Autoencoders

Jian Li

2017-10-17

Contents

- Stochastic Autoencoders
- Denoising Autoencoders
- Contractive Autoencoders
- Application of Autoencoders

Stochastic autoencoders

Usual autoencoder:

- $x \to h$
- $h \to x$

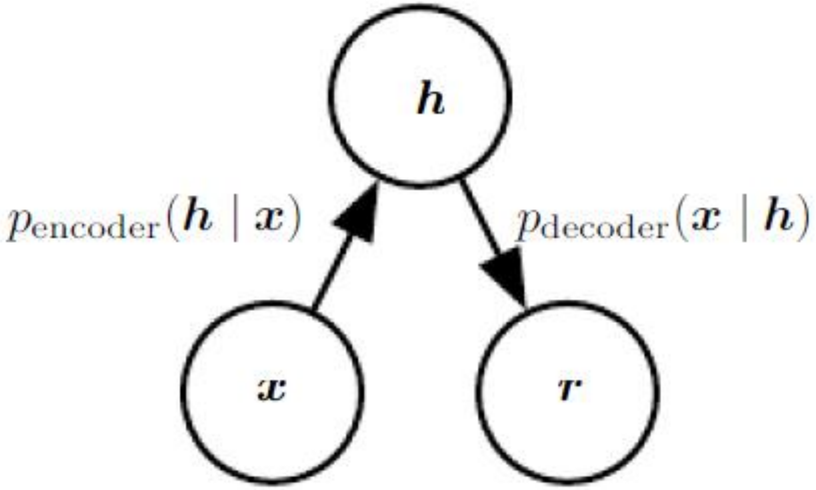
Stochastic encoder:

- $x \to p_{encoder}(h|x)$

Stochastic decoder:

- $h \to p_{decoder}(x|h)$

Stochastic autoencoder may be estimated with maximum likelihood.



Stochastic autoencoders

- $p_{decoder}(x|h)$ determines the ***output units*** and the ***loss function***.
- Train the autoencoder by minimizing $-\log p_{decoder}(x|h)$
- E.g., $p_{decoder}(x|h) = N(x;\ \hat{x}(h), \sigma^2)$
- Since the examples are assumed to be i.i.d., the loss is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$-\log p_{decoder}(x|h) = -\sum_{i=1}^{m} \log p_{decoder}\left(x^i \middle| h^i\right)$$

$$= m \log \sigma + \frac{m}{2}\log(2\pi) + \sum_{i=1}^{m} \frac{\left\|\hat{x}^i - x^i\right\|^2}{2\sigma^2}$$

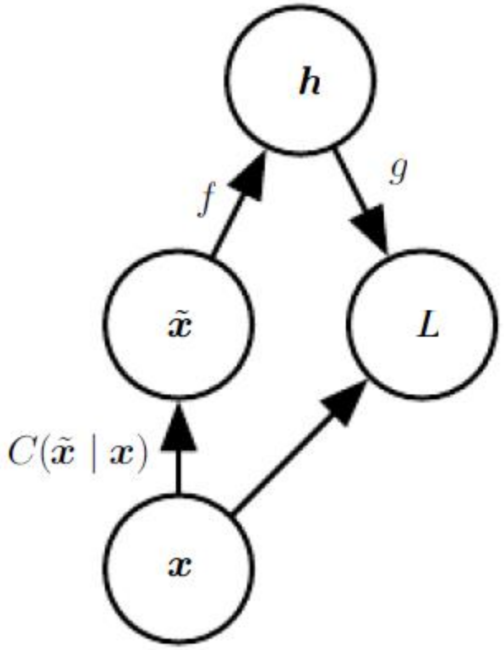
- The MSE error: $MSE_{decoder} = \frac{1}{m}\sum_{i=1}^{m}\left\|\hat{x}^i - x^i\right\|^2$

