TCP Modeling

Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED

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Ordinary Differential Equations in Matlab

Remember to put the ODE in the **initial value problem format**. For example:

$$
\begin{aligned}
\frac{dy_1}{dt} &= f_1(t, y_1, y_2, \ldots, y_n) && \text{ODE 1} \\
&\vdots \\
\frac{dy_n}{dt} &= f_n(t, y_1, y_2, \ldots, y_n) && \text{ODE } n \\
y_1(0) &= y_{10} && \text{initial condition 1} \\
&\vdots \\
y_n(0) &= y_{n0} && \text{initial condition } n
\end{aligned}
$$

Example

$$\frac{d^2x}{dt} - \mu(1 - x^2)\frac{dx}{dt} + x = 0$$

Let us define $y_1 = x$ and $y_2 = \frac{dx}{dt}$ and use these as inputs. By putting them in the initial value problem format, we have:

$$\begin{aligned} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= \mu(1 - y_1^2)y_2 - y_1 \end{aligned}$$

Matlab Example

```
function ydot=vdpol(t,y)
%VDPOL van der Pol equation
% ydot=ydpol(t,y)
% ydot(1) = y(2)
% ydot(2) = mu*(1-y(1)^2*y(2) - y(1)
% mu =2
mu=2;
%ydot=[y(2);mu*(1-y(1)^2)*y(2)-y(1)];
ydot(1,1)=y(2);
ydot(2,1)=mu*(1-y(1)^2)*y(2)-y(1);
```

Testing Program

```
tspan = [0 40]; % time span for integration
yo = [2;0]; % initial conditions must be a column
[t,y] = ode45('ydot',tspan,yo);
size(t)       % number of time points
size(y)       % i^th column is y(i) at time t(i)

plot(t,y(:,1))
xlabel ('time')
title ('solution y')
```

Output

```
>> john
ans =

   621      1

ans =

   621      2
```

Output Graph

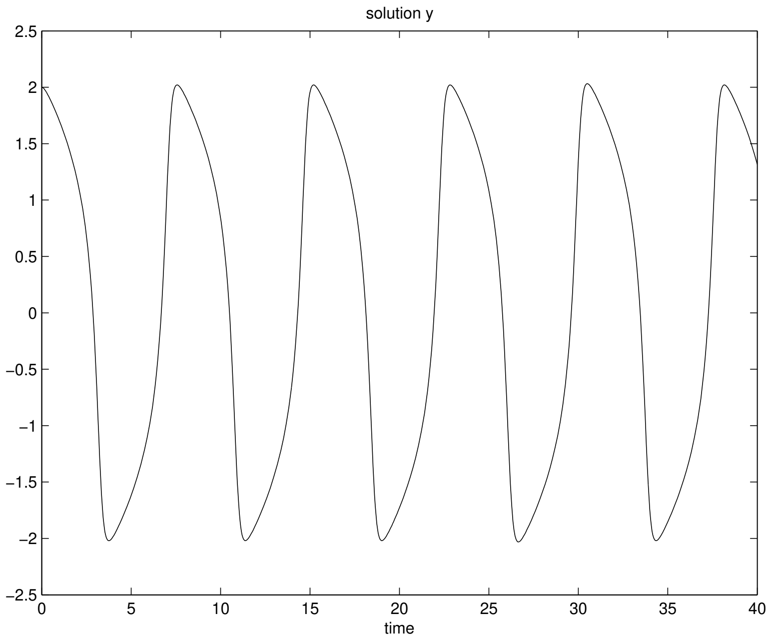


Figure 1: Output graph for Example

Goals

- provide formal understanding TCP and other congestion control algorithms
- introduce stochastic differential equations to model TCP
  - transient analysis
  - steady state analysis
- interaction of TCP with AQM (e.g., RED)
- Illustrate flaw in the RED averaging mechanism.

