# CSCI2100: Quiz 1

Name: Student ID:

Each multiple-choice question has only one correct answer unless otherwise stated.

**Problem 1 (16%).** Which *two* of the following are *not* atomic operations of the RAM model? No marks are given unless you correctly identify both.
Answer: [ ]
A. Calculate $a/b$ where integers $a$ and $b$ are stored in two registers.
B. Calculate $\sqrt{a}$ where integer $a$ is stored in a register.
C. Suppose that a register $A$ stores a value 275; write the content of a register $B$ into the memory cell at address 275.
D. Calculate $a^b$ for integers $a, b$ stored in two registers.

**Answer:** $BD$

**Problem 2 (8%).** Which of the following functions is *not* $O(n^{2.5})$. Answer: [ ]
A. $\frac{n^{1000}}{2^n}$
B. $(\log_2 n)^{98}$
C. $938593729n^2$
D. None of the above.

**Answer:** $D$

**Problem 3 (8%).** Which of the following functions is $O(n \log \sqrt{n})$. Answer: [ ]
A. $358 \cdot n \log_2 n$ B. $n^{1.2}/\log^5 n$ C. $(1.01)^n$ D. $n \cdot (\log_2 n)^{1.0003}$

**Answer:** $A$

**Problem 4 (8%).** Which of the following functions is *not* $\Theta(n \log n)$. Answer: [ ]
A. $n \log_{395} n$ B. $895n \log_2 n + 98\sqrt{n}$ C. $n^{1.07} + 8n \log_{207} n$. D. $n \log_2 n - n^{0.99}$.

**Answer:** $C$

**Problem 5 (20%).** Complete the blanks of the following proof of $n^2 + 25n = O(n^3)$.

**Proof.** We need to find constants $c_1, c_2$ such that _____ for all _____.
Towards this purpose, let us inspect the inequality:

$$\begin{aligned} & n^2 + 25n \leq c_1 \cdot n^3 \\ \Leftarrow \quad & n^3 + 25n \leq c_1 \cdot n^3 \\ \Leftrightarrow \quad & 25n \leq (c_1 - 1)n^3 \\ \Leftrightarrow \quad & 25 \leq (c_1 - 1)n^2. \end{aligned}$$

By setting $c_1 =$ _____ and $c_2 = 1$, the above holds for all _____.
This concludes the proof. □

**Answer:** "such that $n^2 + 25n \leq c_1 \cdot n^3$ for all $n \geq c_2$". "By setting $c_1 = 26$". "holds for all $n \geq c_2$".

**Problem 6 (15%).** Given an array $A$ of $n$ integers, describe an algorithm to check whether $A$ is already sorted in ascending order. Your algorithm must finish in $O(n)$ time.

**Solution.** For each $i \in [1, n-1]$, check whether $A[i] \leq A[i+1]$. If this is false for any $i$, output "no". Otherwise, output "yes".

**Problem 7 (25%).** Suppose that $\mathcal{A}$ is a deterministic sorting algorithm with worst-case running time $\Theta(n^2)$. Prof. Goofy asserts that merge sort must be faster than $\mathcal{A}$ on every possible input. Is Prof. Goofy's claim correct? If you believe so, provide a proof of the claim. Otherwise, give a specific algorithm $\mathcal{A}$ that serves as a counterexample. (Hint: You may find your solution to Problem 6 useful.)

**Solution.** No. For a counterexample, let $\mathcal{A}$ be the following algorithm:

- Check whether the input array is already sorted (e.g., using your solution to Problem 6).
- If so, the algorithm terminates immediately.
- Otherwise, the algorithm applies selection sort.

The worst-case running time of $\mathcal{A}$ is $\Theta(n^2)$. However, $\mathcal{A}$ finishes in $O(n)$ time when the input array is sorted, whereas merge sort still runs in $O(n \log n)$ time regardless of whether the input array is sorted.