Transparent Anonymization: Thwarting Adversaries Who Know the Algorithm

Xiaokui Xiao[1] Yufei Tao[2] Nick Koudas[3]

[1]Nanyang Technological University
[2]Chinese University of Hong Kong
[3]University of Toronto

Abstract

Numerous generalization techniques have been proposed for privacy preserving data publishing. Most existing techniques, however, implicitly assume that the adversary knows little about the anonymization algorithm adopted by the data publisher. Consequently, they cannot guard against privacy attacks that exploit various characteristics of the anonymization mechanism. This paper provides a practical solution to the above problem. First, we propose an analytical model for evaluating disclosure risks, when an adversary knows *everything* in the anonymization process, except the sensitive values. Based on this model, we develop a privacy principle, *transparent l-diversity*, which ensures privacy protection against such powerful adversaries. We identify three algorithms that achieve transparent l-diversity, and verify their effectiveness and efficiency through extensive experiments with real data.

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1 Introduction

Privacy protection is highly important in the publication of sensitive personal information (referred to as *microdata*), such as census data and medical records. A common practice in anonymization is to remove the identifiers (e.g., social security numbers or names) that uniquely determine entities of interest. This, however, is not sufficient because an adversary may utilize the remaining attributes to identify individuals [31]. For instance, consider that a hospital publishes the microdata in Table 1, without disclosing the patient names. Utilizing the publicly-accessible voter registration list in Table 2, an adversary can still discover Ann's disease, by joining Tables 1 and 2. The joining attributes $\{Age, Zipcode\}$ are called the *quasi-identifiers* (QI).

Name	**Age**	**Zipcode**	**Disease**
Ann	21	10000	dyspepsia
Bob	27	18000	flu
Cate	32	35000	gastritis
Don	32	35000	bronchitis
Ed	54	60000	gastritis
Fred	60	63000	flu
Gill	60	63000	dyspepsia
Hera	60	63000	diabetes

Table 1: Microdata T_1

Name	**Age**	**Zipcode**
Ann	21	10000
Bob	27	18000
Bruce	29	19000
Cate	32	35000
Don	32	35000
Ed	54	60000
Fred	60	63000
Gill	60	63000
Hera	60	63000

Table 2: Voter List E_1

Age	**Zipcode**	**Disease**
[21, 27]	[10k, 18k]	dyspepsia
[21, 27]	[10k, 18k]	flu
32	35000	gastritis
32	35000	bronchitis
[54, 60]	[60k, 63k]	gastritis
[54, 60]	[60k, 63k]	flu
[54, 60]	[60k, 63k]	dyspepsia
[54, 60]	[60k, 63k]	diabetes

Table 3: Generalization T_2^*

Generalization [31] is a popular solution to the above problem. It works by first assigning tuples to *QI-groups*, and then transforming the QI values in each group to an identical form. As an example, Table 3 illustrates a generalized version of Table 1 with three QI-groups. Specifically, the first, second, and third QI-groups contain the tuples {Ann, Bob}, {Cate, Don}, and {Ed, Fred, Gill, Hera}, respectively. Even with the voter registration list in Table 2, an adversary still cannot decide whether Ann owns the first or second tuple in Table 3, i.e., Ann's disease cannot be inferred with absolute certainty.

Generalizations can be divided into *global recoding* and *local recoding* [19]. The former demands that if two tuples have identical QI values, they must be generalized to the same QI-group. Without this constraint, the generalization is said to use local recoding. For instance, Table 3 obeys global recoding. Notice that Cate and Don have equivalent QI-values in the microdata (Table 1), and therefore must be included in the same QI-group. This is also true for Fred, Gill, and Hera.

The privacy-preservation power of generalization relies on the underlying *privacy principle*, which determines what is a publishable QI-group. Numerous principles are available in the literature, offering different degrees of privacy protection. One popular, intuitive and effective principle is l*-diversity* [23]. It requires that, in each QI-group, at most $1/l$ of the tuples can have the same sensitive value[1]. This ensures that an adversary can have at most $1/l$ confidence in inferring the sensitive information of an individual. For example, Table 3 is 2-diverse. Thus, an adversary can discover the disease of a person with at most 50% probability.

Interestingly, none of the existing privacy principles (except those in [36] and [42]) specifies any requirement on the algorithm that produce the generalized tables. Instead, they impose constraints only on the formation of the QI-groups (like l-diversity does), which, unfortunately, leaves open the

[1] There also exist other formulations of l-diversity, as will be discussed in Section 2.1

opportunity for an adversary to breach privacy by exploiting the characteristics of the generalization algorithm. This problem is first pointed out by [36], who demonstrate a *minimality attack*[2] that (i) can compromise most existing generalization techniques, and (ii) requires only a small amount of knowledge about the generalization algorithm. As a solution, they propose an anonymization approach that can guard against minimality attacks.

The work by Wong et al. reveals an essential issue in publishing microdata: a generalization method should preserve privacy, even against adversaries with knowledge of the anonymization algorithm. Towards addressing this issue, the techniques in [36] establish the first step by dealing with minimality attacks, which, however, is still insufficient for privacy protection. Specifically, given information about the anonymization method, an adversary can easily devise other types of attacks to circumvent a generalized table. To explain this, in the following we first clarify how minimality attacks work, and then, elaborate the deficiencies of [36].

Minimality Attacks. Good generalization should keep the QI values as accurate as possible. Towards this objective, the previous algorithms [6, 12, 14, 19, 20, 40] produce *minimal generalizations*, where no QI-group can be divided into smaller groups without violating the underlying privacy principle. For example, Table 3 is a minimal 2-diverse generalization of Table 1 under global recoding. In particular, the first (second) QI-group in Table 3 cannot be divided, since any split of the group results in two QI-groups with a single tuple, which apparently cannot be 2-diverse. On the other hand, as Fred, Gill, and Hera have identical QI values, their tuples must be in the same QI-group, as demanded by global recoding. Therefore, the only way to partition the third QI-group is to break it into {Ed} and {Fred, Gill, Hera}, which also violate 2-diversity.

Minimal generalizations can lead to severe privacy breach. Consider that a hospital holds the microdata in Table 4, and releases the 2-diverse Table 5, which is a minimal generalization under global recoding. Assume that an adversary has access to the voter registration list in Table 2. Then, s/he can easily identify the six individuals in the second QI-group $G_2 = \{$Cate, Don, Ed, Fred, Gill, Hera$\}$ in Table 5. After that, the adversary can infer the diseases of Cate and Don by reasoning as follows (i.e., a minimality attack). First, there exist only two tuples in G_2 with the same disease, which is *gastritis*. Second, since Table 5 is minimal, if we split G_2 into two parts $G_3 = \{$Cate, Don$\}$ and $G_4 = \{$Ed, Fred, Gill, Hera$\}$, either G_3 or G_4 must violate 2-diversity. Assume that G_4 is not 2-diverse. In that case, at least three tuples in G_4 should have an identical sensitive value, contradicting the fact that, in G_2, the maximum number of tuples with the same *Disease* value is 2. It follows that G_3 cannot be 2-diverse, indicating that both Cate and Don have the same disease, which must be *gastritis* (as mentioned earlier, no other disease is possessed by two tuples in G_2).

Motivation. [36] advance the other solutions by assuming that an adversary has one extra piece of knowledge: *whether* the anonymization algorithm produces a minimal generalization (note: the adversary is not allowed to have other details of the algorithm). Under this assumption, minimality attacks can be prevented using a simple solution — just deploy non-minimal generalizations. Nevertheless, given knowledge of the algorithm, can the adversary employ other types of attacks to compromise non-minimal generalizations? The answer, unfortunately, is positive, as can be demonstrated in a simple example as follows.

[2]Note that minimality attack can be effective only when the microdata is anonymized with generalization or a similar methodology called *anatomy* [38]. There exist other anonymization methods that are immune to attacks based on knowledge of the anonymization algorithm, as will be discussed in Section 5.

Name	Age	Zipcode	Disease
Ann	21	10000	dyspepsia
Bob	27	18000	flu
Cate	32	35000	gastritis
Don	32	35000	gastritis
Ed	54	60000	bronchitis
Fred	60	63000	flu
Gill	60	63000	dyspepsia
Hera	60	63000	diabetes

Table 4: Microdata T_3

Age	Zipcode	Disease
[21, 27]	[10k, 18k]	dyspepsia
[21, 27]	[10k, 18k]	flu
[32, 60]	[35k, 63k]	gastritis
[32, 60]	[35k, 63k]	gastritis
[32, 60]	[35k, 63k]	bronchitis
[32, 60]	[35k, 63k]	flu
[32, 60]	[35k, 63k]	dyspepsia
[32, 60]	[35k, 63k]	diabetes

Table 5: Generalization T_4^*

Algorithm *Vul-Gen* (T)
1. if T is the microdata T_1 in Table 1
   return the generalization T_4^* in Table 5
2. otherwise, return a generalization of T that is different from T_4^*

Figure 1: The *Vul-Gen* algorithm

Example 1 Consider the conceptual anonymization algorithm *Vul-Gen* in Figure 1. The algorithm takes as input a microdata table T, and generates a generalization T^* of T. In particular, *Vul-Gen* outputs the generalization T_4^* in Table 5, if and only if T equals the microdata T_1 in Table 1. Notice that, T_4^* is not a minimal 2-diverse version of T_1. This is because, the second QI-group of T_4^*, including the tuples {Cate, Don, Ed, Fred, Gill, Hera}, can be divided into 2-diverse QI-groups {Cate, Don} and {Ed, Fred, Gill, Hera}, which conform to global recoding.

Assume that a data publisher applies *Vul-Gen* on T_1, and releases the resulting 2-diverse generalization T_4^*. Since T_4^* is not minimal, it does not suffer from minimality attacks. However, imagine an adversary who knows that *Vul-Gen* is the generalization algorithm adopted by the publisher. Once T_4^* is released, the adversary immediately concludes that T_1 is the microdata, because *Vul-Gen* outputs T_4^* if and only if the input is T_1. Hence, the adversary learns the exact disease of every individual, i.e., releasing T_4^* causes a severe privacy breach. □

It is clear from the above discussion that preventing minimality attacks alone is insufficient for privacy preservation, since an adversary (with understanding about the generalization algorithm) may employ numerous other types of attacks to infer sensitive information. This leads to a challenging problem: how can we anonymize the microdata in a way that proactively prevents *all* privacy attacks that may be launched based on *knowledge of the algorithm*?

[42] present the first theoretical study on the above problem. The core of their solution is a privacy model in which the anonymization algorithm (adopted by the publisher) is assumed to be public knowledge[3]. As will be discussed in Section 2.3, however, Zhang et al.'s privacy model is only applicable on a small subset of anonymization algorithms that (i) are deterministic, (ii) adopt global recoding generalization, and (iii) follow a particular algorithmic framework. This severely restricts the design of new anonymization approaches under the model, and makes it impossible to verify the privacy guarantees of existing randomized or local-recoding-based algorithms. Furthermore,

[3]This is reminiscent of Kerckhoffs' principle (well adopted in cryptography): a cryptographic system should be secure, even if everything about the system, except the key, is public knowledge.

the anonymization algorithms proposed by Zhang et al. have high time complexities: All but one algorithm run in time exponential in the number n of tuples in the microdata, while the remaining one has a time complexity that is polynomial in n and the total number m of possible generalizations of the microdata. Note that, in practice, m can be an exponential of n, since there may exist an exponential number of ways to divide the tuples in the microdata into QI-groups. As a consequence, the algorithms developed by Zhang et al. are rather inapplicable in practice.

Contributions. This paper develops a practical solution for data publishing against an adversary who knows the anonymization algorithm. First, we propose a model for evaluating the degree of privacy protection achieved by an anonymized table, assuming that the adversary has knowledge of (i) the anonymization algorithm employed by the publisher, (ii) the algorithmic parameters with which the anonymized table is computed, and (iii) the QI values of all individuals in the microdata. Our model captures all deterministic and randomized generalization algorithms [1, 6, 12, 14, 19, 20, 21, 15, 35, 37, 40, 41, 36, 42], regardless of whether they adopt global recoding or local recoding. The model is even applicable for anonymized tables produced from *anatomy* [38], a popular anonymization methodology that will be clarified in Section 2.1. Based on this model, we develop a new privacy principle called *transparent l-diversity*, which safeguards privacy against the adversary we consider.

As a second step, we identify two sufficient conditions for transparent l-diversity, based on which we propose three anonymization algorithms that achieve transparent l-diversity. None of these algorithms could have been possible under Zhang et al.'s privacy model, as they are either randomized or based on local recoding. We provide detailed analysis on the characteristics of each algorithm, and show that they all run in $O(n^2 \log n)$ time. In addition, we demonstrate the effectiveness and efficiency of our algorithms through extensive experiments with real data. Compared with the existing anonymization techniques that do not ensure transparent l-diversity, our solutions not only provide stronger privacy protection, but also achieve satisfactory performance in terms of data utility and computation overhead.

The rest of the paper is organized as follows. Section 2 presents the theoretical framework that underlies transparent l-diversity. Section 3 presents our generalization algorithms, which are experimentally evaluated in Section 4. Section 5 surveys the previous work related to ours. Finally, Section 6 concludes the paper with directions for future research.

2 Privacy Model

This section presents our analytical model for assessing disclosure risks. In Section 2.1, we formalize several basic concepts. After that, Section 2.2 elaborates the derivation of disclosure risks. Section 2.3 discusses the differences between our model and the methods in [36] and [42].

2.1 Preliminaries

Let T be a microdata table to be published. We assume that T contains $d + 2$ attributes, namely, (i) an identifier attribute A^{id}, which is the primary key of T, (ii) a sensitive attribute A^s, and (iii) d QI attributes A^q_1, ..., A^q_d. As in most existing work, we require that A^s should be categorical, while

Age	Zipcode	Group ID
21	10000	1
27	18000	1
32	35000	2
32	35000	2
54	60000	3
60	63000	3
60	63000	3
60	63000	3

(a) The QI table

Group ID	Disease
1	dyspepsia
1	flu
2	bronchitis
2	gastritis
3	diabetes
3	dyspepsia
3	flu
3	gastritis

(b) The sensitive table

Table 6: An anonymization of Table 1 produced from anatomy

the other attributes can be either numerical or categorical.

For each tuple t in T, let $t[A]$ be the value of t on the attribute A. We define a *QI-group* as a set of tuples, and a *partition* of T as a set of disjoint QI-groups of T whose union equals T. We say that two QI-groups G_1 and G_2 are *isomorphic*, if (i) G_1 and G_2 contain the same multi-set of sensitive values, and (ii) every tuple $t_1 \in G_1$ shares the same identifier and QI values with a tuple $t_2 \in G_2$, and vice versa. For instance, let G_1 be a QI-group that contains the first two tuples in Table 1. Suppose that we swap the sensitive values of Ann and Bob, such that Ann (Bob) has *flu* (*dyspepsia*). Then, the resulting QI-group G_2 is isomorphic to G_1.

We formalize the *anonymization* of T as follows.