Introduction

- AQM techniques have been proposed to alleviate some congestion problems.
- Modeling and analysis of IP networks and QoS is important to understand their dynamics.
- Model data traffic as *fluid* and use Poisson Counter Driven Stochastic Differential Equations to model sample path description of TCP traffic.
- Derive a set of *differential equations* that describe the AQM policy and the router queueing process.

- Systematic way to derive:
  - queue length
  - round trip time
  - TCP flow throughput
  - various expected transient behavior

Model and Analysis of Single Congested Router

- single congested router with capacity C.
- the router is AQM enabled with *"packet discard function"* $p(x)$, where x is the *"average queue length"* at the router.

$$p(x) = \begin{cases} 0 & 0 \leq x \leq t^{min} \\ \frac{x - t^{min}}{t^{max} - t^{min}} & t^{min} \leq x \leq t^{max} \\ 1 & t^{max} < x \end{cases}$$

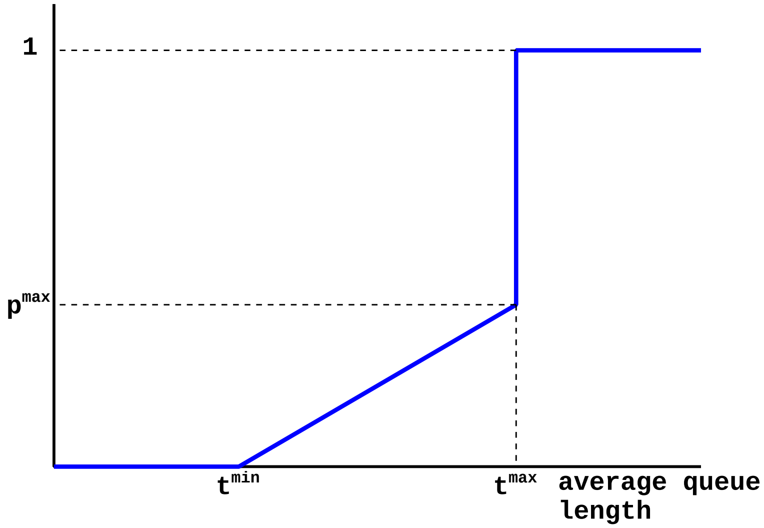


Figure 2: RED drop function

- Instantaneous queueing at time t is denoted by $q(t)$.

- N TCP flows labeled $i = 1, 2, ..., N$.
- W_i the window size of TCP flow i at time t.
- R_i the round trip time of TCP flow i at time t.

$$R_i(t) = a_i + q(t)/C \qquad t \geq 0, i = 1, ..., N$$

  where a_i is the fixed propagation delay for flow i and $q(t)/C$ is the queueing delay.
- $B_i(t)$ is the *instantaneous* throughput of TCP-flow i at time $t \geq 0$.
- Authors assumed that the packet losses to flow i are described by a Poisson process $\{N_i(t)\}$ with time varying rate $\lambda_i(t)$. where $N_i(t)$ is the number of losses at the time point when the flow detects losses, which is different from when the actual dropping at the router.

- Let $W_i(t)$ be the window size of TCP-flow i:

$$dW_i(t) = \frac{dt}{R_i(q(t))} - \frac{W_i(t)}{2} dN_i(t)$$

Explain additive increase and multiplicative decrease. Taking expectation:

$$\begin{aligned} E[dW_i(t)] &= E\left[\frac{dt}{R_i(q(t))}\right] - \frac{E[W_i(t)dN_i(t)]}{2} \\ &\approx E\left[\frac{dt}{R_i(\bar{q})}\right] - \frac{E[W_i(t)]\lambda_i(t)}{2} \end{aligned}$$