Denoising autoencoders

- Solved task:

$$\sum_{n=1}^{N} \mathcal{L}(x, g_\theta(f_\theta(\tilde{x}))) \to \min_\theta$$

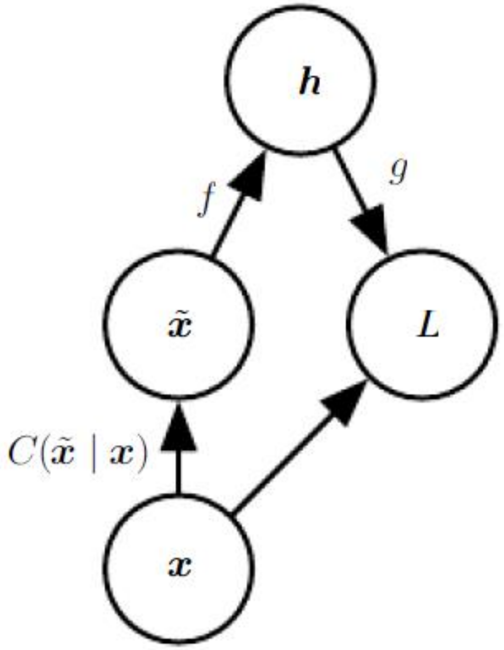
  where $\tilde{x}$ corresponds to x with added random noise.
- Autoencoder needs to reconstruct structure of the data.
  - recover density $p(x)$
    - to move x away from improbable regions
  - recover typical dependencies between features
    - to reconstruct one feature using other features



Denoising autoencoders

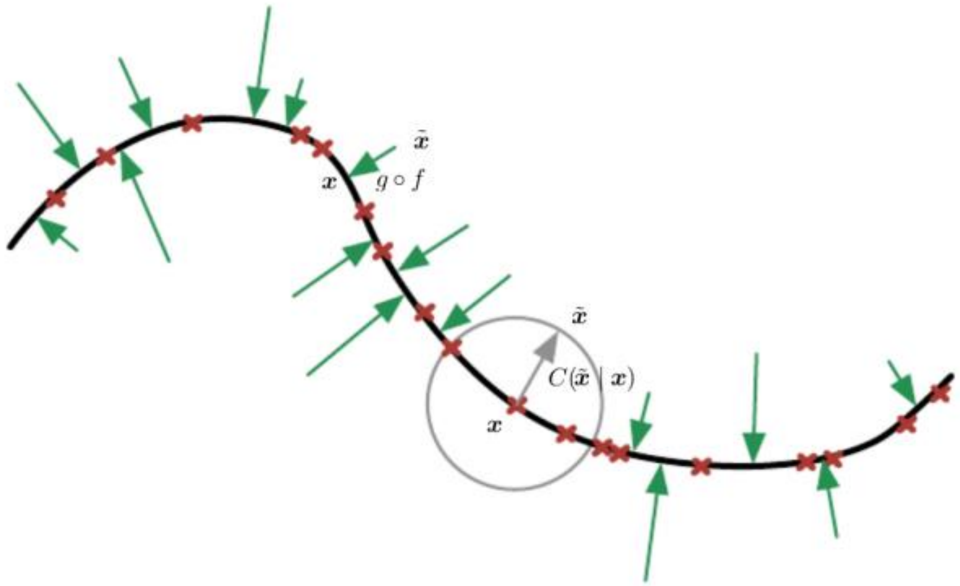
The autoencoder learns a reconstruction distribution $p_{reconstruct}(x|\tilde{x})$ estimated from training pairs $(x, \tilde{x})$, as follows:

1. Sample a training example $\boldsymbol{x}$ from the training data.
2. Sample a corrupted version $\tilde{\boldsymbol{x}}$ from $C(\tilde{\mathrm{x}} \mid \mathrm{x} = \boldsymbol{x})$.
3. Use $(\boldsymbol{x}, \tilde{\boldsymbol{x}})$ as a training example for estimating the autoencoder reconstruction distribution $p_{\text{reconstruct}}(\boldsymbol{x} \mid \tilde{\boldsymbol{x}}) = p_{\text{decoder}}(\boldsymbol{x} \mid \boldsymbol{h})$ with $\boldsymbol{h}$ the output of encoder $f(\tilde{\boldsymbol{x}})$ and p_{decoder} typically defined by a decoder $g(\boldsymbol{h})$.



If encoder is deterministic, $L = -\log p_{\text{decoder}}(\boldsymbol{x} \mid \boldsymbol{h} = f(\tilde{\boldsymbol{x}}))$

What it learns



- Gray circle: corruption area
- Black line: manifold, where objects are concentrated.
- Red crosses: training set.
- Green lines: $g(f(x)) - x$

Contractive autoencoders

- minimize $L(\boldsymbol{x}, g(f(\boldsymbol{x}))) + \Omega(\boldsymbol{h}, \boldsymbol{x})$,