Definition 1 (Anonymization) *An* **anonymization function** *f is a function that maps a QI-group to another set of tuples, such that for any two isomorphic QI-groups G_1 and G_2, $f(G_1) = f(G_2)$ always holds. Given a partition P of T and an anonymization function f, a table T^* is an* **anonymization** *of T decided by P and f, if and only if $T^* = \bigcup_{G \in P} f(G)$.*

There exist two popular types of anonymization methodologies, namely, generalization [31] and anatomy [38]. Specifically, generalization employs an anonymization function that maps a QI-group G to a set G^* of tuples, such that (i) for any tuple $t^* \in G^*$, $t^*[A_i^q]$ ($i \in [1, d]$) is an interval containing all A_i^q values in G, and (ii) any two tuples in G^* have the same QI values. Anatomy, on the other hand, adopts an anonymization function that transforms a QI-group G to two separate sets of tuples, such that first (second) set contains only the QI (sensitive) values in G. For example, given a partition of Table 1 that contains three QI-groups {Ann, Bob}, {Cate, Don}, and {Ed, Fred, Gill, Hera}, Table 6 illustrates an anonymization of Table 1 produced from anatomy. Observe that Table 6a (6b) contains only the QI (sensitive) values in Table 1.

The techniques developed in this paper can be incorporated with any anonymization method that conforms to Definition 1. For ease of exposition, in the rest of the paper we will adopt a specific anonymization function, namely, the *MBR (Minimum Bounding Rectangle)* generalization function [6, 14, 20, 40]. This function anonymizes a QI-group G by replacing each A_i^q ($i \in [1, d]$) value with the tightest interval that contains all A_i^q values in G. For instance, Table 3 is obtained by applying the MBR function to a partition of Table 1 with three QI-groups {Ann, Bob}, {Cate, Don}, and {Ed, Fred, Gill, Hera}.

Let T^* be the anonymization of T released by the publisher. T^* should satisfy *l-diversity*:

Definition 2 (l-Diversity [23]) *A QI-group G is* **l-diverse***, if and only if it contains at most $|G|/l$ tuples with the same sensitive value. A partition is l-diverse, if and only if each of its QI-groups is l-diverse. An anonymization is l-diverse, if and only if it is produced from an l-diverse partition.*

It is noteworthy that there exist several different definitions of l-diversity [23]. For example, *entropy l-diversity* requires that the entropy of sensitive values in each QI-group should be at least $\ln l$; *recursive (c, l)-diversity* demands that, even if we remove $l - 2$ arbitrary sensitive values in a QI-group G, at most c fraction of the remaining tuples should have the same sensitive value. Definition 2 corresponds to a simplified version of recursive (c, l)-diversity, and has been widely adopted previously [14, 38, 37, 36].

Let $\mathcal{G}$ be the anonymization algorithm adopted by the publisher. $\mathcal{G}$ can be either deterministic or randomized, but it should be an *l-diversity algorithm*. That is, $\mathcal{G}$ should take as input any microdata T' and any positive integer l, and output either $\emptyset$ or an l-diverse anonymization of T'. In particular, $\mathcal{G}$ may return $\emptyset$, when no l-diverse anonymization exists for T'. For instance, given the microdata T_1 in Table 1, no algorithm can generate a 10-diverse anonymization, since T_1 contains only 8 tuples.

Consider an adversary who tries to infer sensitive information from T^*. As demonstrated in Section 1, the adversary may employ an *external source* (e.g., a voter registration list) to identify the individuals involved in T^*. More formally, we define an external source E as a table that contains all attributes in T, except A^s. In addition, for each tuple $t \in T$, there should exist a unique record $e \in E$, such that t and e coincide on all identifier and QI attributes. In other words, each individual in T should appear in E, but not necessarily vice versa. For example, the external source E_1 in Table 2 involves all individuals in the microdata T_1 in Table 1, but it also contains the information of *Bruce*, who does not appear in T_1.

In addition to E and T^*, we also assume that the adversary knows the details of the anonymization algorithm $\mathcal{G}$ and the value of l used by the publisher (in practice, l can be inferred from T^* [36]). We quantify the disclosure risks incurred by the publication of T^* as:

Definition 3 (Disclosure Risk) *For any individual o, the* **disclosure risk** *$risk(o)$ of o in T^* is the tight upper-bound of the adversary's posterior belief in the event that "o appears in T and has a sensitive value v", given T^*, any sensitive value v, the external source E, the algorithm $\mathcal{G}$, and the value of l:*

$$risk(o) = \max_{v \in A^s} Pr\big\{o \text{ appears in } T \text{ and has a sensitive value } v \mid T^* \wedge E \wedge \mathcal{G} \wedge l\big\}, \tag{1}$$

where $Pr\{X \mid Y\}$ denotes the conditional probability of event X given the occurrence of event Y.

2.2 Disclosure Risks in Anonymized Tables

Next, we present a detailed analysis of disclosure risks. Before examining T^*, the adversary has no information about (i) which individuals in the external source E appear in T, and (ii) what is the sensitive value of each person. Thus, from the adversary's perspective, there exist many *possible instances* of the microdata. In particular, each instance $\hat{T}$ may involve any individuals in E, and each person in $\hat{T}$ can have an arbitrary sensitive value. We formally define such instances as:

Name	Age	Zipcode	Disease
Bruce	29	19000	bronchitis
Cate	32	35000	flu
Fred	60	63000	dyspepsia

Table 7: A Possible Microdata Instance Based on Table 2

Algorithm *Opt-Gen* (T, l)
1. S_p = a set containing all partitions P of T, such that P and the MBR function decide an l-diverse global recoding generalization
2. if $S_p = \emptyset$ then return $\emptyset$
3. among all $P \in S_p$, select the one that minimizes $\sum_{G \in P} |G|^2$
4. return the generalization determined by P and the MBR function

Figure 2: The *Opt-Gen* algorithm

Definition 4 (Possible Microdata Instance) *Given an external source E, a* **possible microdata instance** *based on E is a microdata table $\hat{T}$ that contains a subset of the individuals in E, such that each of these individuals have the same QI values in E and $\hat{T}$ (the sensitive value of each individual in $\hat{T}$ can be arbitrary).*

For example, given the external source in Table 2, Table 7 is a possible microdata instance. Note that, the microdata T itself is also a possible instance. In general, possible instances may be completely different from T, e.g., Table 1 and Table 7 do not even have the same cardinality. Nevertheless, it is reasonable to assume that, before inspecting T^*, the adversary considers each possible instance to be equally likely. This assumption is referred to as the *random worlds assumption* [5], and is adopted by most existing work on data anonymization[4] [7, 8, 18, 21, 22, 25, 27, 34, 37, 36, 39, 40, 43, 42].

Let S be the set of all possible microdata instances based on E. Now, consider that the adversary has obtained T^*, the anonymization algorithm $\mathcal{G}$, and the parameter l. For simplicity, assume for the moment that $\mathcal{G}$ is deterministic. The adversary can utilize the algorithm $\mathcal{G}$ to refine S. Specifically, s/he can apply $\mathcal{G}$ on each instance $\hat{T} \in S$, and inspect the output of $\mathcal{G}$. If $\hat{T}$ leads to an anonymization different from T^*, the adversary asserts that, $\hat{T}$ is not the real microdata T. Let S' be the set of instances that pass the sanity check, i.e., for each $\hat{T} \in S'$, $\mathcal{G}(\hat{T}, l) = T^*$ (apparently, $T \in S'$).

The adversary then uses S' to infer the sensitive information in T. As a special case, if an individual o is associated with an A^s value v in all instances in S', then v must be the A^s value of o in T. In general, the probability that o has v in T depends on the portion of instances in S' where o has v. We refer to the above inference approach as a *reverse engineering attack*.

Example 2 Consider the l-diversity generalization algorithm *Opt-Gen*, as shown in Figure 2. In a nutshell, *Opt-Gen* employs the MBR function, and returns l-diverse generalizations that (i) obey

[4]Recent research [17] shows that, when the random worlds assumption does not hold, some of the existing anonymization methods are vulnerable to privacy attacks based on machine learning techniques. The treatment of such privacy attacks is beyond of the scope of this paper.

global recoding, and (ii) minimize the *discernability metric* [6]. Specifically, the discernability of a generalized table T^* equals $\sum_{G \in P} |G|^2$, where P is the partition that decides T^*.

Suppose that a publisher adopts *Opt-Gen* to anonymize the microdata T_1 in Table 1, setting l to 2. Table 3 illustrates the resulting generalization T_2^*. Assume that an adversary has the external source E_1 in Table 2, and knows *Opt-Gen* and $l = 2$. To launch a reverse engineering attack, s/he first constructs the set S of all possible microdata instances based on E_1 (e.g., Table 7 is one instance in S). As a second step, the adversary invokes *Opt-Gen* on each $\hat{T} \in S$, and verifies whether the output of *Opt-Gen* is T_2^*. Let S' be the maximal subset of S such that *Opt-Gen*$(\hat{T}, 2) = T_2^*$ for each $\hat{T} \in S'$. In the sequel, we will show that every $\hat{T} \in S'$ must associate Ed with *gastritis*. Namely, based on T_2^*, E_1, $l = 2$, and the details of *Opt-Gen*, the adversary can infer the exact disease of Ed.

Let G_1, G_2, and G_3 be the first, second, and third QI-group in T_2^*, respectively. Any $\hat{T} \in S$, which can be generalized to T_2^*, must satisfy the following conditions. First, $\hat{T}$ should not involve Bruce, since his age 29 is not covered by any *Age* interval in T_2^*. Second, $\hat{T}$ should either (i) associate Ann with *dyspepsia* and Bob with *flu*, or (ii) conversely, associate Ann and Bob with *flu* and *dyspepsia*, respectively. This is because, Ann and Bob are the only individuals whose ages fall in the *Age* interval $[21, 27]$ of G_1, while G_1 contains two sensitive values *dyspepsia* and *flu*. By the same reasoning, $\hat{T}$ should assign the diseases in G_2 (G_3) to Cate and Don (Ed, Fred, Gill, and Hera).

We are now ready to prove that, any possible microdata instance in S' must set the sensitive value of Ed to *gastritis*. Assume, on the contrary, that this is not true in a $\hat{T}' \in S'$. Then, since Ed is in G_3, his disease in $\hat{T}'$ must be one of {*flu, diabetes, dyspepsia*}, i.e., the sensitive values in G_3 except *gastritis*. In that case, Ed's disease in $\hat{T}'$ must differ from those of Cate and Don (each of whom suffers from either *gastritis* or *bronchitis* in $\hat{T}'$). Hence, we can construct a 2-diverse QI-group $G_2' =${Cate, Don, Ed}. The other tuples in $\hat{T}'$ can also form two 2-diverse QI-groups $G_1' =$ {Ann, Bob}, and $G_3' =$ {Fred, Gill, Hera}.

Let $P' = \{G_1', G_2', G_3'\}$, which decides a 2-diverse global recoding generalization. Let us refer to that generalization as T'^*. The discernability of T'^* is $2^2 + 3^2 + 3^2 = 22$, which is smaller than the discernability 24 of T_2^*. As *Opt-Gen* minimizes the discernability, given $\hat{T}'$ as the input, it should have output T'^* instead of T_2^*, leading to a contradiction. In conclusion, Ed must be assigned a sensitive value *gastritis* in any $\hat{T} \in S'$. □

The above discussion motivates the following proposition for computing disclosure risks.

Proposition 1 *Let o be any individual, E be an external source, and T^* be an anonymization of T produced with an l-diversity algorithm $\mathcal{G}$ and a parameter l. Let S be the set of possible microdata instances based on E. Let $S_{o,v}$ be the maximal subset of S, such that each instance $\hat{T} \in S_{o,v}$ includes a tuple t, with $t[A^{id}] = o$ and $t[A^s] = v$. Then,*

$$risk(o) = \max_{v \in A^s} \frac{\sum_{\hat{T} \in S_{o,v}} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}}, \quad (2)$$

where $Pr\{\mathcal{G}(\hat{T}, l) = T^*\}$ denotes the probability that, given $\hat{T}$ and l, algorithm $\mathcal{G}$ outputs T^*.

The proofs of all propositions, lemmas, and theorems can be found in the appendix. We are now ready to introduce the *transparent l-diversity* principle, for protecting privacy when the anonymization algorithm is "transparent" to adversaries.

Definition 5 (Transparent l-Diversity) *An anonymization T^* of T is* **transparently l-diverse** *if, given any external source, T^* ensures $risk(o) \leq 1/l$ for any individual o. An l-diversity algorithm $\mathcal{G}$ is* **transparent***, if and only if given any microdata T and any positive integer l, algorithm $\mathcal{G}$ outputs either $\emptyset$ or a transparently l-diverse anonymization of T.*

Intuitively, an l-diversity algorithm $\mathcal{G}$ is transparent, if and only if each output T^* of $\mathcal{G}$ can be generated from a set S of possible microdata instances, such that each individual o is associated with a diverse set of sensitive values in different instances. As the adversary cannot decide which instance in S corresponds to the input microdata, s/he would not be able to infer the exact sensitive value of o from T^*. The fact that each instance in S can lead to T^* implies that the output of $\mathcal{G}$ should not be highly dependent on the sensitive value of any particular individual. For instance, the *Opt-Gen* algorithm fails in Example 2, because it outputs T_2^* (in Table 3) only if Ed has a sensitive value *gastritis*. In general, a transparent algorithm should anonymize data in a manner such that none of the steps of the anonymization process is uniquely decided by the sensitive value of a particular tuple. In Section 3, we will present three transparent algorithms that are developed according to the above principle.

2.3 Comparison with Previous Work

As explained in Section 1, [36] and [42] are the only previous works that do not assume adversaries with no knowledge of the anonymization algorithm $\mathcal{G}$. In this section, we elaborate the solutions in [36] and [42], and point out how they differ from our solution.

Comparison with [36]. The privacy model in [36] assumes that (i) the anonymization algorithm $\mathcal{G}$ is deterministic, and (ii) the adversary knows whether $\mathcal{G}$ produces minimal generalization. To clarify the model, we begin by reviewing several concepts in [36].

Definition 6 (Child Partition) *Let P_1 and P_2 be two partitions of T. P_2 is a* **child** *of P_1, if and only if there exist $G_1 \in P_1$ and $G_2, G_3 \in P_2$, such that (i) $G_1 = G_2 \cup G_3$, and (ii) $P_1 - \{G_1\} = P_2 - \{G_2, G_3\}$.*

Note that we can obtain a child of a partition P, by splitting a QI-group in P into two smaller QI-groups.

Definition 7 (Minimal Generalization) *Let f be a generalization function, P an l-diverse partition, and T^* the generalization decided by f and P. T^* is a* **minimal l-diverse generalization** *under global (local) recoding, if f and any child of P cannot decide an l-diverse generalization under the same recoding.*

For example, Table 3 is a minimal 2-diverse generalization of Table 1 with respect to the MBR function and global recoding, as explained in Section 1. Given a generalization function f and recoding scheme H, we say that an l-diversity algorithm is *minimal*, if it produces only minimal generalizations under f and H. The subsequent discussion will focus on minimal algorithms $\mathcal{G}$, because the results of [36] are inapplicable to non-minimal algorithms (i.e., minimality attacks cannot be performed if $\mathcal{G}$ is non-minimal).