Note that $\lambda_i(t)$ is the loss indication received by the source. It reaches the source approximately one round-trip delay (τ) after the packet has been marked/dropped. To model the delay, let t' be the time of that packet got dropped, we have:

$$\begin{aligned} t &= R(q(t')) + t' \\ t' &= t - \tau \end{aligned}$$

The bandwidth of the flow is $B_i(t-\tau)$ at time $t-\tau$, the rate of loss indication $(\lambda_i(t))$ is $p(\bar{x}(t-\tau))B_i(t-\tau)$, the expected value is $p(\bar(x)(t-\tau))E[W_i(t-\tau)]/R_i(\bar{q}(t-\tau))$. Therefore:

$$dE[W_i] \quad \approx \quad \frac{dt}{R_i(\bar{q})} - \frac{E[W_i]}{2} p(\bar{x}(t-\tau)) \frac{E[W_i(t-\tau)]}{R_i(\bar{q}(t-\tau))}$$

Finally, we have N differential equations:

$$\frac{dE[W_i]}{dt} \approx \frac{1}{R_i(\bar{q})} - \frac{E[W_i]E[W_i(t-\tau)]}{2R_i(\bar{q}(t-\tau))} p(\bar{x}(t-\tau)) \ i = 1, .., N \quad (1)$$

RED uses exponential weighted average to estimate the average queue length $x(t)$, the sample window is δ, we have:

$$x((k+1)\delta) = (1-\alpha)x(k\delta) + \alpha q(k\delta)$$

where $0 < \alpha < 1$. Convert this to differential equation, we have:

$$\frac{dx}{dt} = \frac{\log_e(1-\alpha)}{\delta}x(t) - \frac{\log_e(1-\alpha)}{\delta}q(t)$$

Take the expectation, we have another important differential equation:

$$\frac{dx}{dt} = \frac{\log_e(1-\alpha)}{\delta}\bar{x}(t) - \frac{\log_e(1-\alpha)}{\delta}\bar{q}(t) \qquad (2)$$

We need the final equation which describes the instantaneous queue length. We have:

$$\frac{dq(t)}{dt} = -1_{q(t)}C + \sum_{i=1}^{N} \frac{W_i(t)}{R_i(q(t))}$$

Take the expectation, we have:

$$\frac{d\bar{q}(t)}{dt} \approx -1_{\bar{q}(t)}C + \sum_{i=1}^{N} \frac{E[W_i(t)]}{R_i(\bar{q}(t))} \tag{3}$$

- Illustrate with program in Matlab.

Extension to a network

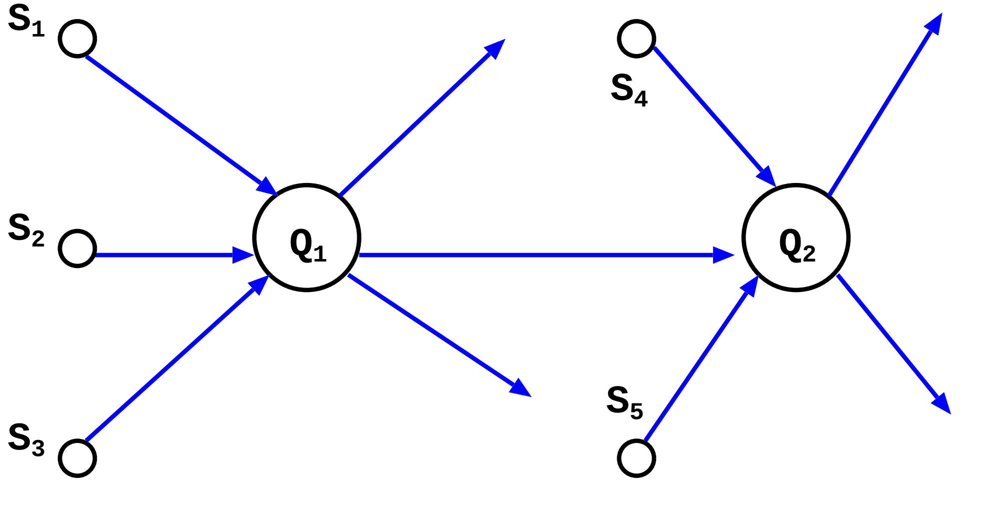
Assume TCP-flow i traverses routers in the set V_i. We have:

$$R_i(\boldsymbol{q}, t) = a_i + \sum_{v \in V_i} q_v(t)/C_v \tag{4}$$

$$\frac{dE[W_i(t)]}{dt} \approx \frac{1}{R_i(\bar{\boldsymbol{q}}(t-\tau))} - \frac{E[W_i(t)]E[W_i(t-\tau)]}{2R_i(\bar{\boldsymbol{q}}(t-\tau))} \left[1 - \prod_{v \in V_i} (1 - p_v(\bar{x}(t-\tau)))\right] \tag{5}$$

$$\frac{d\bar{q}_v}{dt} = -1_{\bar{q}_v(t)} C_v + \sum_{i \in F_v} \frac{E[W_i(t)]}{R_i(\bar{q}_v)} \tag{6}$$

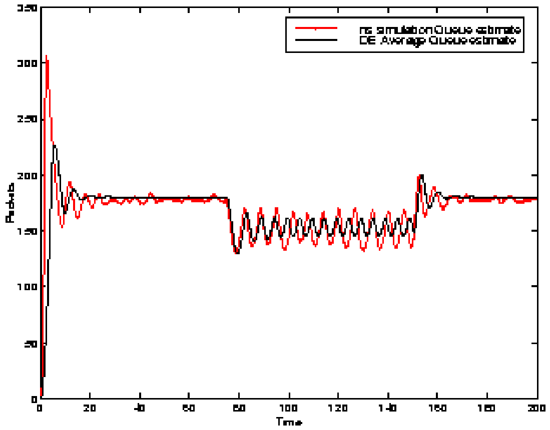
Experiments



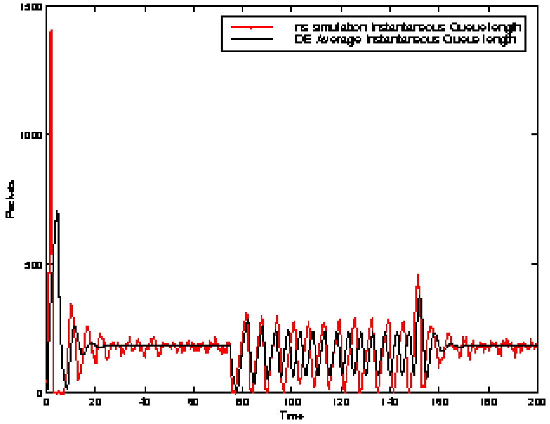
- $t^{min} = 150$ packets; $t^{max} = 200$ packets

Experiment 1 & 2

- $\alpha = 0.001$, C_1 =5 Mb/s, C_2 = 5 Mb/s (or 2.5 Mbps), pkt size=500, $p_{max} = 0.1$.
- Each class has 40 flows.
- At $t = 75$, three fourths of the flows drop out.
- At $t = 150$, three fourths of the flows restart.
- Illustrates good tracking for S_1 and S_2, both in average queue length and instantaneous queue length.

