Integration- Supplement Solution Set

See If You Need This Video!

1. Answer: E. You should be looking for a limit like this

$$\lim_{N\to\infty}\sum_{n=1}^{N} f\left(x_n\right)\Delta x$$

From the limit, we know n is from 1 to N, which means we divided the integration range into N parts.
Then, Δx, which is the width of rectangle should be $\frac{3-1}{N}$.
And the position x for each rectangle, $x_n = 1 + \Delta x$, because x starts from 1.

≪Bridge between area and integration≫
Episode 1- Cantonese: 1:20 English: 1:17 Putonghua: 1:25

2. Answer: C.

    1. It is an reversed path of integration, we get flipped sign.
    2. The original integration from a to b, $-a$ to $-b$ is a totally different range.
    3. It is similar to above.
    4. You may check this with substitution $u = -x$

≪Path of integration≫
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≪Substitution≫
Episode 3- Cantonese: 1:02 English: 0:49 Putonghua: 1:10

3. Answer: A. The upper and lower limits are the range of integration. We integral from lower to upper limit. If the curve lies on positive side ($f(x) \geq 0$) and $a > b$, then the area under the curve is the integral.

   (Think about the case if $f(x) \leq 0$, what is the area "under" the curve we are finding through integration. The picture of summation of rectangles may helps).

   ≪Bridge between area and integration≫
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4. Answer: C.

$$\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$$

   1. $n = 0$ here.
   2. $n = -3$ here.
   3. Please be careful that when $n = -1$, we need this

$$\int \frac{1}{x} \, \mathrm{d}x = \ln x + C$$

   4. $n = 20$ here, the result is wrong.

   $\ll \int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$ (memorization is recommended)≫
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   $\ll \int x^{-1} \, \mathrm{d}x = \ln x + C$ (memorization is recommended)≫
   Episode 2- Cantonese: 3:27 English: 3:28 Putonghua: 3:36

5. Answer: A.

    1. e is a constant only. $\int e \, \mathrm{d}x = e \int \, \mathrm{d}x$
    2. This is why e is a special constant.
    3. This is wrong. Convert 2^x to base e.

    $$2^x = e^{x \ln 2}$$

    Then you need simple substitution.
    4. It is $\int x^{-1} \, \mathrm{d}x = \ln x + C$, please don't get it wrong.

    ≪ $\int e^x \, \mathrm{d}x = e^x + C$ (memorization is recommended)≫
    Episode 2- Cantonese: 3:44 English: 3:44 Putonghua: 3:58

6. Answer: E.

    1. x and u are just dummy variable, you may label them whatever you want.
    2. Definite integral is plugging in the upper and lower limits into the result of indefinite integral and perform subtraction.
    3. Is is definition of indefinite integral, and it is why we say integration likes an inverse of differentiation.
    4. Don't forget the minus sign.

    ≪Dummy variable≫
    Episode 2- Cantonese: 4:40 English: 4:47 Putonghua: 5:10
    ≪Definite integral≫
    Episode 2- Cantonese: 4:40 English: 4:47 Putonghua: 5:10
    ≪Indefinite integral≫
    Episode 2- Cantonese: 0:15 English: 0:18 Putonghua: 0:18
    ≪Integrals of sinusoidal functions (memorization is recommended)≫
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7. Answer: D.

$$\begin{aligned}\int x^{-1} + \cos x \, \mathrm{d}x &= \int x^{-1} \, \mathrm{d}x + \int \cos x \, \mathrm{d}x \\ &= \ln x - \sin x + C\end{aligned}$$

You don't need $2C$, C is just an unknown constant.

≪Linearity of integration≫
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≪Integrals of sinusoidal functions≫
Episode 2- Cantonese: 3:54 English: 3:53 Putonghua: 4:15
≪ $\int x^{-1} \, \mathrm{d}x = \ln x + C$ (memorization is recommended)≫
Episode 2- Cantonese: 3:27 English: 3:28 Putonghua: 3:36

8. Answer: C.
Substitution $u = x^3$ is needed.

$$\begin{aligned}u &= x^3 \\ \mathrm{d}u &= 3x^2 \mathrm{d}x\end{aligned}$$

$$\begin{aligned}\int x^2 e^{x^3+1} \, \mathrm{d}x &= \frac{e}{3} \int e^u \, \mathrm{d}u \\ &= \frac{e}{3} e^u + C \\ &= \frac{e^{x^3+1}}{3} + C\end{aligned}$$

≪Substitution≫
Episode 3- Cantonese: 1:02 English: 1:14 Putonghua: 1:10

9. Answer: B.
Integration by-parts is needed.

$$\begin{aligned}\int \ln x \, \mathrm{d}x &= \int (1)(\ln x) \, \mathrm{d}x \\ &= \ln x \int \mathrm{d}x - \int x \frac{\mathrm{d} \ln x}{\mathrm{d}x} \, \mathrm{d}x \\ &= x \ln x - \int \mathrm{d}x \\ &= x \ln x - x + C\end{aligned}$$

≪Integration by-parts≫
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10. Answer: B.

$$\begin{aligned}u &= x^2 \\ \mathrm{d}u &= 2x\mathrm{d}x\end{aligned}$$

$$\int_0^{\pi} x \sin x^2 \, \mathrm{d}x = \int_0^{\pi^2} 0.5 \sin u \, \mathrm{d}u$$

Please be careful of the upper and lower limits. They changed because they are the range of u now.

$$\begin{aligned}\int_0^{\pi} x \sin x^2 \, \mathrm{d}x &= 0.5 \left[-\cos u \right]_0^{\pi^2} \\ &= 0.5 \left(-\cos \pi^2 + 1 \right)\end{aligned}$$

≪Substitution for definite integral≫
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