In a similar fashion to Definition 1, [36] formulate the disclosure risks (referred to as *credibilities* in [36]) as:

Definition 8 (Credibility) *Let o be any individual, and V be a predefined subset of the values in A^s. The* **credibility** *of o in T^* is the adversary's maximum posterior belief in the event that "o appears in T and has a sensitive value v", given T^*, an external source E, generalization function f, recoding scheme H, value of l, and $\mathcal{G}$ being minimal:*

$$cred(o) = \max_{v \in V} Pr\{o \text{ } has \text{ } v \text{ } in \text{ } T \mid T^* \wedge E \wedge f \wedge H \wedge l \wedge \mathcal{G} \text{ } is \text{ } minimal\}.$$

Note that the credibility model quantifies disclosure risks based only on a subset V of the A^s values. To facilitate the comparison between the credibility model and our privacy model, we assume $V = A^s$ in the rest of the paper.

Credibilities can be derived as:

Proposition 2 ([36]) *Let o, E, f, H, l be as introduced in Definition 8. Let S^+ be the set including any possible microdata instance $\hat{T}$ based on E, such that T^* is a minimal l-diverse generalization of $\hat{T}$ with respect to f and H. Let $S^+_{o,v}$ be the maximal subset of S^+, such that in each instance in S^+, o is associated with a sensitive value v. We have*

$$cred(o) = \max_{v \in A^s} |S^+_{o,v}|/|S^+|. \tag{3}$$

The following analysis will confirm the intuition that credibilities underestimate the actual privacy risks, when an adversary knows everything about $\mathcal{G}$. Towards this, let us revisit the scenario in Example 2, where the adversary can precisely find out Ed's disease with a reverse engineering attack, i.e, the disclosure risk of Ed equals the maximum value 1. In the sequel, we will show that $cred(\text{Ed}) = 1/4$.

Lemma 1 *The Opt-Gen algorithm (in Figure 2) is a minimal algorithm.*

Example 3 Consider the settings in Example 2, where $T = T_1$, $T^* = T_2^*$, $E = E_1$, $\mathcal{G} = \textit{Opt-Gen}$, $l = 2$, $o =$ Ed. Since *Opt-Gen* is a minimal algorithm (see Lemma 1), by Proposition 2, the credibility of Ed in T_2^* is calculated as $\max_{v \in A^s} |S^+_{o,v}|/|S^+|$, where S^+ is the set of all possible microdata instances that have T_2^* as a minimal generalization, and $S^+_{o,v}$ is the subset of instances in S^+ that associate Ed with a certain sensitive value v.

Let $\hat{T}$ be any possible microdata instance based on E_1. As demonstrated in Example 2, if $\hat{T}$ can be generalized to T_2^*, then $\hat{T}$ must not involve Bruce. Furthermore, $\hat{T}$ should assign the sensitive values in the first, second, and third QI-groups in T_2^* to {Ann, Bob}, {Cate, Don}, and {Ed, Fred, Gill, Hera}, respectively. Totally, there are $2! \times 2! \times 4! = 96$ different combinations between the sensitive values and individuals. This leads to a set S_m of 96 possible microdata instances. For any $v =$

gastritis, *flu*, *dyspepsia*, or *diabetes*, there exist 24 instances in S_m that associate Ed with v. Since S_m includes all possible microdata instances that can be generalized to T_2^*, we have $S^+ \subseteq S_m$.

Next, we will prove $S^+ = S_m$. For this purpose, it suffices to establish that, for any instance $\hat{T} \in S_m$, T_2^* is a minimal 2-diverse generalization with respect to global recoding and the MBR function f. Let $G_1 = \{\text{Ann, Bob}\}$, $G_2 = \{\text{Cate, Don}\}$, $G_3 = \{\text{Ed, Fred, Gill, Hera}\}$. The partition underlying T_2^* is $P_1 = \{G_1, G_2, G_3\}$. Assume, on the contrary, that T_2^* is not minimal for some $\hat{T} \in S_m$. Then, there exists a partition P_2 of $\hat{T}$ such that (i) P_2 is a child of P_1, and (ii) P_2 and f decide a 2-diverse global recoding generalization.

As P_2 is a child of P_1, by Definition 6, we can obtain P_2 from P_1 by splitting G_1, G_2, or G_3. However, it is impossible to split G_1 (G_2) into 2-diverse QI-groups, since it contains only two tuples. On the other hand, G_3 cannot be divided either. This is because, Fred, Gill, and Hera have identical QI values, and thus, have to be in the same QI-group (due to global recoding); meanwhile, the remaining tuple Ed itself does not make a 2-diverse QI-group. Hence, under global recoding, no child of P_1 can lead to a 2-diverse generalization of $\hat{T}$. It follows that T_2^* is a minimal generalization of every $\hat{T} \in S_m$, i.e., $S^+ = S_m$.

Finally, since (as mentioned earlier) there exist exactly 24 instances in S_m that assign the same sensitive value to Ed, $cred(\text{Ed}) = \max_{v \in A^s} |S^+_{o,v}|/|S^+| = 24/96 = 1/4$. □

Since the credibility model cannot secure privacy against an adversary who knows the anonymization algorithm, any method developed based on the model is susceptible to revere engineering attacks. To demonstrate this, we exemplify in Appendix II an attack against *Mask*, an anonymization approach devised in [36] under the credibility model.

Comparison with [42]. [42] consider the publication of microdata using deterministic algorithms that adopt global recoding. They model a global recoding generalization as a projection of the microdata into a "coarsened" multi-dimensional domain. For example, given the microdata T_1 in Table 1, we can coarsen the *Age* domain, so that it contains only seven values: "≤ 20", "$[21, 27]$", "$(27, 32)$", "32", "$(32, 54)$", "$[54, 60]$", and "≥ 60". Similarly, we can define a coarsened *Zipcode* domain that has only seven values: "< 10k", "[10k, 18k]", "(18k, 35k)", "35k", "(35k, 60k)", "[60k, 63k]", and "> 63k". Accordingly, the global recoding generalization T_2^* in Table 3 can be regarded as the projection of T_1 into the three-dimensional domain spanned by *Disease* and the coarsened *Age* and *Zipcode*. Let C be the set of all coarsened multi-dimensional domains that can be constructed from the attributes in the microdata. Zhang et al. assume that the domains in C can be totally ordered by their *information loss*, which measures the degree of coarseness of the domains. For example, the information loss of a domain is (i) minimized if no coarsening is applied, and (ii) maximized if every attribute is maximally coarsened.

Zhang et al. consider that the publisher adopts a deterministic generalization algorithm $\mathcal{G}$ as follows. Given a microdata T and a privacy principle, $\mathcal{G}$ first examines the multi-dimensional domains in C in ascending order of their information loss. For each domain D^*, $\mathcal{G}$ projects T into D^*, and checks whether the resulting generalization satisfies the given privacy principle. If the principle is satisfied, $\mathcal{G}$ returns the generalization and terminates; otherwise, $\mathcal{G}$ moves on to the next domain in C. In other words, $\mathcal{G}$ always outputs the first generalization that conforms to the adopted privacy principle. Alternatively, $\mathcal{G}$ may also traverse C in descending order of information loss, and returns the last generalization on which the given principle is satisfied. The adversary is assumed to (i) have an external source E that contains only the individuals in the microdata, and (ii) know the privacy principle as well as the order in which $\mathcal{G}$ traverses C.

Under the above problem setting, Zhang et al. present a theoretical study on how $\mathcal{G}$ should be designed to prevent the adversary from inferring private information. Let n_p be the total number of possible microdata instances based on E. Zhang et al. first prove that it is NP-hard (with respect to n_p) to compute a generalization that both ensures privacy and incurs the minimum information loss. After that, they investigate three special cases of the problem by imposing various constraints on C and the privacy principle. For each case, they show that the optimal generalization can be computed in time polynomial in n_p and the size of C. Finally, they propose a generalization algorithm that ensures *entropy l-diversity* (see Section 2.1), and prove that its time complexity is polynomial in $|C|$ and independent of n_p. Note that, in practice, both $|C|$ and n_p are usually exponential in the number n of tuples in the microdata.

Compared with the solution in [36], Zhang et al.'s techniques achieve a higher level of privacy protection, as they can guard against an adversary who has full knowledge of the anonymization algorithm. Nevertheless, Zhang et al.'s work has the following limitations. First, the privacy model in [42] is restricted to a particular type of deterministic algorithms that adopt global recoding. Consequently, the model cannot be used to evaluate the privacy guarantee of any existing anonymization algorithm that is randomized or local-recoding-based, nor does it support the development of new anonymization approaches of those kinds. Second, all algorithms proposed in [42] have time complexities exponential in the number n of tuples in the microdata, and there is no experimental evaluation included in [42] to demonstrate the effectiveness or efficiency of the algorithms. This leaves open the question of whether or not the algorithms in [42] are applicable in practice.

Our work remedies the deficiencies of [42]. In particular, our privacy model captures all (deterministic or randomized) anonymization algorithms that adopt generalization or anatomy. This general model enables us to design three transparent anonymization algorithms, all of which fall beyond Zhang et al.'s model as they rely on random choices and/or local recoding. In addition, as will be shown in Section 3, our algorithms run in $O(n^2 \log n)$ time, which significantly improves over the exponential time complexities of Zhang et al.'s techniques. Finally, we will present in Section 4 an extensive experimental study that demonstrates the practical performance of our algorithms in terms of data utility and computation time.

3 Achieving Transparent l-Diversity

Equipped with the analytical model in Section 2, our next step is to develop transparent anonymization algorithms for l-diversity. Ideally, an algorithm should produce anonymizations with minimum information loss, according to a certain *penalty metric* h. Specifically, h is a function that, given a QI-group G, calculates a penalty $h(G)$ based on the tuples in G. Given h, the information loss of an anonymization T^* is computed as $\sum_{G \in P} h(G)$, where P is the partition underlying T^*. For example, the discernability metric deployed in Example 2 corresponds to a function h_d such that $h_d(G) = |G|^2$ for any QI-group G.

In the following, we will elaborate three transparent algorithms, each of which can be combined with any penalty metric h, as long as the metric (i) does not rely on the sensitive values in the input QI-group, and (ii) is *superadditive*, i.e., $h(G_1 \cup G_2) \geq h(G_1) + h(G_2)$ holds for any disjoint QI-groups

G_1 and G_2. For our discussion, we use the *perimeter function* h_p [14, 15] as a representative:

$$h_p(G) = |G| \cdot \sum_{i=1}^{d} \frac{\max_{t \in G} \{t[A_i^q]\} - \min_{t \in G} \{t[A_i^q]\}}{\max \{A_i^q\} - \min \{A_i^q\}}. \tag{4}$$

Given a set S_G of QI-groups, we refer to $\sum_{G \in S_G} h_p(G)$ as the *perimeter* of S_G.

3.1 The *Tailor* Algorithm

3.1.1 Algorithm Description

This section presents a transparent algorithm, *Tailor*, which produces anonymized tables in a manner similar to the construction of kd-trees [11]. *Tailor* requires the microdata T to be *l-eligible*. That is, at most $|T|/l$ tuples in T have the same sensitive value. If T is not l-eligible, *Tailor* returns $\emptyset$, since no l-diverse anonymization of T exists [23].

Given an l-eligible T, *Tailor* first creates a partition P with only one QI-group G_0, which includes all tuples in T. As a second step, *Tailor* tries to split G_0 into two l-diverse subsets G_1 and G_2 subject to certain constraints to be clarified later. If splitting is possible, *Tailor* removes G_0 from P, and inserts G_1 and G_2 in P. This decreases the perimeter of P. After that, *Tailor* recursively splits a QI-group in P, until no QI-group can be divided further, i.e., the perimeter of P has reached a local minimum. Then, *Tailor* terminates, and outputs the anonymization decided by P and an anonymization function (e.g., the MBR function).

Whenever *Tailor* divides a QI-group G into subsets G_a and G_b, $\{G_a, G_b\}$ must be an *l-cut*:

Definition 9 (l-Cut) *Let G be a QI-group, l be a positive integer, and c be the maximum number of tuples in G with the same sensitive value. An* **l-cut** *of G on A_i^q ($i \in [1, d]$) is an ordered set $\{G_a, G_b\}$ of QI-groups, such that:*

1. *$G_a \cup G_b = G$, and $G_a \cap G_b = \emptyset$.*
2. *$|G_a| \geq l \cdot c$ and $|G_b| \geq l \cdot c$.*
3. *For any $t_a \in G_a$ and $t_b \in G_b$, either (i) $t_a[A_i^q] < t_b[A_i^q]$, or (ii) $t_a[A_i^q] = t_b[A_i^q]$ and $t_a[A^{id}] < t_b[A^{id}]$.*

The **perimeter** *of the l-cut is the total perimeter of G_a and G_b.*

Condition 2 in Definition 9 implies that G (on which the l-cut is performed) is $2l$-diverse. Condition 3 requires, intuitively, that all tuples in G_a must precede those in G_b, along the dimension A_i^q on which G is divided.

Interestingly, *as long as G is $2l$-diverse, there exists at least one l-cut on any QI-attribute A_i^q ($i \in [1, d]$).* Such a cut can be found as follows. First, we sort the tuples in G in ascending order of their A_i^q values. In case two tuples have the same value on A_i^q, the tuple with a smaller identifier precedes the other. Then, we create G_a by including the first k tuples in the sorted sequence (for any $k \in [l \cdot c, |G| - l \cdot c]$), and construct G_b using the remaining tuples.

Algorithm *Tailor* (T, l)
1. if T is not l-eligible then return $\emptyset$
2. G_0 = a QI-group containing all tuples in T, and $P = \{G_0\}$
3. while there exists a $2l$-diverse QI-group G in P
4. $\quad \{G_a, G_b\}$ = the canonical l-cut of G
5. $\quad P = P - \{G\} + \{G_a, G_b\}$
6. return the anonymization decided by P and an anonymization fucntion

Figure 3: The *Tailor* algorithm

The above strategy yields totally $d \cdot (|G| + 1 - 2l \cdot c)$ different l-cuts. Among them, *Tailor* always selects the *canonical* one:

Definition 10 (Canonical l-Cut) *The* **canonical l-cut** *of a QI-group G is the l-cut with the smallest perimeter. In case multiple l-cuts have the smallest parameter, the canonical l-cut $\{G_a, G_b\}$ is uniquely decided as follows. Assume $\{G_a, G_b\}$ is on dimension A_i^q ($i \in [1, d]$); then:*

1. *No l-cut on any A_j^q ($j < i$) has the same perimeter as $\{G_a, G_b\}$.*
2. *For any l-cut $\{G'_a, G'_b\}$ on A_i^q, if $\{G'_a, G'_b\}$ and $\{G_a, G_b\}$ have the same perimeter, it must hold that $|G_a| < |G'_a|$.*

Note that the canonical l-cut of a QI-group G is determined by (i) the identifiers and QI values in G, as well as (ii) the maximum number c of tuples in G with the same sensitive value – all of this information is independent of the concrete sensitive value of any particular tuple. This property is the key to ensuring transparent l-diversity, as will be discussed in Section 3.1.2.

Figure 3 shows the pseudo-code of *Tailor*. We demonstrate the algorithm with an example, assuming that the MBR function is adopted.

