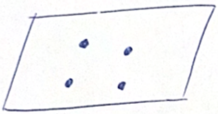
MATH 2010

Tutorial 2

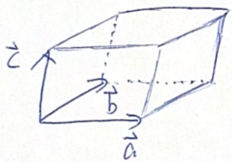
Exercises 11.4

Q55 Determine whether the given points are coplanar.

$A(1,1,1)$, $B(-1,0,4)$, $C(0,2,1)$, $D(2,-2,3)$



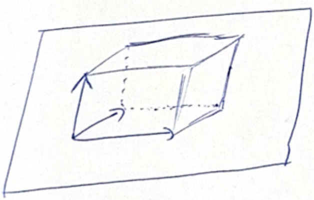
coplanar
(lying on the same plane)



Triple product

$(\vec{a} \times \vec{b}) \cdot \vec{c}$

$|(\vec{a} \times \vec{b}) \cdot \vec{c}|$ = volume of the parallelopiped spanned by $\vec{a}$, $\vec{b}$ & $\vec{c}$

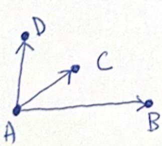


Coplanar

$\Leftrightarrow$ the parallelopiped is flat

$\Leftrightarrow$ Volume $= 0$

$\Leftrightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$



$\vec{AB} = (-2, -1, 3)$

$\vec{AC} = (-1, 1, 0)$

$\vec{AD} = (1, -3, 2)$

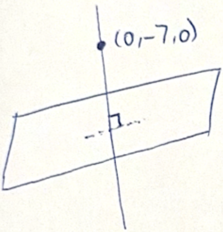
$$(\vec{AB} \times \vec{AC}) \cdot \vec{AD} = \begin{vmatrix} -2 & -1 & 3 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{vmatrix} = 3\begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} + 2\begin{vmatrix} -2 & -1 \\ -1 & 1 \end{vmatrix}$$

$$= 3(3-1) + 2(-2-1) = 0$$

$\therefore$ The given points are coplanar.

Exercises 11.5

Q9 The line through $(0,-7,0)$ perpendicular to the plane $x+2y+2z=13$



A normal vector of the plane is

$$\vec{n} = (1,2,2).$$

A parametrization of the line is

$$\vec{x} = (0,-7,0) + t(1,2,2), \quad t \in \mathbb{R}$$

//

Q27 Find the point of intersection of the lines
$x = 2t+1,\ y = 3t+2,\ z = 4t+3$, and
$x = s+2,\ y = 2s+4,\ z = -4s-1$,
and then find the plane determined by these lines.

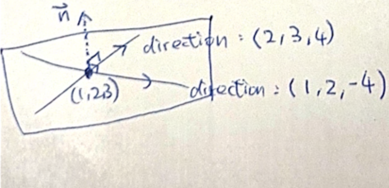
$$\begin{cases} 2t+1 = s+2 \\ 3t+2 = 2s+4 \\ 4t+3 = -4s-1 \end{cases} \Rightarrow \begin{cases} 2t-s=1 \\ 3t-2s=2 \end{cases} \Rightarrow \begin{cases} 4t-2s=2 \\ 3t-2s=2 \end{cases}$$

$$\Rightarrow t=0,\ s=-1$$

$4t+3 = 3$, $-4s-1 = 4-1 = 3$ $\Rightarrow$ $4t+3 = -4s-1$ is satisfied.

$\Rightarrow$ The lines intersect when $t=0$ and $s=-1$.

$\Rightarrow$ The point of intersection $(x,y,z) = (1,2,3)$.



A normal vector of the plane is

$$\vec{n} = (2,3,4) \times (1,2,-4)$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix} = -20\vec{i} + 12\vec{j} + \vec{k}$$

The equation of the plane is

$$((x,y,z)-(1,2,3))\cdot(-20,12,1)=0$$

$$-20(x-1)+12(y-2)+(z-3)=0$$

$$-20x+12y+z=7$$

Q35 Find the distance from the point to the line.

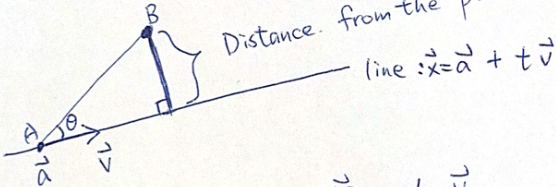
$(2,1,3)$; $x=2+2t$, $y=1+6t$, $z=3$

The line : $\vec{x}=(2,1,3)+t(2,6,0)$

The point $(2,1,3)$ lies on the line.

$\Rightarrow$ Distance $=0$.

In general,



θ : angle between $\vec{AB}$ and $\vec{v}$

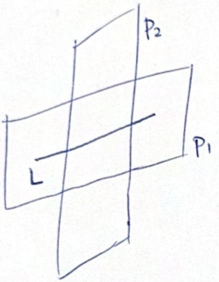
$$\text{Distance} = \|\vec{AB}\|\sin\theta$$

$$\|\vec{AB}\times\vec{v}\| = \|\vec{AB}\|\,\|\vec{v}\|\sin\theta$$

$$\Rightarrow \text{Distance} = \frac{\|\vec{AB}\times\vec{v}\|}{\|\vec{v}\|}$$

Q73 Find two different planes whose intersection is the line $x = 1+t$, $y = 2-t$, $z = 3+2t$.

Write equations for each plane in the form $Ax + By + Cz = D$.



$$t = x-1 = 2-y = \frac{z-3}{2}$$

$$\Rightarrow \begin{cases} x-1 = 2-y \\ 2-y = \frac{z-3}{2} \end{cases}$$

$$\Rightarrow \begin{cases} x+y = 3 \\ 2y+z = 7 \end{cases}$$

$x+y=3$ and $2y+z=7$ are two such planes.

(many different possible answers).