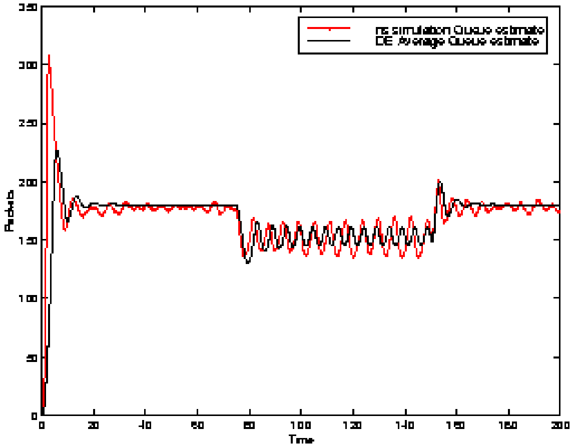


(a) Queue estimate

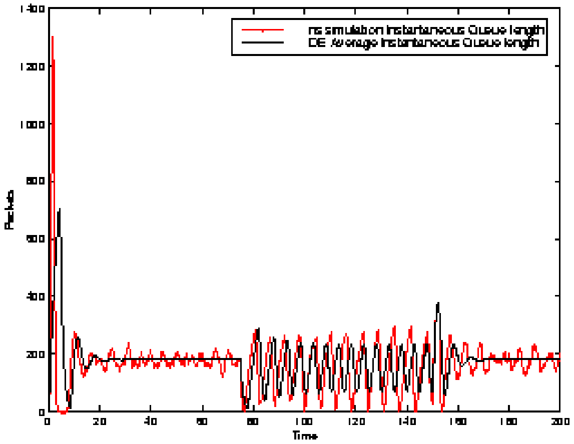


(b) Instantaneous Queue Length

Figure 3: Symmetric case, Plots for Queue 1, Experiment 1

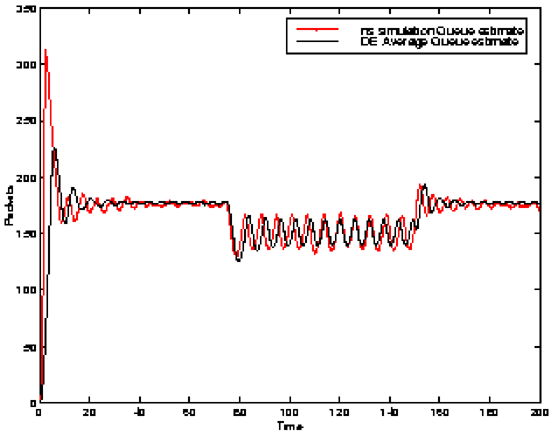


(a) Queue estimate

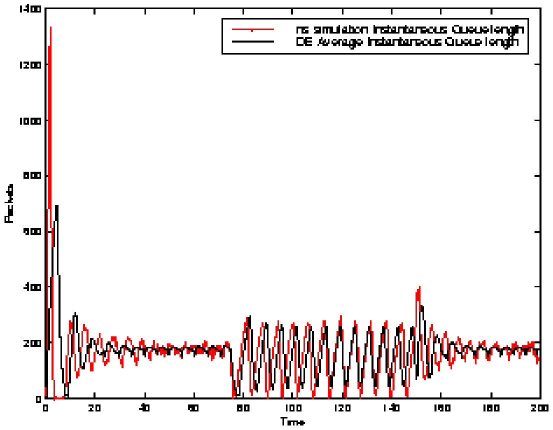


(b) Instantaneous Queue Length

Figure 4: Symmetric case, Plots for Queue 2, Experiment 1

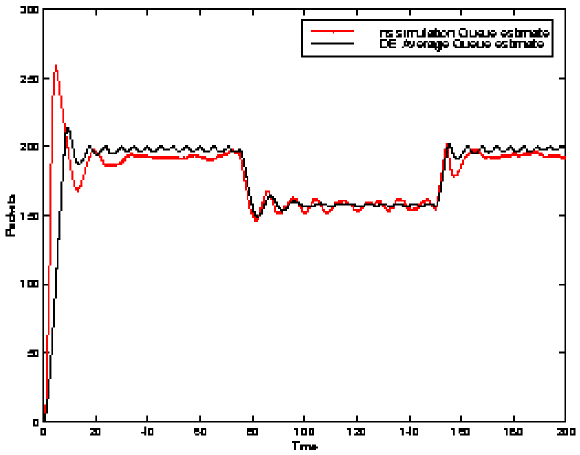


(a) Queue estimate

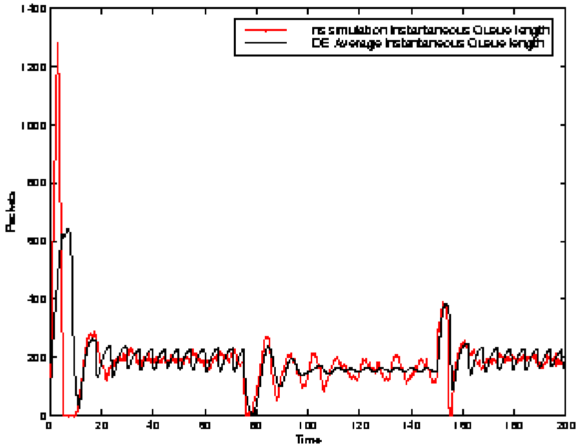


(b) Instantaneous Queue Length

Figure 5: Asymmetric case, Plots for Queue 1, Experiment 2



(a) Queue estimate

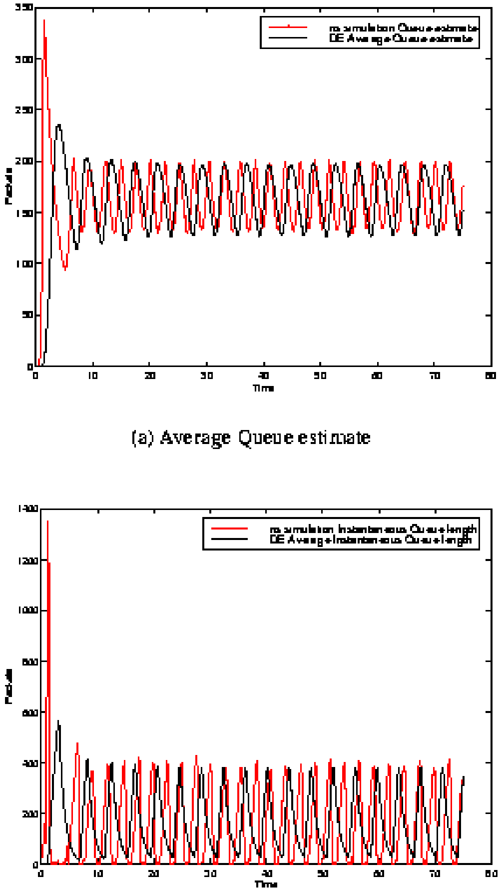


(b) Instantaneous Queue Length