Example 4 Let us use *Tailor* to obtain a transparently 2-diverse generalization of the microdata T_5 in Table 8 (i.e., $T = T_5$ and $l = 2$). *Tailor* first verifies that T_5 is 2-eligible (Line 1 in Figure 3), and then initializes a partition $P = \{G_0\}$, where $G_0 = T_5$ (Line 2). The subsequent execution of *Tailor* is in iterations (Lines 3-5). In each iteration, *Tailor* looks for a 4 ($= 2l$) diverse QI-group G in P (Line 3). If G does not exist, *Tailor* terminates, and returns the generalization decided by P (Line 6). Otherwise, *Tailor* splits G using its canonical l-cut (Lines 4-5), and replaces G with the new QI-groups.

Specifically, in the first iteration, the only QI-group G_0 in P is 4-diverse, and hence, is chosen to be split. *Tailor* identifies $c = 2$, which, as in Definition 9, is the largest number of tuples in G_0 having the same sensitive value. Then, *Tailor* proceeds to find the canonical 2-cut of G_0. For this purpose, it needs to obtain the best 2-cut (with the smallest perimeter) along every dimension. Dealing with *Age* first, *Tailor* sorts the tuples in G_0 by their *Age* values, and tries all possibilities of dividing the sorted list into two parts, each with at least 4 ($= 2c$) tuples (required by condition 2 in Definition 9). There is only possibility: $\{G_2, G_3\}$, where G_2 = {Ann, Bob, Cate, Don}, and G_3 = {Ed, Fred, Gill, Hera}. Hence, $\{G_2, G_3\}$ is the best 2-cut on *Age*. Switching to dimension *Zipcode*, *Tailor* sorts the tuples in G_0 by their *Zipcode* values, and again, attempts all division possibilities. Again, $\{G_2, G_3\}$

Name	Age	Zipcode	Disease
Ann	21	10000	dyspepsia
Bob	27	18000	flu
Cate	32	35000	gastritis
Don	32	35000	gastritis
Ed	54	60000	flu
Fred	60	63000	bronchitis
Gill	60	63000	dyspepsia
Hera	60	63000	diabetes

Table 8: Microdata T_5

Age	Zipcode	Disease
[21, 32]	[10k, 35k]	dyspepsia
[21, 32]	[10k, 35k]	flu
[21, 32]	[10k, 35k]	gastritis
[21, 32]	[10k, 35k]	gastritis
[54, 60]	[60k, 63k]	flu
[54, 60]	[60k, 63k]	bronchitis
60	63000	dyspepsia
60	63000	diabetes

Table 9: Generalization T_6^*

is the only possibility, and hence, is also the best 2-cut on *Zipcode*. Hence, $\{G_2, G_3\}$ is the canonical 2-cut. *Tailor* thus replaces G_0 with G_2 and G_3 in P.

In the second iteration, $P = \{G_2, G_3\}$. As G_2 is not 4-diverse, it cannot be split. But G_3 is 4-diverse, and thus, is split using its canonical cut $\{G_4, G_5\}$, where $G_4 = \{\text{Ed, Fred}\}$ and $G_5 = \{\text{Gill, Hera}\}$. Now, P becomes $\{G_2, G_4, G_5\}$. Since no QI-group is 4-diverse, *Tailor* returns the generalization T_6^* determined by P, as shown in Table 9. □

Tailor is deterministic, i.e., for any T, l, and T^*, $Pr\{Tailor(T, l) = T^*\}$ (see Proposition 1) equals either 0 or 1. In addition, *Tailor* has an $O(n^2 \log n)$ time complexity, where n is the number of tuples in T. This follows from the facts that (i) *Tailor* performs at most n/l l-cuts on T, and (ii) each l-cut takes $O(n \log n)$ time.

3.1.2 Proof of Transparent l-Diversity

In this section, we will prove that *Tailor* ensures transparent l-diversity. The core of our proof is an analysis on the set S of all possible microdata instances based on the adversary's external source E. We will show that S can be divided into several subsets, such that for each subset S_{sub}, (i) all instances in S_{sub} can be transformed to the same anonymization T^* by *Tailor*, and (ii) each individual in E is assigned many different sensitive values in different instances in S_{sub}. Intuitively, when the adversary observes T^*, s/he would not be able to infer which instance in S_{sub} is the real microdata, and hence, the sensitive value of each individual can be concealed.

More specifically, our analysis exploits the *isomorphism* between partitions. We say that a partition P_1 of a possible microdata instance is isomorphic to a partition P_2 of another instance, if and only if each QI-group in P_1 is isomorphic to a QI-group in P_2, and vice versa (see Section 2.1 for the definition of QI-group isomorphism).

Example 5 Consider the partition P of T_5 (in Table 8) generated by *Tailor* in Example 4. P contains three QI-groups, namely, $G_2 = \{\text{Ann, Bob, Cate, Don}\}$, $G_4 = \{\text{Ed, Fred}\}$, and $G_5 = \{\text{Gill, Hera}\}$. The sensitive values of Ed and Fred are *flu* and *bronchitis*, respectively. Suppose that we modify the two tuples in G_4 by swapping their *Disease* values, such that Ed has *bronchitis* and Fred has *flu*. The resulting QI-group G_4' is isomorphic to G_4, while the partition $P' = \{G_2, G_4', G_5\}$ isomorphic to P. Note that P' is *not* a partition of T_5, but is in fact a partition of the microdata T_3 in Table 4 (this will be useful in demonstrating Lemma 3 later). □

Recall that, for any anonymization function f and any two isomorphic QI-groups G_1 and G_2, we have $f(G_1) = f(G_2)$ (see Definition 1). Therefore, once f is fixed, isomorphic partitions always lead to the same anonymization. For instance, consider the partitions P and P' in Example 5. Notice that, P' and the MBR function decide T_6^* (in Table 9), which is determined by P and the MBR function as well. In addition, isomorphic QI-groups have a crucial property:

Lemma 2 *Let G and G' be two isomorphic QI-groups, and $\{G_1, G_2\}$ ($\{G'_1, G'_2\}$) be the canonical l-cut of G (G'). Then, G_1 and G'_1 (G_2 and G'_2) must involve the same set of individuals.*

The above lemma is fairly intuitive. Recall that, the canonical l-cut of a QI-group G depends *only* on the identifiers and QI values in G, and is independent of the sensitive values. Since isomorphic QI-groups contain equivalent identifiers and QI values, their canonical l-cuts divide them in the same way, and thus Lemma 2 holds. Based on Lemma 2, we derive the following result, which shows an important characteristic of *Tailor*.

Lemma 3 *Let T_1 be a microdata table, l be an integer, and $T^* = Tailor(T_1, l)$. Let P_1 be the partition of T_1 that decides T^*, P_2 be a partition isomorphic to P_1, and $T_2 = \bigcup_{G \in P_2} G$. Then, $Tailor(T_2, l) = T^*$, and P_2 is the partition of T_2 that decides T^*.*

For instance, consider the microdata T_3, T_5 and the partitions P, P' in Example 5. We have shown in Example 4 that $Tailor(T_5, 2) = T_6^*$, where T_6^* is decided by P. Recall that P' is isomorphic to P, and $T_3 = \bigcup_{G \in P'} G$. According to Lemma 3, we have $Tailor(T_3, 2) = T_6^*$, i.e., given $l = 2$, *Tailor* transforms both T_3 and T_5 into T_6^*.

The following theorem shows a sufficient condition for transparent l-diversity.

Theorem 1 *An l-diversity algorithm $\mathcal{G}$ is transparent if it satisfies the following condition: For any microdata T_1 such that $\mathcal{G}(T_1, l) = T^*$, we have $\mathcal{G}(T_2, l) = T^*$ for a microdata table T_2, if T_2 has a partition isomorphic to the partition of T_1 that decides T^*.*

By Lemma 3, *Tailor* satisfies the sufficient condition in Theorem 1, which proves that *Tailor* is a transparent algorithm.

3.2 The *Ace* Algorithm

This section discusses another algorithm, *Ace* (assign and slice), which first appeared in [40] as part of a solution to anonymizing dynamic datasets. Here, we present non-trivial proofs on the privacy guarantee of *Ace* against adversaries who have full knowledge of the algorithm.

3.2.1 Algorithm Description

Let us first introduce several concepts. Given a QI-group B, we define the *signature* of B as the set of sensitive values in B. A *column* of B refers to a maximal set of tuples in B with the same

Ann	Bob
Gill	Ed
dyspepsia	*flu*

B_1

Don	Fred
gastritis	*bronchitis*

B_2

Hera	Cate
diabetes	*gastritis*

B_3

Figure 4: Bucket Partition U_1

sensitive value. B is a *bucket*, if all of its columns contain an equal number of tuples. A partition U is a *bucket partition*, if each QI-group in U is a bucket.

For example, consider a QI-group B_1 of the microdata T_5 in Table 8, where $B_1 = \{$Ann, Bob, Ed, Gill$\}$. The signature of B_1 is $\{$*dyspepsia*, *flu*$\}$. B_1 contains two columns, $L_1 = \{$Ann, Gill$\}$ and $L_2 = \{$Bob, Ed$\}$, where all tuples in L_1 (L_2) have sensitive value *dyspepsia* (*flu*). Since $|L_1| = |L_2|$, B_1 is a bucket. Let B_2 and B_3 be another two QI-groups of T_5, such that $B_2 = \{$Don, Fred$\}$ and $B_3 = \{$Cate, Hera$\}$. It can be verified that, B_2 and B_3 are also buckets. Therefore, the partition $U_1 = \{B_1, B_2, B_3\}$ is a bucket partition of T_5. Figure 4 illustrates U_1.

Apparently, U_1 is 2-diverse. Suppose that we divide B_1 into two smaller buckets, $B_4 = \{$Ann, Bob$\}$ and $B_5 = \{$Gill, Ed$\}$, both having the same signature as B_1. The partition $U_1' = \{B_2, B_3, B_4, B_5\}$ is also 2-diverse, and has a lower perimeter than U_1. In general, given any l-diverse bucket partition U, we may reduce its perimeter by splitting the buckets in U, without violating l-diversity. This strategy is adopted by *Ace*. In particular, whenever *Ace* splits a bucket B, the resulting sub-buckets always constitute a *division* of B, as defined below:

Definition 11 (Division) *A* **division** *of a bucket B on A_i^q ($i \in [1, d]$) is an ordered set $\{B_a, B_b\}$ of buckets, such that:*

1. *$B_a \cup B_b = B$, and $B_a \cap B_b = \emptyset$.*
2. *B, B_a and B_b have an identical signature.*
3. *For any two tuples $t_a \in B_a$ and $t_b \in B_b$ with the same sensitive value, we have either (i) $t_a[A_i^q] < t_b[A_i^q]$, or (ii) $t_a[A_i^q] = t_b[A_i^q]$ and $t_a[A^{id}] < t_b[A^{id}]$.*

The **perimeter** *of the division equals the perimeter of $\{B_a, B_b\}$. A bucket is* **divisible***, if each of its columns has at least two tuples.*

Given a bucket B with x columns, we can obtain a division $\{B_a, B_b\}$ of B on A_i^q ($i \in [1, d]$) as follows. First, we sort the tuples in each column of B in ascending order of their A_i^q values. Whenever two tuples have an identical value on A_i^q, the tuple with a smaller identifier precedes the other. This results in x sorted sequences. To construct B_a, we can remove an equal number of tuples from the top of each sequence, and insert them into B_a. After that, B_b can be formed using the remaining tuples.

A bucket may have multiple divisions. In a way similar to canonical l-cuts, we formulate *canonical division* as:

Definition 12 (Canonical Division) *The* **canonical division** *of a bucket B is the division with the smallest perimeter. In case multiple divisions have the smallest perimeter, the canonical division $\{B_a, B_b\}$ is uniquely decided as follows. Assume $\{B_a, B_b\}$ is on dimension A_i^q ($i \in [1, d]$); then:*

Algorithm *Ace* (T, l)
1. if T is not l-eligible then return $\emptyset$
2. $U = Assign(T, l)$
3. $U' = Slice(U)$
4. return the generalization decided by U' and an anonymization function

Figure 5: The *Ace* algorithm

1. *No division on any A_j^q $(j < i)$ has the same perimeter as $\{B_a, B_b\}$.*
2. *For any division $\{B'_a, B'_b\}$ on A_i^q, if $\{B_a, B_b\}$ and $\{B'_a, B'_b\}$ have the same perimeter, it must hold that $|B_a| < |B'_a|$.*

As with canonical l-cuts, the canonical division of a bucket B is irrelevant to the sensitive values in B. Instead, it is decided only by the identifiers and QI values in each column. In Section 3.2.2, we will exploit this property to prove that *Ace* is transparent.

Figure 5 illustrates the pseudo-code of *Ace*. Given a microdata T and a positive integer l, *Ace* first verifies whether T is l-eligible. After that, it invokes a subroutine *Assign* (in Figure 6) to construct an l-diverse bucket partition U of T. Next, *Ace* employs the *Slice* algorithm (in Figure 8) to split the buckets in U, and obtains a refined partition U' of T. In particular, the construction of U is performed without inspecting the QI values of the tuples, while the split of each bucket in U is based on canonical divisions, which are independent of the sensitive value in each column of the bucket. In other words, *Assign* and *Slice* do not rely on the correlations between the QI and sensitive values, which helps achieve transparent l-diversity. Finally, *Ace* returns the generalization decided by U'. In the following, we explain the details of *Ace* with an example, assuming that the MBR function is adopted.

Example 6 Assume that we apply *Ace* on the microdata T_5 in Table 8, with $l = 2$. *Ace* begins by checking whether T_5 is l-eligible. Since T_5 is 2-eligible, *Ace* invokes *Assign* to construct a bucket partition U of T_5.

Assign first sets $U = \emptyset$, and creates a set S_t containing all tuples in T_5 (Lines 1-3 in Figure 6). After that, *Assign* iteratively removes tuples from S_t to construct buckets in U, until S_t is empty (Lines 4-13). In each iteration, *Assign* first counts the frequency of each sensitive value in S_t (Lines 5-6), and then builds a bucket B, such that (i) the signature of B consists of the β most frequent sensitive values in S_t, and (ii) each column of B contains α tuples in S_t. The values of α and β are decided in Lines 7-10, which, as explained in [40], guarantee that (i) $\beta \geq l$, (ii) $\alpha \geq 1$, and (iii) *Assign* always terminates[5]. For our discussion, it suffices to know that, α and β depend only on the size of S_t and the sensitive values in S_t. Since $\beta \geq l$, any bucket B created by *Assign* is l-diverse.

In the first iteration, $S_t = T_5$, and $\alpha = \beta = 2$ (calculated by Lines 7-10). Figure 7(a) illustrates the tuples in S_t. *Assign* first creates a bucket B_1 whose signature consists of the $\beta = 2$ most frequent sensitive values in S_t. As shown in Figure 7(a), there exist three sensitive values in S_t, *dyspepsia*,

[5]Intuitively, *Assign* always terminates, because (i) each iteration of *Assign* removes $\alpha \cdot \beta > 0$ tuples from S_t, and hence, (ii) S_t will become empty after a certain number of iterations, in which case *Assign* stops by returning the bucket partition U it constructs (see Lines 3 and 13 in Figure 6).