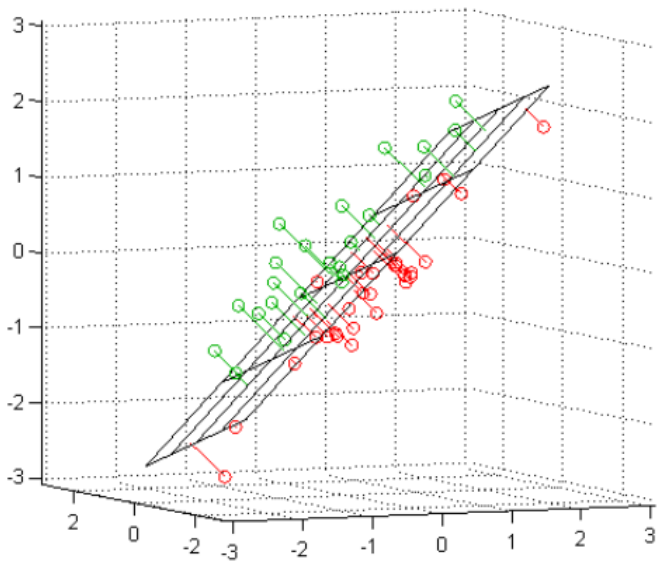
$$\Omega(\boldsymbol{h}) = \lambda \left\| \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} \right\|_F^2 . \tag{14.18}$$

The penalty $\Omega(\boldsymbol{h})$ is the squared Frobenius norm (sum of squared elements) of the Jacobian matrix of partial derivatives associated with the encoder function.

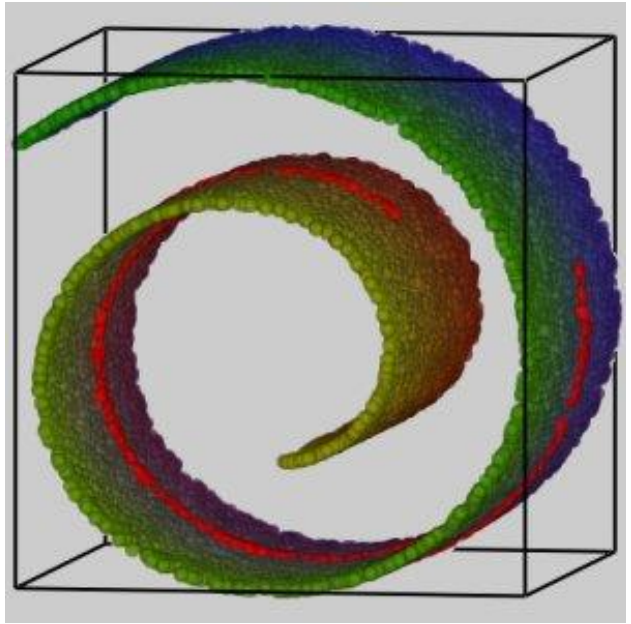
- Jacobian reduces sensitivity of h to x
- Such regularization $\uparrow$robustness of representation to small variations in x.
- λ controls tradeoff between reconstruction error and robustness.

Manifold learning

- Reconstruction error alone: learn an identity function.
- Contractive penalty alone: learn features that are constant w.r.t. input *x*.
- The compromise between the two forces makes representation *h* only sensitive to changes along the manifold directions.



PCA example



Non-linear Manifold

Contractive autoencoders

- Manifold learning
  - $h = f(x)$ maps input to coordinates in embedded space.
  - $f(x + \Delta x) \approx f(x) + J_f(x)\Delta x$
  - Directions Δx:
    - with large $\|J_f(x)\Delta x\|$ are tangent to manifold
    - with small $\|J_f(x)\Delta x\|$ are perpendicular to manifold

Contractive VS denoising autoencoders

Denoising autoencoder becomes equivalent to contractive autoencoder under 2 conditions:

- denoising autoencoder: for infinitesimal Gaussian noise
- contractive autoencoder: for penalty on reconstruction $r(x)$ rather than on $f(x)$.

[1] See Alain and Bengio (2013).

Application of autoencoders

- Applications:
  - dimensionality reduction
    - visualization
    - feature extraction
    - $\uparrow$ prediction accuracy
    - $\uparrow$ speed of prediction
    - $\downarrow$ memory requirements
  - semantic hashing
  - unsupervised pretraining

Semantic hashing

- Map complicated objects (e.g. texts, images) to binary codes.
- Objects with the same binary code are similar
  - may also consider objects which have almost the same binary code
    - by flipping several bits

Semantic hashing

- To make binary codes:
  - use sigmoid non-linearity
  - before this non-linearity add noise
    - to confront this noise model will need to make activations very large or small - sigmoid will saturate in both cases.