Figure 6: Asymmetric case, Plots for Queue 2, Experiment 2

Experiment 3

- $\alpha = 0.001$, $C_1 = 15$ Mb/s, $C_2 = 15$ Mb/s, pkt size=500, $p_{max} = 0.1$.
- Observe **instability** of average queue length and instantaneous queue length at S_1. **INSTABILITY IS BAD**.
- Reason, the *sampling rate* of RED is on every packet. This makes the system sampling too fast and include higher order signal.
- Proposal, increase δ. But δ should not be set too high or else it has high rise time and increasing initial overshoot.



(a) Average Queue estimate

(b) Instantaneous Queue Length

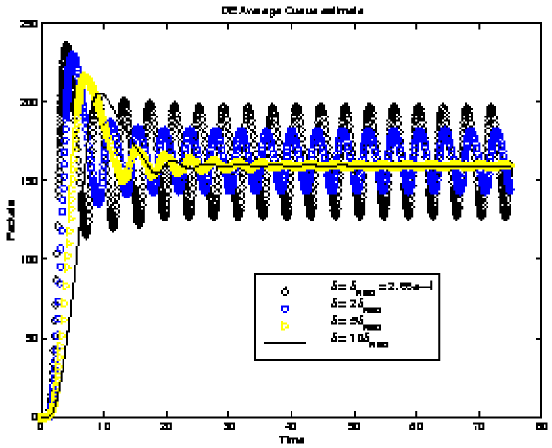
Figure 8: Link speed 15 Mb/s, Packet size 500 Bytes, Experiment 3

