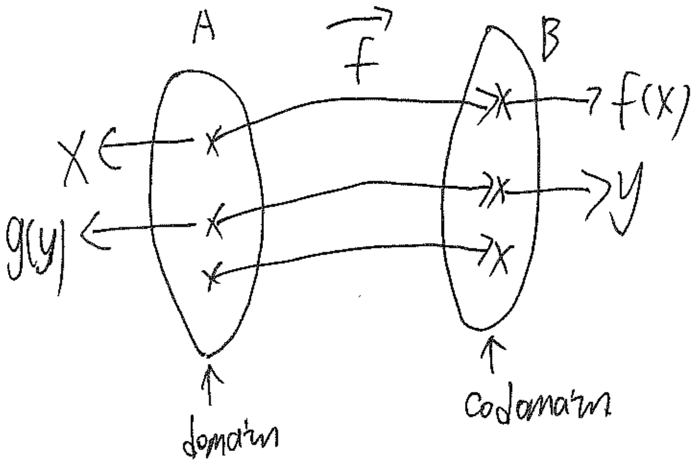
(2) Surjective:

$y = f(x) = 2x+3 \in \mathbb{R}$, we have $x = \frac{y-3}{2}$, $f(x) = f(\frac{y-3}{2}) = 2(\frac{y-3}{2}) + 3$

$= y$

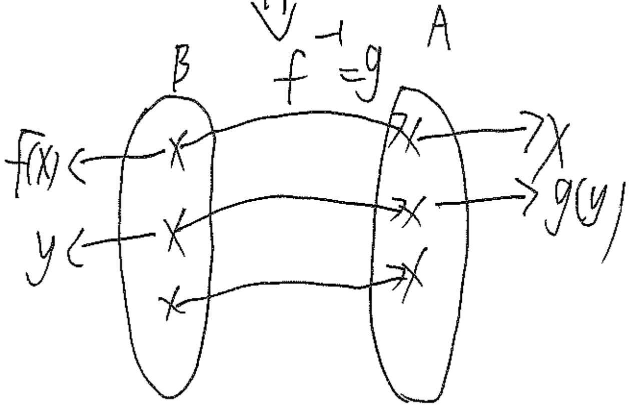
$\therefore f$ is surjective: For any $y \in \mathbb{R}$, we can find $x = \frac{y-3}{2}$ such that $f(x) = y$ holds.

When a function is bijective, we can claim that this function has an inverse.



You can image such a mapping

$\Downarrow$



We call f^{-1}: inverse of f

Definition 1.3.2

Let $f: A \to B$ be a function. If $g: B \to A$ is a function such that

(1) $g(f(x)) = x, \ \forall x \in A$

(2) $f(g(y)) = y, \ \forall y \in B$

Then g is said to be an inverse of f.

Fact: 1) once an inverse of f exists, it is unique, so we denote it by f^{-1}.

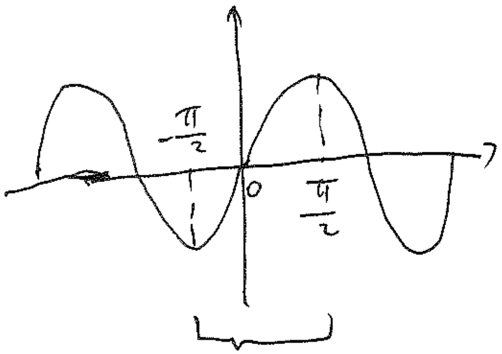
2) f has an inverse if and only if f is bijective

Example 1.3.10

	Injective	Surjective
① $\sin x: \mathbb{R} \to [-1,1]$	✗	✓
② $\sin x: [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1,1]$	✓	✓

For the case ②, we can define inverse function, arcsin

$$\arcsin = \sin^{-1} : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$$



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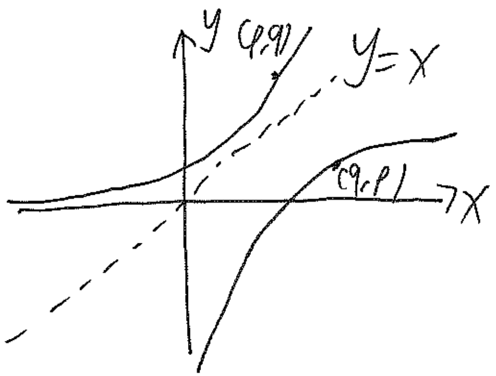
Fact: The graph of f^{-1} is the reflection of the graph of f along $y=x$.



$$f(p) = q \Leftrightarrow p = f^{-1}(q)$$

Even, odd and periodic Functions

Even function: $f(-x) = f(x)$

odd function: $f(-x) = -f(x)$

periodic function: $f(x) = f(x+T)$ for some $T>0$

$\sin x = \sin(x+2\pi)$, we consider $\sin x : \mathbb{R} \to [-1,1]$

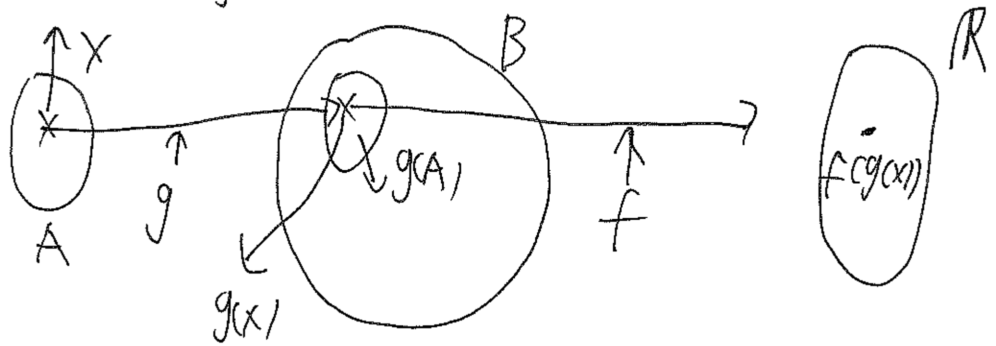
$x=1$, $\sin(1+8\pi) = \sin 1$

How to find $\tan(400\pi + 1.2) = \tan(1.2)$

"period of $\tan x = \frac{\sin x}{\cos x}$ is π"

Composite Function

If $f: B \to \mathbb{R}$ and $g: A \to \mathbb{R}$ are functions such that $g(A) \subseteq B$. We can construct a new function $f \circ g : A \to \mathbb{R}$ which is defined by $(f \circ g)(x) = f(g(x))$. We call it "f composite with g"



$f(x) = \sin x$, $g(x) = e^x$, what ~~is~~ is $(f \circ g)(x)$ and $(g \circ f)(x)$?

$(f \circ g)(x) = \sin e^x$

$(g \circ f)(x) = e^{\sin x}$

$f(x) = -x^2 - 1$, $g(x) = \ln x$, what is $(f \circ g)(x)$ and $(g \circ f)(x)$?

$(f \circ g)(x) \Downarrow -(\ln x)^2 - 1$

$(g \circ f)(x) \Downarrow \ln(-x^2-1)$ (Impossible)