Algorithm *Assign* (T, l)

1. initialize a partition $U = \emptyset$
2. $w =$ the number of distinct A^s value in T
3. $S_t = T$
   /* The tuples in S_t will be iteratively removed to construct buckets in U */
4. while $S_t \neq \emptyset$
      /* Lines 5-12 create a new bucket in U using tuples from S_t */
5.   let v_i $(i \in [1, w])$ be the i-th most frequent A^s value in the current S_t
      /* Ties are resolved by a total ordering on A^s (see Example 6) */
6.   let n_i $(i \in [1, w])$ be the number of tuples in S_t with sensitive value v_i
7. $\beta = l$
      /* the new bucket's signature will contain the β most frequent A^s values in S_t */
8. $\alpha =$ the largest positive integer satisfying three inequalities:
         $\alpha \leq n_\beta, \quad n_1 - \alpha \leq \frac{|S_t| - \alpha \cdot \beta}{l}, \quad \text{and } n_{\beta+1} \leq \frac{|S_t| - \alpha \cdot \beta}{l}$
      /* the new bucket will contain α tuples for each sensitive value in its signature */
8.   if α does not exist
9. $\beta = \beta + 1$; goto Line 7
10.   create in U a bucket B with a signature $\{v_1, ..., v_\beta\}$
11.   for $i = 1$ to β
12.   from S_t, randomly remove α tuples whose sensitive values equal v_i, and insert those tuples into B
13. return U

Figure 6: The *Assign* algorithm

flu, and *gastritis*, that have the same highest frequency. To pick two of the three diseases, *Assign* resorts to a total ordering. In general, any total ordering works, but for our illustration, we use the alphabetic order, in which case the signature of B_1 is selected as $\{$*dyspepsia*, *flu*$\}$. Next, for each disease in the signature, *Assign* adds $\alpha = 2$ tuples to B_1. As a consequence, B_1 contains four tuples {Ann, Bob, Gill, Ed}, as illustrated in Figure 4. The tuples in B_1 are then removed from S_t.

In the second iteration, S_t contains four tuples, as shown in Figure 7(b). This time, $\alpha = 1$ and $\beta = 2$. Hence, *Assign* yields a bucket B_2 with signature $\{$*gastritis*, *bronchitis*$\}$ (*gastritis* is picked as it has the highest frequency in S_t; *bronchitis* is chosen because it alphabetically ranks before *diabetes*). Accordingly, *Assign* inserts two tuples into B_2: one with a sensitive value *gastritis*, and the other one with *bronchitis*. As there are two tuples having *bronchitis*, the one to appear in B_2 is randomly chosen; suppose that we pick Don. This leads to $B_2 =$ {Don, Fred}, as illustrated in Figure 4. Don and Fred are then evicted from S_t, as shown in Figure 7(c).

Similarly, the third iteration constructs a bucket $B_3 =$ {Hera, Cate} (see Figure 4). Then, S_t becomes empty, and hence, *Assign* terminates with a bucket partition $U = \{B_1, B_2, B_3\}$.

As the second step, *Ace* applies *Slice* to divide the buckets in U into smaller QI-groups. *Slice* also runs in iterations. In each iteration, it first identifies a divisible bucket B in U (Line 1 in Figure 8), and then, splits B using its canonical division $\{B_a, B_b\}$. This is repeated until no bucket in U is divisible.

In our example, the input to *Slice* is the bucket partition $U = \{B_1, B_2, B_3\}$ in Figure 4. B_1 is the only divisible bucket. To determine the canonical division of B_1, *Slice* finds the best division on each dimension (with the lowest perimeter). It turns out that, on both dimensions *Age* and *Zipcode*, the

Tuples in S_t	
Fred	(*bronchitis*)
Hera	(*diabetes*)
Ann, Gill	(*dyspepsia*)
Bob, Ed	(*flu*)
Cate, Don	(*gastritis*)

(a) Before B_1 Is Constructed

Tuples in S_t	
Fred	(*bronchitis*)
Hera	(*diabetes*)
Cate, Don	(*gastritis*)

(b) Before B_2 Is Constructed

Tuples in S_t
Hera (*diabetes*)
Cate (*gastritis*)

(c) Before B_3 Is Constructed

Figure 7: Changes in S_t During the Execution of *Assign* in Example 6

```
Algorithm Slice (U)
while there exists a divisible bucket B in U
    {B_a, B_b} = the canonical division of B
    U = U − {B} + {B_a, B_b}
return U
```

Figure 8: The *Slice* algorithm

best division is $\{B_4, B_5\}$, where $B_4 = \{\text{Ann, Bob}\}$ and $B_5 = \{\text{Gill, Ed}\}$. Thus, $\{B_4, B_5\}$ becomes the canonical division. Therefore, *Slice* removes B_1 from U, and inserts B_4 and B_5 instead, leading to $U = \{B_2, B_3, B_4, B_5\}$. As no bucket in U is divisible, *Slice* returns U to *Ace*. Finally, *Ace* reports the generalization T_7^* (in Table 10) decided by U. □

Ace is a randomized algorithm, due to the randomness in its component *Assign*. Furthermore, *Ace* has an $O(n^2 \log n)$ time complexity, where n is the number of tuples in T. To understand this, observe that *Assign* runs in $O(n)$ time (we regard the number of distinct A^s values in T as a constant). On the other hand, *Slice* has an $O(n^2 \log n)$ time complexity, since (i) each bucket B generated from *Assign* is divided by *Slice* exactly $|B|/l$ times, (ii) each division of B incurs $O(|B| \log |B|)$ overhead, and (iii) the sizes of all buckets add up to n. Since *Ace* is a composition of *Assign* and *Slice*, its time complexity is $O(n^2 \log n)$.

3.2.2 Proof of Transparent l-Diversity

This section proves that *Ace* achieves transparent l-diversity. Our analysis utilizes a crucial concept, the *symmetry* between buckets.

Definition 13 (Symmetry) *Two buckets B_1 and B_2 are* **symmetric***, if and only if (i) B_1 and B_2 have the same signature, and (ii) for any column $L_1 \subseteq B_1$, there exists a column $L_2 \subseteq B_2$, such that L_1 and L_2 involve the same set of individuals. Two bucket partitions U_1 and U_2 are symmetric, if each bucket in U_1 is symmetric to a bucket in U_2, and vice versa.*

Consider, for example, the bucket partition U_1 in Figure 4. Bucket $B_1 \in U_1$ contains two columns $L_1 = \{\text{Ann, Gill}\}$ and $L_2 = \{\text{Bob, Ed}\}$. Suppose that we exchange the sensitive values between L_1 and L_2, by setting the sensitive values of the tuples in L_1 (L_2) to *flu* (*dyspepsia*). Then, we obtain a bucket B_1' symmetric to B_1, as shown in Figure 9. The bucket partition $U_2 = \{B_1', B_2, B_3\}$ is

Age	Zipcode	Disease
[21, 27]	[10k, 18k]	dyspepsia
[21, 27]	[10k, 18k]	flu
[54, 60]	[60k, 63k]	dyspepsia
[54, 60]	[60k, 63k]	flu
[32, 60]	[35k, 63k]	gastritis
[32, 60]	[35k, 63k]	bronchitis
[32, 60]	[35k, 63k]	diabetes
[32, 60]	[35k, 63k]	gastritis

Table 10: Generalization T_7^*

Name	Age	Zipcode	Disease
Ann	21	10000	flu
Bob	27	18000	dyspepsia
Cate	32	35000	gastritis
Don	32	35000	gastritis
Ed	54	60000	dyspepsia
Fred	60	63000	bronchitis
Gill	60	63000	flu
Hera	60	63000	diabetes

Table 11: Microdata T_8

Bob, Ed	Ann, Gill
dyspepsia	*flu*

B'_1

Don	Fred
gastritis	*bronchitis*

B_2

Hera	Cate
diabetes	*gastritis*

B_3

Figure 9: Bucket partition U_2

symmetric to U_1. In general, we can obtain any symmetric counterpart of a bucket B, by swapping the sensitive values between different columns of B.

Interestingly, the canonical division of a symmetric bucket always results in symmetric sub-buckets:

Lemma 4 *Let B and B' be two symmetric buckets, and $\{B_1, B_2\}$ ($\{B'_1, B'_2\}$) be the canonical division of B (B'). Then, B_1 and B'_1 (B_2 and B'_2) are symmetric.*

The rationale behind Lemma 4 is similar to that of Lemma 2. Specifically, since B and B' are symmetric, each column L in B can be mapped to a column L' in B', such that L and L' involve an identical set of identifiers and QI values. Recall that the canonical division of a bucket depends only on identifiers and QI-values, and is irrelevant to sensitive values. Hence, the canonical division of B has the same effect as that of B', thus establishing Lemma 4. The lemma naturally leads to the following result.

Lemma 5 *Let U_1 and U_2 be two symmetric bucket partitions. Let $U'_1 = Slice(U_1)$ and $U'_2 = Slice(U_2)$. Then, U'_1 and U'_2 are symmetric.*

Assign also has an interesting property related to symmetric buckets:

Lemma 6 *Let T_1 be a microdata table, l an integer, and U_1 a possible output of $Assign(T_1, l)$. Let U_2 be a bucket partition symmetric to U_1, and $T_2 = \bigcup_{B \in U_2} B$. Then, $Pr\{Assign(T_1, l) = U_1\} = Pr\{Assign(T_2, l) = U_2\}$.*

For instance, consider the symmetric bucket partitions U_1 and U_2 in Figures 4 and 9, respectively. U_1 (U_2) is a partition of the microdata T_5 in Table 8 (T_8 in Table 11). By Lemma 6, the probability that $Assign(T_5, 2)$ returns U_1 equals the probability that $Assign(T_8, 2)$ outputs U_2.

We prove that *Ace* ensures transparent l-diversity by combining Lemmas 5 and 6 with the following theorem, which states a sufficient condition for transparent l-diversity.

Theorem 2 *Let $\mathcal{G}_A$ and $\mathcal{G}_B$ be two algorithms as follows:*

1. *$\mathcal{G}_A$ takes as input a microdata table T_1 and a positive integer l, and outputs a bucket partition U_1 of T_1, such that for any bucket partition U_2 symmetric to U_1, we have $Pr\{\mathcal{G}_A(T_1, l) = U_1\}$ $= Pr\{\mathcal{G}_A(T_2, l) = U_2\}$, where $T_2 = \bigcup_{B \in U_2} B$.*

2. *$\mathcal{G}_B$ is a deterministic algorithm that takes as input a bucket partition U and outputs another bucket partition, such that for any bucket partition U' symmetric to U, $\mathcal{G}_B(U)$ is always symmetric to $\mathcal{G}_B(U')$.*

Let $\mathcal{G}$ be an l-diversity algorithm that first applies $\mathcal{G}_A$ on the input microdata, then invokes $\mathcal{G}_B$ on the bucket partition output from $\mathcal{G}_A$, and finally returns the anonymization decided by the bucket partition generated from $\mathcal{G}_B$. $\mathcal{G}$ is transparent.

By Lemma 6 (Lemma 5), *Assign* (*Slice*) satisfies the requirements for $\mathcal{G}_A$ ($\mathcal{G}_B$) stated in Theorem 2; therefore, *Ace* (as a combination of *Assign* and *Slice*) is a transparent algorithm.

3.3 The *Hybrid* Algorithm

This section develops a new algorithm *Hybrid* that combines *Tailor* and *Ace*. *Hybrid* is motivated by, and overcomes the drawbacks of, *Tailor* and *Ace*. We will first explain those drawbacks, and then, elaborate the details of *Hybrid*.

Given a microdata T and an integer l, *Tailor* initiates a partition $P = \{T\}$, and then iteratively refines P, by splitting the QI-groups of P into smaller ones. However, once a QI-group violates $2l$-diversity, it is ignored by *Tailor*, even if it can be further divided. As a result, *Tailor* sometimes spawns QI-groups with many tuples, entailing high information loss. For example, consider the 2-diverse generalization T_6^* (Table 9), which is produced by *Tailor* in Example 4. The first QI-group G_1 in T_6^* has four tuples {Ann, Bob, Cate, Don} in T_5 (Table 8). In fact, G_1 can be further split into 2-diverse QI-groups {Ann, Cate} and {Bob, Don}. *Tailor* fails to see the split because G_1 is not 4-diverse.

Ace does not suffer from the above defect, but its random nature may occasionally create poor QI-groups. Recall that, *Ace* employs *Assign* to obtain an l-diverse bucket partition U of T. Let us revisit the way *Assign* builds a bucket B in U: *Assign* first decides the signature of B, and then determines each column in B, using tuples *randomly* selected from T. The distribution of QI values in each column of B may vary significantly. For instance, in Example 6, *Assign* generates a bucket B_3 with signature {*diabetes*, *gastritis*}. Diabetes usually affects people over 40, while gastritis is common for all ages. Therefore, when *Assign* constructs the *diabetes* column, the random samples from T are likely to have large *Age* values. In contrast, the *gastritis* column may contain individuals with any ages.

This (QI-distribution) difference becomes problematic in *Slice*, which *Ace* deploys to refine the bucket partition U output by *Assign*. As explained in Section 3.2.1, *Slice* splits each bucket $B \in U$ into non-divisible buckets (a.k.a QI-groups), each of which has exactly one tuple from every column of B. If the columns of B have diverse QI-distributions, the tuples in a final non-divisible QI-group may have dissimilar QI values. After anonymization, such a QI-group would incur large information loss.

Algorithm *Hybrid* (T, l)
1. if T is not l-eligible then return $\emptyset$
2. G_0 = a QI-group containing all tuples in T, and $P = \{G_0\}$
3. while there exists a $2l$-diverse QI-group G in P
4. $\quad \{G_a, G_b\}$ = the canonical l-cut of G
5. $\quad P = P - \{G\} + \{G_a, G_b\}$
6. $T^* = \emptyset$
7. for each QI-group $G_i \in P$
8. $\quad T_i^* = Ace(G_i, l)$
9. $\quad T^* = T^* \cup T_i^*$
10. return T^*

Figure 10: The *Hybrid* algorithm

Hybrid, as in Figure 10, remedies the deficiencies of *Tailor* and *Ace* by running the two algorithms consecutively. Specifically, *Hybrid* first computes a partition P of T using *Tailor*. In particular, Lines 1-5 in Figure 10 are identical to Lines 1-5 in Figure 3. As the second step, *Hybrid* treats each QI-group in P as a tiny microdata table, and invokes *Ace* to generalize the QI-group (Lines 6-10).

By employing *Ace* to refine P, *Hybrid* outputs QI-groups with (much) fewer tuples than *Tailor*, thus avoiding the defect of *Tailor*. Meanwhile, compared to *Ace*, *Hybrid* incurs lower information loss, by executing *Ace* on each QI-group in P, where tuples already have similar QI values. The following theorem shows that *Hybrid* is transparent.

Theorem 3 *Let T be a microdata table, l be a positive integer, and T^* be any possible output of Hybrid(T, l). Given any external source E for T, we have $risk(o) \leq 1/l$ for any individual o.*

Finally, we point out that *Hybrid* has an $O(n^2 \log n)$ time complexity, where n is the number of tuples in T. This follows from the $O(n^2 \log n)$ complexity of both *Tailor* and *Ace*.

4 Experiments

In the earlier sections, we have proved the privacy guarantees of our transparent algorithms. A natural question is, how do they compare with the existing solutions in terms of data utility and computation overhead (remember that no previous solution is transparent, i.e., it does not ensures anonymity, when an adversary knows the algorithm details)? In the sequel, we answer this question with empirical evidence that validates the effectiveness and efficiency of our algorithms. First, Section 4.1 clarifies the experiment settings, and then Sections 4.2 and 4.3 present detailed results.