Experiment 4 & 5

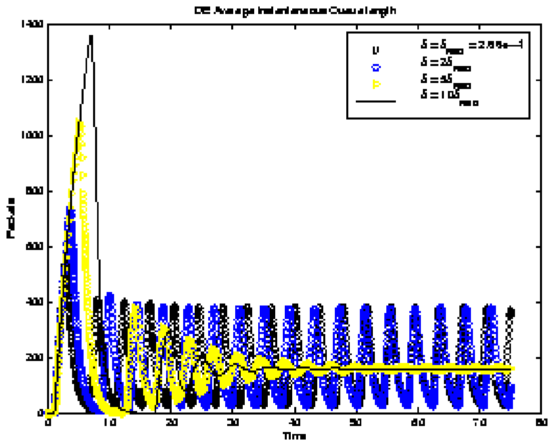
- In Experiment 4, if capacity of both routers are set to 15 Mb/s, then we see stabilities.
- In Experiment 5, setting $p_{max} = 1$, as suggested by other authors on controlling RED, will **NOT** remove instability.

Conclusion: the authors provide a control theoretic approach to solve the AQM problem in an INFOCOM 2001 paper.

This is a good research because it provides some theoretical underpinning and gives designers the understanding on how to perform congestion control and QoS delivery.

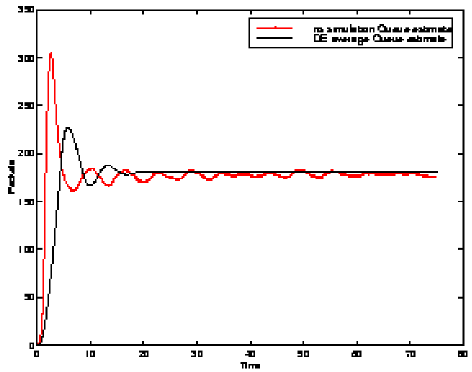


(a) Average Queue estimate

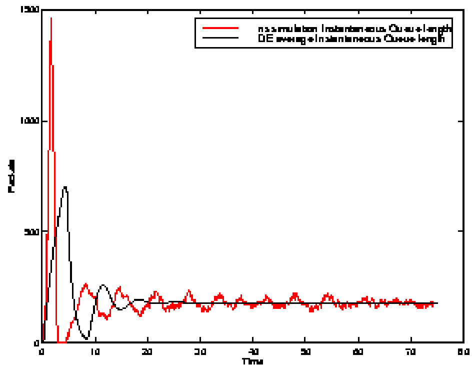


(b) Instantaneous Queue Length

Figure 9: Link speed 15 Mb/s, Packet size 500 Bytes

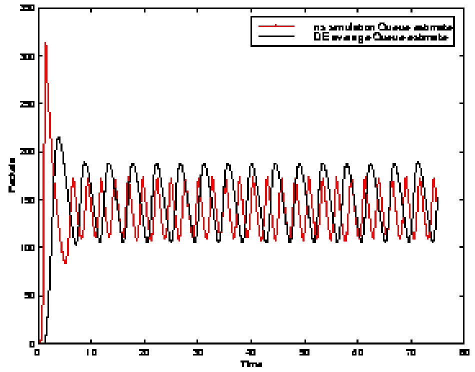


(a) Average Queue estimate

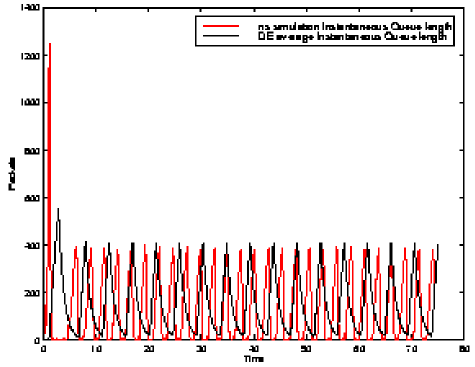


(b) Instantaneous Queue Length

Figure 10: Link speed 15 Mb/s, Packet size 1500 Bytes, Experiment 4



(a) Average Queue estimate



(b) Instantaneous Queue Length

Figure 11: Link speed 15 Mb/s, Packet size 500 bytes, p_{max} = 1, Experiment 5