4.1 Experimental Setting

Following previous work [14, 40], we employ two real-world datasets, OCC and SAL, extracted from the *Integrated Public Use Microdata Series* [30]. Both datasets consist of 600k tuples, each containing the information of an American adult. OCC has a sensitive attribute *Occupation*, and four QI attributes, *Age*, *Gender*, *Education*, and *Birthplace*. SAL has the same QI attributes, but

	Age	*Gender*	*Education*	*Birthplace*	*Occupation*	*Income*
Size	79	2	17	57	50	50

Table 12: Attribute domain sizes

Parameter	**Values**
l	6, 7, **8**, 9, 10
Query dimensionality qd	2, 3, 4, **5**
Expected selectivity s	2%, 4%, **6**%, 8%, 10%

Table 13: Parameters and Tested Values

a different sensitive attribute *Income*. All attributes have integer domains. Table 12 presents their domain sizes.

We compare our techniques (adopting the MBR function) against two l-diversity generalization algorithms, *Mondrian* [20] and *Mask* [36]. The former is a popular technique in the literature [7, 21, 27, 29], due to its simplicity and effectiveness. *Mask*, on the other hand, is an existing approach that does not assume adversaries with zero algorithm knowledge (nevertheless, as explained in Sections 1 and 2.3, *Mask* is not transparent, as it can prevent only minimality attacks). We apply each algorithm to compute l-diverse generalizations of OCC and SAL, using various values of l[6]. Note that the generalizations produced by our solutions are guaranteed to be transparent l-diverse, whereas those by the other methods are not.

In accordance with [14, 36, 40], we evaluate the utility of a generalized table T^* by using it to answer count queries about the underlying microdata T. Each query has the form:

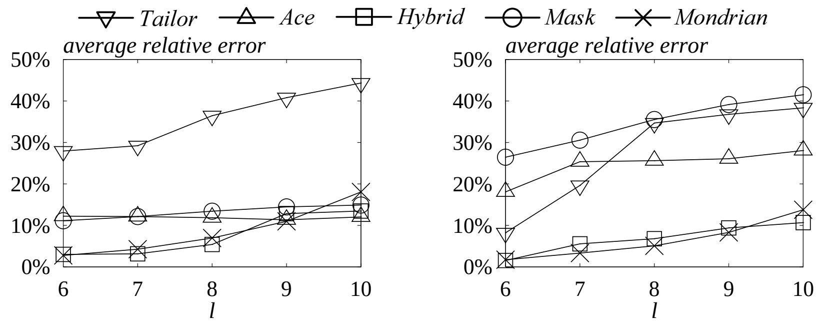
SELECT COUNT(*) FROM T
WHERE $pred(A_1^q)$ AND ... AND $pred(A_4^q)$ AND $pred(A^s)$

where $pred(A)$ denotes a predicate on A. Predicates are generated based on two parameters: *query dimensionality* qd and *expected selectivity* s. Specifically, given $qd \in [2, 5]$ and $s \in (0, 1)$, we create a set S_A that contains the sensitive attribute A^s of T, and $qd - 1$ QI attributes randomly selected. Then, for each $A \in S_A$, we set $pred(A)$ to "$A \in I$", where I is a random interval on A, enclosing a fraction $s^{1/qd}$ of the values in A. Finally, for each $A' \notin S_A$, $pred(A')$ is "$A' = *$". By requiring $qd \geq 2$ and $A^s \in S_A$, we aim to examine how well T^* preserves the correlation between the QI and sensitive attributes.

On each generalized table, we process several query workloads, each of which contains 1000 queries with identical qd and s. We gauge the utility of T^* by the average workload error computed as follows. For each query, we derive its exact result act from T, and compute an estimated answer est from T^* using the approximation technique in [20]. The error of est is defined as $\frac{|act - est|}{\max\{act, \delta\}}$, where δ is set to 0.5% of the dataset cardinality. Then, the workload error equals the average error of all queries in the workload. Note that δ is introduced to prevent the workload error from being dominated by queries with exceedingly small results (similar approaches are adopted in [13, 33]).

Table 13 summarizes the experiment parameters. Unless otherwise specified, we always set the

[6] *Mask* requires two parameters k and l ($k \geq l$) to generate an l-diverse table. We set $k = l$ in our experiments, since a smaller k leads to a generalized table with higher utility, as shown in [36].



(a) OCC (b) SAL

Figure 11: Query Accuracy vs. l

	Age	*Gender*	*Education*	*Birthplace*
Occupation	0.49	0.62	0.28	0.38
Income	0.71	0.87	0.41	0.50

Table 14: Correlation Ratio between Attributes

parameters to their default values, i.e., the bold numbers in Table 13. All experiments are performed on a computer with a 1.8GHz CPU and 1GB memory.

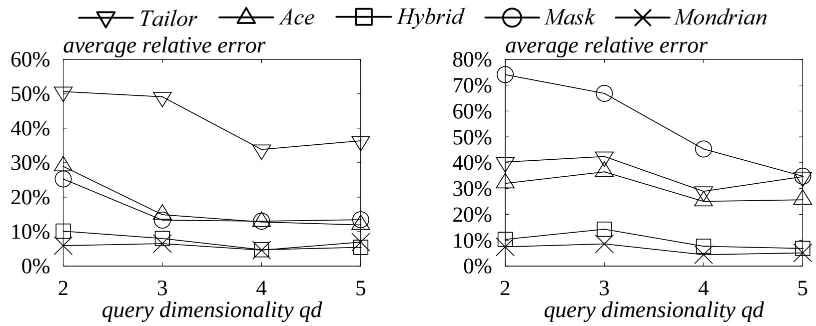
4.2 Utility of Generalization

The first set of experiments evaluates the information loss incurred by each algorithm. Figure 11 illustrates the results as a function of l. As expected, the error of all methods escalates with l, since a larger l implies a more stringent anonymity requirement, which, in turn, demands more aggressive generalization. *Hybrid* and *Mondrian* have the best overall performance. This is a strong evidence indicating that the heuristics of *Hybrid* are highly effective. In particular, even though *Hybrid* must guarantee transparency, it still offers almost the same utility compared to *Mondrian* (which is non-transparent).

Tailor and *Ace* exhibit worse performance than *Hybrid*. This is not surprising because, as mentioned in Section 3.3, *Hybrid* is designed to overcome the shortcomings of *Tailor* and *Ace*. *Mask* incurs larger error than *Hybrid* in all cases, even though the former is vulnerable to adversaries with full algorithm knowledge (recall that *Mask* prevents only minimality attacks).

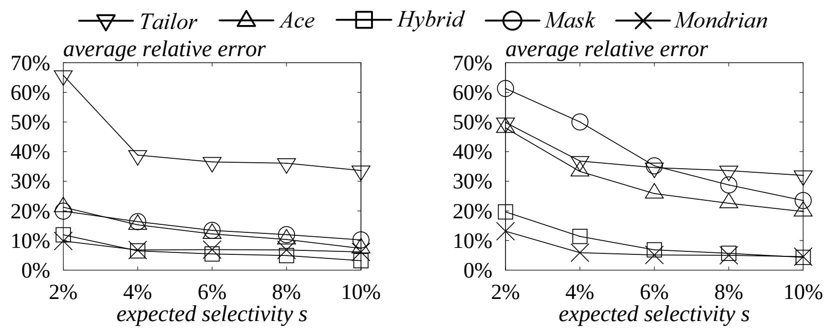
Each algorithm demonstrates similar behavior regardless of the dataset, except that *Ace* performs worse on SAL than on OCC. To explain this, we observe that the incomes depend heavily on people's ages and education. Hence, when *Ace* employs *Assign* to create a partition U of SAL, each bucket in U contains tuples with very different QI values, due to the reason explained in Section 3.3. As a result, the QI-groups returned by *Ace* have long generalized intervals, rendering low data utility. The above phenomenon does not exist on OCC because occupation is much less correlated to the QI-attributes. To support our analysis, Table 14 shows the correlation ratios [16] between the QI and sensitive attributes of OCC and SAL. A larger ratio indicates stronger correlation.

To study the influence of query dimensionality qd, Figure 12 plots the workload error as a function of qd. The relative performance of alternative algorithms remains the same as in Figure 11. In



(a) OCC (b) SAL

Figure 12: Query Accuracy vs. Query Dimensionality qd



(a) OCC (b) SAL

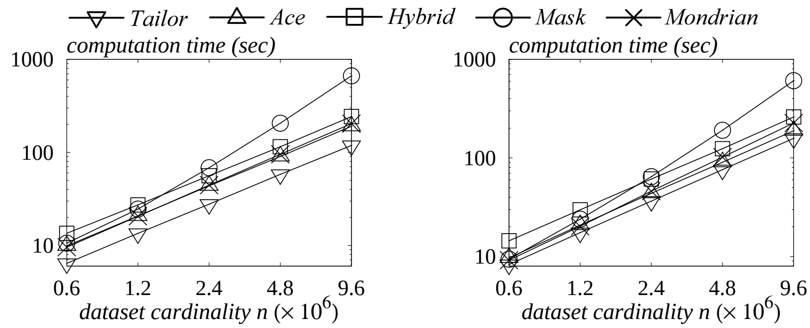
Figure 13: Query Accuracy vs. Expected Selectivity s

particular, *Hybrid* and *Mondrian* permit highly accurate counting analysis; their maximum error is less than 10%. Each algorithm has better query precision when the query dimensionality qd is higher. To understand this, recall that each query predicate either includes the whole domain of an attribute, or is an interval covering $s^{1/qd}$ of the domain. When s is fixed but qd increases, $s^{1/qd}$ becomes greater, implying wider query intervals, which lead to smaller error, as explained in [38]. Figure 13 shows the error when the expected selectivity s grows from 2% to 10%. Again, the relative superiority of different algorithms is the same. Their error decreases when s increases, as is consistent with the experiment results in [14, 36, 40].

In summary, *Hybrid* and *Mondrian* produce generalizations with similar data utility, and both significantly outperform *Tailor*, *Ace*, and *Mask*. Therefore, overall *Hybrid* is the best anonymization technique, since it promises much stronger privacy guarantee than *Mondrian*.

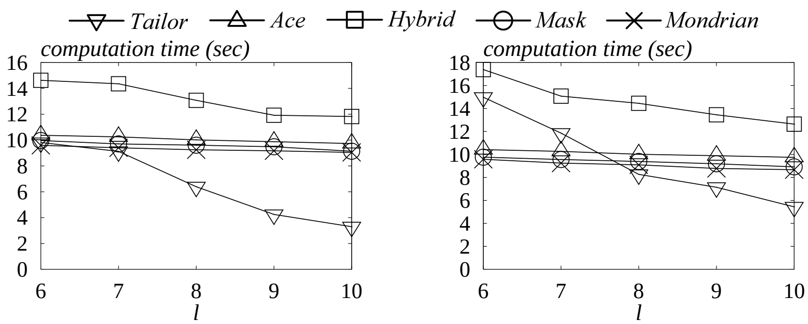
4.3 Computation Overhead

Having examined the effectiveness of the proposed solutions, we proceed to evaluate their efficiency. In order to inspect their scalability with the dataset cardinality, based on OCC (SAL), we generate microdata tables with various cardinalities. Specifically, given a multiple n of 600k, a table with n tuples is synthesized by including n/600k copies of OCC (SAL). Figure 14 shows the generalization time of each method, as a function of n. The running time of *Mask* exhibits a superlinear increase with n, while the other algorithms scale almost linearly. *Hybrid* requires slightly higher overhead



(a) OCC (b) SAL

Figure 14: Computation Time vs. Dataset Cardinality n



(a) OCC (b) SAL

Figure 15: Computation Time vs. l

than *Mondrian*. This is not a serious disadvantage because (i) the difference is not large, (ii) the disadvantage is the compensated by the transparency of *Hybrid*, and (iii) anonymization is an offline process, so it is reasonable to spend a little more time preparing a publication that safeguards privacy better.

Utilizing the 600k datasets, in Figure 15, we inspect the computation overhead as a function of l. The running time of *Ace*, *Mondrian*, and *Mask* is insensitive to l. In contrast, the processing cost of *Tailor* and *Hybrid* decreases rapidly as l grows. Recall that, *Tailor* works by iteratively dividing QI-groups, until all QI-groups violate $2l$-diversity. As l increases, fewer $2l$-diverse QI-groups exist; hence *Tailor* terminates earlier. *Hybrid* has similar behavior because it deploys *Tailor* as the first step.

In summary, *Hybrid* is ideal for practical applications because its computation cost enjoys linear scalability to the dataset cardinality. In particular, it anonymizes a dataset with nearly 10 million tuples within 5 minutes (see Figure 14).

5 Related Work

The works closest to ours are due to [36] and [42]. Since they has been discussed extensively in Sections 1 and 2.3, the following review concentrates on the rest of the literature on privacy preserving data publishing.

A bulk of the literature focuses on designing privacy principles. The earliest principle, k-anonymity [31], requires that every QI-group should contain at least k tuples. [23] point out that a k-anonymous table may still incur privacy breach, unless each QI-group includes sufficiently diverse sensitive values. This observation leads to the concept of l-diversity, which has several instantiations, e.g., entropy l-diversity, recursive (c, l)-diversity, as discussed in Section 2.1. Besides k-anonymity and l-diversity, numerous other privacy principles [7, 8, 18, 21, 22, 25, 27, 34, 37, 39, 40, 36, 43, 42] have been developed to offer different flavors of privacy protection, by placing various constraints on the contents of QI-groups. Our transparent l-diversity principle distinguishes itself from all the previous principles, in that it guarantees privacy even when the anonymization process is public knowledge.

Generalization algorithms is another well-explored topic [1, 6, 12, 14, 19, 20, 21, 15, 35, 37, 40, 41, 36, 42]. These solutions aim at minimizing the information loss, according to different anonymization constraints (e.g., global/local recoding) and measurements of loss (e.g., discernibility). Many of them are initially devised for k-anonymity, but can be modified to support l-diversity and other principles, as explained in [23]. However, except the algorithms proposed in [40, 42], none of these algorithms is transparent. In other words, they can no longer ensure the privacy guarantee of the underlying principle, when an adversary is aware of the details of the algorithm.

Other problems related to generalization have also attracted considerable research efforts. Specifically, optimal k-anonymous generalization has been shown to be NP-hard in [3, 26, 28], which also develop approximation algorithms with provable worst-case quality guarantees. [2] shows that when the number of QI attributes is large, it is simply impossible to achieve k-anonymity without substantial information loss (even when k is small). [38] develop anatomy as an alternative anonymization technique that achieves higher data utility than generalization does.

In addition, there exist several anonymization techniques [4, 9, 10, 24, 32] that do not adopt generalization. Instead, they anonymize microdata by adding random "noise" into the data, i.e., by replacing a fraction of tuples in the microdata with randomly generated tuples [4, 10, 32], or by deriving the tuple distribution in the microdata and then publishing a noisy version of the distribution [9, 24]. These techniques are designed by assuming that the process for generating random "noise" is known to the public, and hence, they do not suffer from reverse engineering attacks.

6 Conclusions

Most existing anonymization techniques fail to protect privacy against adversaries with full knowledge of the anonymization mechanism. In this paper, we remedy the problem with two important contributions. First, we provide a thorough analysis on the disclosure risks in the anonymized tables, assuming that the anonymization algorithm is public knowledge. This analysis leads to the formulation of transparent l-diversity, which ensures small disclosure risks in an anonymized table, even if everything involved in the anonymization process, except the microdata, is revealed to the public. Second, we identify three anonymized algorithms that can enforce transparent l-diversity, and demonstrate their practical usefulness through extensive experiments.

This work also lays down a solid foundation for future research. First, our analysis focuses on l-diversity due to its popularity in the literature. However, the concept of transparent anonymization is general, and can be integrated with any other principle (e.g., t-closeness [22], δ-presence [27]).

It is an interesting direction to design transparent generalization algorithms for those principles. Second, the proposed solutions are heuristic in nature, and do not have attractive asymptotical performance guarantees. It is a challenging problem to study theoretical transparent algorithms. Note that the existing findings (including the complexity results, approximation algorithms, etc.) were derived for conventional generalization, and hence, are not immediately applicable to transparent anonymization.

Acknowledgements

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APPENDIX I: Detailed proofs

Proof of Proposition 1. Observe that, the adversary's knowledge about the external source E can be expressed as $T \in S$, since S consists of all microdata tables that involve the individuals in

E. Furthermore, if $T \in S_{o,v}$, then o has a sensitive value v in T, and vice versa. Hence,

$$\begin{aligned}
risk(o) &= \max_{v \in A^s} Pr\{o \text{ has } v \text{ in } T \mid E \wedge \mathcal{G} \wedge T^* \wedge l\} \\
&= \max_{v \in A^s} Pr\{T \in S_{o,v} \mid T \in S \wedge \mathcal{G} \wedge T^* \wedge l\} \\
&= \max_{v \in A^s} Pr\{T \in S_{o,v} \mid T \in S \wedge \mathcal{G}(T,l) = T^*\} \\
&= \max_{v \in A^s} \frac{Pr\{T \in S_{o,v} \wedge T \in S \wedge \mathcal{G}(T,l) = T^*\}}{Pr\{T \in S \wedge \mathcal{G}(T,l) = T^*\}} \\
&= \max_{v \in A^s} \frac{Pr\{T \in S_{o,v} \wedge \mathcal{G}(T,l) = T^*\}}{Pr\{T \in S \wedge \mathcal{G}(T,l) = T^*\}} \quad (\text{since } S_{o,v} \subseteq S) \\
&= \max_{v \in A^s} \frac{\sum_{\hat{T} \in S_{o,v}} \left(Pr\{T = \hat{T}\} \cdot Pr\{\mathcal{G}(\hat{T},l) = T^*\}\right)}{\sum_{\hat{T} \in S} \left(Pr\{T = \hat{T}\} \cdot Pr\{\mathcal{G}(\hat{T},l) = T^*\}\right)}.
\end{aligned}$$

Recall that, each possible microdata instance in S is equally likely for the adversary, before s/he observes T^*. That is, for any $\hat{T}_1, \hat{T}_2 \in S$, we have $Pr\{T = \hat{T}_1\} = Pr\{T = \hat{T}_2\}$. Thus,

$$\begin{aligned}
risk(o) &= \max_{v \in A^s} \frac{\sum_{\hat{T} \in S_{o,v}} \left(Pr\{T = \hat{T}\} \cdot Pr\{\mathcal{G}(\hat{T},l) = T^*\}\right)}{\sum_{\hat{T} \in S} \left(Pr\{T = \hat{T}\} \cdot Pr\{\mathcal{G}(\hat{T},l) = T^*\}\right)} \\
&= \max_{v \in A^s} \frac{\sum_{\hat{T} \in S_{o,v}} Pr\{\mathcal{G}(\hat{T},l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{\mathcal{G}(\hat{T},l) = T^*\}},
\end{aligned}$$

which completes the proof. □

Proof of Lemma 1. Assume by contradiction that *Opt-Gen* is not a minimal algorithm. Then, there exists a microdata table T and a positive integer l, such that $T_1^* = \textit{Opt-Gen}(T,l)$ is not a minimal l-diverse generalization of T, with respect to the MBR function f and the global recoding scheme. Let P_1 be the partition of T that decides T_1^*. By Definition 7, there should be a child P_2 of P_1, such that P_2 and f decide a generalization T_2^* that conforms to the global recoding scheme.

According to Definition 6, (i) there exists a unique QI-group G_1 in P that does not appear in P_2, and (ii) P_2 contains only two QI-groups G_2 and G_3 that are not included in P_1. Furthermore, since $G_1 = G_2 \cup G_3$ and $G_2 \cap G_3 = \emptyset$, we have $|G_1| = |G_2| + |G_3|$. Thus,

$$\begin{aligned}
\sum_{G \in P_1} |G|^2 &= |G_1|^2 + \sum_{G \in P_1 - \{G_1\}} |G|^2 \\
&\geq |G_2|^2 + |G_3|^2 + \sum_{G \in P_1 - \{G_1\}} |G|^2 \\
&= \sum_{G \in P_1 - \{G_1\} + \{G_2, G_3\}} |G|^2 \\
&= \sum_{G \in P_2} |G|^2,
\end{aligned}$$

which contradicts the fact that *Opt-Gen* minimizes the discernability of the generalized tables. Hence, the lemma is proved. □

Proof of Lemma 2. Let G'_3 (G'_4) be the set of tuples in G', such that G'_3 and G_1 (G'_4 and G_2) involve the same set of individuals. To prove the lemma, it suffices to show that $\{G'_3, G'_4\}$ is the canonical l-cut of G'.

Without loss of generality, assume that $\{G_1, G_2\}$ is an l-cut of G on A_i^q ($i \in [1, d]$). We will first prove that $\{G'_3, G'_4\}$ is an l-cut of G' on A_i^q, i.e., $\{G'_3, G'_4\}$ satisfies the three conditions in Definition 9. Observe that the first condition trivially holds. Let v be the most frequent A^s value in G, and c be the number of tuples in G with a sensitive value v. Since G and G' are isomorphic, they contain the same multi-set of A^s values. Therefore, c is also the maximum number of tuples in G' with an identical sensitive value. Since $|G'_3| = |G_1| \geq c \cdot l$ and $|G'_4| = |G_2| \geq c \cdot l$, $\{G'_3, G'_4\}$ fulfills the second condition in Definition 9.

Assume by contradiction that, $\{G'_3, G'_4\}$ violates the third condition in Definition 9. There should exist $t'_3 \in G'_3$ and $t'_4 \in G'_4$, such that (i) $t'_3[A_i^q] > t'_4[A_i^q]$, or (ii) $t'_3[A_i^q] = t'_4[A_i^q]$ and $t'_3[A^{id}] = t'_4[A^{id}]$. Let t_1 (t_2) be the tuple in G_1 (G_2), such that t_1 and t_3 (t_2 and t_4) concern the same individual. Then, t_1 and t'_3 (t_2 and t'_4) should have the same QI values. As a result, we have either (i) $t_1[A_i^q] > t_2[A_i^q]$, or (ii) $t_1[A_i^q] = t_2[A_i^q]$ and $t_1[A^{id}] > t_2[A^{id}]$. This contradicts the assumption that $\{G_1, G_2\}$ is an l-cut of G. Therefore, $\{G'_3, G'_4\}$ is an l-cut of G' on A_i^q.

Next, we will show that $\{G'_3, G'_4\}$ is canonical. Assume that this is not true. Then, by Definition 10, at least one of the following three conditions must hold:

1. Among the l-cuts of G', the perimeter of $\{G'_3, G'_4\}$ is not the smallest.
2. There exists an l-cut $\{G'_5, G'_6\}$ of G' on A_j^q ($j < i$), such that $h_p(G'_5) + h_p(G'_6) = h_p(G'_3) + h_p(G'_4)$.
3. There exists an l-cut $\{G'_5, G'_6\}$ of G' on A_i^q, such that $|G'_5| < |G'_3|$, and $h_p(G'_5) + h_p(G'_6) = h_p(G'_3) + h_p(G'_4)$.

Consider that Condition 3 is satisfied. Let G_5 (G_6) be the set of tuples in G, such that G_5 and G'_5 (G_6 and G'_6) contain the same set of individuals. It can be verified that $\{G_5, G_6\}$ is an l-cut of G on A_i^q, and $h_p(G_5) + h_p(G_6) = h_p(G'_5) + h_p(G'_6)$. Then,

$$h_p(G_5) + h_p(G_6) = h_p(G'_5) + h_p(G'_6) \quad = \quad h_p(G'_3) + h_p(G'_4) = h_p(G_1) + h_p(G_2).$$

Furthermore, $|G_5| = |G'_5| < |G'_3| = |G_1|$. This contradicts the assumption that $\{G_1, G_2\}$ is the canonical l-cut of G.

Similarly, it can be shown that when Condition 1 or 2 holds, $\{G_1, G_2\}$ cannot be the canonical l-cut of G, leading to a contradiction. Thus, $\{G'_3, G'_4\}$ should be the canonical l-cut of G', which completes the proof. $\square$

Proof of Lemma 3. Let $T_2^* = \textit{Tailor}(T_2, l)$, and P_3 be the partition of T_2 that decides T_2^*. We will prove the lemma, by showing that (i) P_1 and P_3 are isomorphic, and (ii) $P_2 = P_3$. The former guarantees that $T_2^* = T^*$, since isomorphic partitions always lead to the same anonymization.

To facilitate our proof, we construct a binary tree R_1 of QI-groups as follows. First, we set the root of R_1 to T_1. Then, we apply *Tailor* on T_1 with the given l value, and monitor the execution of *Tailor*. As shown in Figure 3, *Tailor* will first construct a partition $P = \{G_0\}$, with $G_0 = T_1$. Then, each time *Tailor* computes the canonical l-cut $\{G_1, G_2\}$ of QI-group $G \in P$, we insert G_1 and G_2 (into R) as the child nodes of G. As such, after *Tailor* terminates, each leaf of R_1 is a QI-group in P_1, and vice versa. We refer to R_1 as the *split history* of T_1. Following the same methodology, we also construct the split history R_2 of T_2, such that the leaves of R_2 constitute P_3.

Next, we will prove that P_1 is isomorphic to P_3, by showing that each leaf of R_1 is isomorphic to a leaf of R_2, and vice versa. Our proof is by induction. For the base case, let us consider the roots of R_1 and R_2. Let G_1 (G_2) denote the root of R_1 (R_2). We have $G_1 = T_1$ and $G_2 = T_2$. Since P_1 and P_2 are isomorphic, T_1 and T_2 should also be isomorphic, because $T_1 = \bigcup_{G \in P_1} G$ and $T_1 = \bigcup_{G \in P_2} G$. Therefore, G_1 is isomorphic to G_2.

As a second step, assume that two nodes $G_3 \in R_1$ and $G_4 \in R_2$ are isomorphic. We will establish two propositions:

- Proposition 1. G_3 is a leaf of R_1, if and only if G_4 is a leaf of R_2.
- Proposition 2. If G_3 is not a leaf, then each child of G_3 is isomorphic to a child of G_4.

Observe that, G_3 (G_4) is a leaf of R_1 (R_2), if and only if it is not $2l$-diverse, otherwise it would have been divided into smaller parts by *Tailor*. Since G_3 and G_4 are isomorphic, if G_3 is not $2l$-diverse, G_4 must violate $2l$-diversity, and vice versa. Therefore, Proposition 1 holds.

Now assume that G_3 is not a leaf. Let $\{G_a, G_b\}$ and $\{G'_a, G'_b\}$ be the canonical l-cuts of G_3 and G_4, respectively. By Lemma 2, G_a and G'_a (G_b and G'_b) contain the same set of individuals. We will show that G_a (G_b) is isomorphic to G'_a (G'_b).

Consider the set S_a of leaves under the subtree of G_a. We have $\bigcup_{G \in S_a} G = G_a$. Since P_1 and P_2 are isomorphic, there exists a subset S'_a of P_2, such that each $G \in S_a$ is isomorphic to some $G' \in S'_a$, and vice versa. Let $G_5 = \bigcup_{G' \in S'_a} G'$. Then, G_5 is isomorphic to G_a, which indicates that G_5 and G_a involve the same set of individuals. Recall that G_a and G'_a also contain an identical set of individuals. Hence, each individual in G'_a appears in G_5, and vice versa. Because both G'_a and G_5 are subsets of T_2, we have $G'_a = G_5$. Consequently, G'_a is isomorphic to G_a. Similarly, it can be verified that the G_b and G'_b are isomorphic. Thus, Proposition 2 is valid. By induction, each leave of R_1 is isomorphic to a leaf of R_2, and vice versa. Hence, P_1 is isomorphic to P_3.

To complete the proof, it remains to show that $P_2 = P_3$. Since both P_2 and P_3 are isomorphic to P_1, P_2 must be isomorphic to P_3. Therefore, for each QI-group $G \in P_2$, there exists $G' \in P_3$, such that G and G' involve the same set of individuals. This indicates that $G = G'$, since the both G and G' are subsets of T_2. Therefore, $P_2 = P_3$, which proves the lemma. □

Proof of Theorem 1. Let T be any microdata table, l be any positive integer, and $T^* = \mathcal{G}(T, l)$. Let E be any external source, and C be the set of possible microdata instances based on E, such that $\mathcal{G}(\hat{T}, l) = T^*$ for any $\hat{T} \in C$. Let o be any individual, v be an arbitrary sensitive value, and C' the subset of C, such that each $\hat{T} \in C'$ contains a tuple t with $t[A^{id}] = o$ and $t[A^s] = v$. By Proposition 1, we can prove Theorem 1 by showing that

$$\frac{|C'|}{|C|} \leq \frac{1}{l}. \tag{5}$$

For each $\hat{T} \in C$, we define the *essential partition* of $\hat{T}$, as the partition of $\hat{T}$ generated by $\mathcal{G}$, when taking $\hat{T}$ and l as input. We divide C into disjoint clusters, such that each cluster is a maximal set of instances (in C) whose essential partitions are isomorphic. Let n be the total number of clusters in C, and C_j ($j \in [1, n]$) the j-th cluster. Let C'_j be a set containing the instances in C_j that associate o with v. In the following, we will show that $|C'_j|/|C_j| \le 1/l$ for any $j \in [1, n]$, which will prove the theorem, as it leads to

$$\frac{|C'|}{|C|} = \frac{\sum_{j=1}^{n} |C'_j|}{\sum_{j=1}^{n} |C_j|} \le \frac{\sum_{j=1}^{n} |C_j|/l}{\sum_{j=1}^{n} |C_j|} = \frac{1}{l}. \tag{6}$$

Consider any $\hat{T} \in C_j$ for some $j \in [1, n]$. Let $\hat{P}$ be the essential partition of $\hat{T}$, $m = |\hat{P}|$, and G_k ($k \in [1, m]$) the k-th QI-group in $\hat{P}$. Let $\hat{P}'$ be a partition isomorphic to $\hat{P}$, and $\hat{T}' = \bigcup_{G' \in \hat{P}'}$. Since $\hat{T}'$ and $\hat{T}$ involve the same set of individuals, $\hat{T}'$ is a possible microdata instance based on E. By the assumption on $\mathcal{G}$, we have $\mathcal{G}(\hat{T}', l) = T^*$. Therefore, $\hat{T}' \in C_j$. In other words, for any partition $\hat{P}'$ isomorphic to $\hat{P}$, the microdata corresponding to $\hat{P}'$ is contained in C_j. Then, by the definition of C_j, $|C_j|$ should equal the total number of distinct partitions isomorphic to $\hat{P}$, including $\hat{P}$ itself. According to the definition of partition isomorphism, we can obtain any partition isomorphic to $\hat{P}$, by replacing any QI-groups in $\hat{P}$ with their isomorphic counterparts. Let a_k be the number of distinct QI-groups isomorphic to G_k. Then, the total number of partitions isomorphic to $\hat{P}$ should be $\prod_{k=1}^{m} a_k$. That is, $|C_j| = \prod_{k=1}^{m} a_k$.

Next, we will derive the value of $|C'_j|$. Without loss of generality, assume that o appears in the first QI-group G_1 of P_i. Among the QI-groups isomorphic to G_1, let a'_1 be the number of QI-groups that associate o with a sensitive value v. Then, we have $|C'_j| = a'_1 \cdot \prod_{k=2}^{m} a_k$. Therefore, $|C'_j|/|C_j| = a'_1/a_1$.

If v does not appear in G_1, then $a'_1 = 0$. Otherwise, assume that G_1 contains x sensitive values v_1, v_2, ..., v_x, such that $v_1 = v$. Further assume that, there exist b_i ($i \in [1, x]$) tuples in G_1 with a sensitive value v_i. Then, there are $\frac{|G_1|!}{\sum_{i=1}^{x}(b_i!)}$ different combinations between the sensitive values and the individuals in G_1. Since each combination corresponds to a QI-group isomorphic to G_1, we have

$$a_1 = \frac{|G_1|!}{\sum_{i=1}^{x}(b_i!)}. \tag{7}$$

Observe that, among the a_1 combinations, there exist $\frac{(|G_1|-1)!}{\sum_{i=2}^{x}(b_i!)}$ combinations that assigns a sensitive value v_1 to o. Therefore,

$$a'_1 = \frac{(|G_1| - 1)!}{\sum_{i=2}^{x}(b_i!)}. \tag{8}$$

Hence, we have

$$\frac{|C'_j|}{|C_j|} = \frac{a'_1}{a_1} = \frac{(|G_1| - 1)!/\sum_{i=2}^{x}(b_i!)}{(|G_1|!)/\sum_{i=1}^{x}(b_i!)} = \frac{b_1}{|G_1|}. \tag{9}$$

Since G_1 is l-diverse, we have $b_1/|G_1| \le 1/l$. Consequently, $|C'_j|/|C_j| \le 1/l$, which completes the proof. □

Proof of Lemma 4. Given any two sets S_t and S'_t of tuples, we say that they are *cousins*, if S_t and S'_t involve the same set of individuals. To prove the lemma, we first establish the following proposition:

- Proposition 3. Let B and B' be two symmetric buckets, and $\{B_a, B_b\}$ be a division of B on A_i^q ($i \in [1, d]$). Let B'_a (B'_b) be the subset of B', such that B_a and B'_a (B_b and B'_b) are cousins. Then, $\{B'_a, B'_b\}$ is a division of B' on A_i^q. Furthermore, B'_a and B_a (B'_b and B_b) are symmetric.

Let V be the signature of B, and $x = |V|$. Since B and B' are symmetric, V should also be the signature of B'. Because B'_a and B'_b are subsets of B', their signatures should be subsets of V. In the following, we will first show that B'_a is a bucket with a signature V. Assume that this is not true. Then, there must exist a column L'_1 of B'_a, such that $|L'_1| \geq |B'_a|/x$. Let L_1 be the subset of B_a, such that L_1 and L'_1 are cousins. Because B and B' are symmetric, if any two individuals have the same sensitive value in B', they should also have an identical A^s value in B. This indicates that all tuples in L_1 share the same A^s value. Since $|L_1| \geq |B_a|/x$, B_a should have a column with more than $|B_a|/x$ tuples. This contradicts the assumption that B_a is a bucket. Therefore, B'_a must be a bucket with a signature V. By the same reasoning, it can be proved that B'_b is also a bucket with a signature V.

Assume by contradiction that, $\{B'_a, B'_b\}$ is not a division of B' on A_i^q. Then, by Definition 11, there must exist two tuples $t'_a \in B'_a$ and $t'_b \in B'_b$, such that (i) $t'_a[A_i^q] > t'_b[A_i^q]$, or (ii) $t'_a[A_i^q] = t'_b[A_i^q]$ and $t'_a[A^{id}] > t'_b[A^{id}]$. Let t_a and t_b be the tuples in B_a, such that t_a and t'_a (t_b and t'_b) concern the same individual. Then, we have either (i) $t_a[A_i^q] > t_b[A_i^q]$, or (ii) $t_a[A_i^q] = t_b[A_i^q]$ and $t_a[A^{id}] > t_b[A^{id}]$. In that case, $\{B_a, B_b\}$ is not a division of B, leading to a contradiction. Hence, $\{B'_a, B'_b\}$ must be a division of B'.

To prove Proposition 3, it remains to show that B'_a and B_a (B'_b and B_b) are symmetric. Consider any column $L_2 \in B_a$. We have $|L_2| = |B_a|/x = |B'_a|/x$. Let L'_2 be the cousin of L_2 in B'_a. Then, $|L'_2| = |L_2| = |B'_a|/x$. Since B and B' are symmetric, for any individuals with the same sensitive value in B, they should also share an identical A^s value in B'. Therefore, all tuples in L'_2 have the same sensitive value. Observe that, each column in B'_a should contain exactly $|B'_a|/x$ tuples, which indicates that L'_2 is a column in B'_a. In summary, for any column L_2 in B_a, there exists a column L'_2 in B'_a, such that L_2 and L'_2 involve an identical set of individuals. Hence, B'_a is symmetric to B_a. Similarly, it can be shown that B'_b and B_b are symmetric. Thus, Proposition 3 holds.

Now we are ready to prove the lemma. Without loss of generality, assume that $\{B_1, B_2\}$ is a division of B on A_i^q ($i \in [1, d]$). Let B'_3 (B'_4) be the subset of B', such that B'_3 and B_1 (B'_4 and B_2) are cousins. By Proposition 3, $\{B'_3, B'_4\}$ is a division of B' on A_i^q, and B'_3 (B'_4) is symmetric to B_1 (B_2). To establish the lemma, it suffice to show that $\{B'_3, B'_4\}$ is the canonical division of B'. Assume, on the contrary, that $\{B'_3, B'_4\}$ is not canonical. Then, by Definition 12, $\{B'_3, B'_4\}$ should satisfy at least one of the following three conditions:

1. $\{B'_3, B'_4\}$ is not a division of G' with the smallest perimeter.
2. There exists a division $\{B'_5, B'_6\}$ of G' on A_j^q ($j < i$), such that $h_p(B'_5) + h_p(B'_6) = h_p(B'_3) + h_p(B'_4)$.
3. There exists a division $\{B'_5, B'_6\}$ of G' on A_i^q, such that $h_p(B'_5) + h_p(B'_6) = h_p(B'_3) + h_p(B'_4)$.

Assume that $\{B'_3, B'_4\}$ fulfills Condition 3. Let B_5 (B_6) be subset of B, such that B_5 and B'_5 (B_6 and B'_6) are cousins. By Proposition 3, $\{B_5, B_6\}$ is a division of B on A_i^q. Then, $|B_5| = |B'_5| < |B'_3| = |B_1|$. Since each individual has the same QI values in B and B',

$$h_p(B_5) + h_p(B_6) = h_p(B'_5) + h_p(B'_6) \quad = \quad h_p(B'_3) + h_p(B'_4) = h_p(B_1) + h_p(B_2).$$

In that case, $\{B_1, B_2\}$ cannot be the canonical division of B (due to the existence of $\{B_5, B_6\}$), leading to a contradiction. Therefore, $\{B'_3, B'_4\}$ must violate Condition 3. Similarly, it can be verified that $\{B'_3, B'_4\}$ must also violate Conditions 1 and 2, i.e., $\{B'_3, B'_4\}$ should be the canonical division of B'. Thus, the lemma is proved. □

Proof of Lemma 5. Consider that we apply *Slice* on U_1, with the given l value. As shown in Figure 8, *Slice* will iteratively retrieve a bucket $B \in U_1$, compute the canonical division $\{B_a, B_b\}$ of B, and then replace B with B_a and B_b. This process is carried on, until the bucket partition U'_1 is obtained. Let Q_1 be the union of the canonical divisions computed by *Slice* in each iteration, and $Q'_1 = Q_1 \cup U_1$. We organize the buckets in Q'_1 into $|U_1|$ binary trees as follows:

1. For the i-th ($i \in [1, |U_1|]$) binary tree R_i, the root of R_i is the i-th bucket B_i in U_1.
2. For any three buckets $B_1, B_2, B_3 \in Q'$, B_2 and B_3 are the child nodes of B_1, if and only if $\{B_2, B_3\}$ is a division of B_1.

We refer to R_i as the *split history* of B_i. Notice that, U'_1 equals the union of the leaves of each R_i ($i \in [1, |U_1|]$). Next, assume that we apply *Slice* on U_2. Let B'_i denote the bucket in U_2 that is symmetric to B_i ($i \in [1, |U_1|]$). Following the way R_i is generated, we also construct the split history R'_i of B'_i. Then, the leaves of all R'_i ($i \in [1, |U_1|]$) constitute U'_2. To prove the lemma, it suffices to show that, for any $i \in [1, |U_1|]$, each leaf of R_i is symmetric to a leaf of R'_i, and vice versa.

Our proof is by induction. For the base case, the root B_i of R_i is symmetric to the root B'_i of R'_i. Next, assume that two nodes $B \in R_i$ and $B' \in R'_i$ are symmetric. We will show that (i) B is a leaf of R_i, if and only if B' is a leaf of R'_i; (ii) if B is not a leaf, then each child node of B is symmetric to a child node of B'.

As shown in Figure 8, a bucket in R_i or R'_i is a leaf, if and only if it is not divisible, otherwise it would have been split into two smaller buckets by *Slice*. Because B and B' are symmetric, all columns in B and B' have an equal size. Thus, B is not divisible, if and only if B' is not divisible. Hence, B is a leaf, if and only if B' is a leaf.

Next, consider that B is not a leaf. Let $\{B_a, B_b\}$ and $\{B'_a, B'_b\}$ be the canonical divisions of B and B', respectively. By Lemma 4, B_a and B'_a (B_b and B'_b) must be symmetric. Therefore, each child node of B is symmetric to a child node of B'. By induction, it can be shown that each leaf of R_i is symmetric to a leaf of R'_i, and vice versa. Hence, the lemma is proved. □

Proof of Lemma 6. Consider that we apply *Assign* on T_1 with the given l value. As shown in Figure 6, *Assign* first initializes a set $S_t = T_1$, and then iteratively creates buckets using tuples in S_t. Let U be the partition returned by *Assign* at the end, B_i the bucket constructed in the i-th iteration, and S_i the set of tuples in S_t right before the i-th iteration. Next, assume that we run *Assign* on T_2. Let U' be the partition of T_2 generated by *Assign*, B'_i the bucket created in the i-th iteration, and S'_i the set of tuples in S_t prior to the i-th iteration. For simplicity, we say that two buckets are *siblings*, if and only if they have the same size and the same signature. In the following, we will first prove a proposition:

- Proposition 4. for any $i \in [1, |U|]$, B_i and B'_i are siblings.

Consider that $i = 1$. By Lines 4-12 in Figure 6, the signature of B_1 should contain the β most frequent sensitive values in S_t, and $|B_1| = \alpha \cdot \beta$, where the values of α and β are decided by $|S_1|$ and the frequencies of sensitive values in S_1. The above statement still holds, if we change B_1 to B'_1, and S_1 to S'_1. Recall that $S_1 = T_1 = \bigcup_{B \in U_1} B$ and $S'_1 = T_2 = \bigcup_{B' \in U_2} B'$. Since U_1 and U_2 are symmetric, S_1 and S'_1 should have the same size, and include an identical multi-set of sensitive values. Therefore, *Assign* should employ the same α and β values to construct B_1 and B'_1. Thus, B_1 and B'_1 are siblings. Furthermore, because $S_2 = S_1 - B_1$ and $S'_2 = S'_1 - B'_1$, S_2 and S'_2 should have an equal size, and contain the same multi-set of sensitive values. In turn, this indicates that, *Assign* should use identical α and β values to generate B_2 and B'_2, i.e., B_2 and B'_2 are also siblings. By an induction on i, it can be shown that Proposition 4 holds.

To prove the lemma, we regard U and U' as random variables, and show that $Pr\{U = U_1\} = Pr\{U' = U_2\}$. Let us derive $Pr\{U = U_1\}$ first. Recall that each bucket in U is constructed using tuples randomly selected from T_1. Therefore, $Pr\{U = U_1\}$ should equal $1/m$, where m is the total number of possible ways to assign the tuples in T_1 into the buckets in U. Assume that T_1 contains w sensitive values v_1, v_2, ..., v_w. Let n_j ($j \in [1, w]$) be the frequency of v_j in in T_1. Let d_{ij} denote the number of tuples in B_i with sensitive value v_j. For simplicity, define $0! = 1$. We have

$$m = \frac{\prod_{j=1}^{w}(n_j!)}{\prod_{i=1}^{|U|}\prod_{j=1}^{w}(d_{ij}!)}. \tag{10}$$

Next, we will calculate $Pr\{U' = U_2\}$. Since T_1 and T_2 contain the same multi-set of sensitive values, for any $j \in [1, w]$, the frequency of v_j in T_2 is also n_j. Furthermore, because B_i and B'_i ($i \in [1, |U|]$) are siblings, there should exist d_{ij} tuples in B'_i that have a sensitive value v_j. As a result, there are also m distinct ways to assign the tuples in T_2 to the buckets in U'. Therefore, $Pr\{U' = U_2\} = 1/m = Pr\{U = U_1\}$, which completes the proof. □

Proof of Theorem 2. Let T be any microdata, l be any positive integer, and T^* be a possible output of $\mathcal{G}$. Let E be any external source, and S be the set of possible microdata instances based on E. Let o be any individual, v be an arbitrary sensitive value, and $S_{o,v}$ be the subset of S, such that each $\hat{T} \in S_{o,v}$ associates o with v. According to Proposition 1, we can prove Theorem 2 by showing that

$$\frac{\sum_{\hat{T} \in S_{o,v}} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}} \leq \frac{1}{l}. \tag{11}$$

We say that a bucket partition U is a *valid partition*, if T^* can be decided by the partition $U' = \mathcal{G}_B(U)$. Let M be the set of all valid partitions, such that for each $U \in M$, we have $Pr\{\mathcal{G}_A(\hat{T}, l) = U\} > 0$ for some $\hat{T} \in S$. Then,

$$\frac{\sum_{\hat{T} \in S_{o,v}} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{\mathcal{G}(\hat{T}, l) = T^*\}} = \frac{\sum_{\hat{T} \in S_{o,v}} \sum_{U \in M} Pr\{\mathcal{G}_A(\hat{T}, l) = U\}}{\sum_{\hat{T} \in S} \sum_{U \in M} Pr\{\mathcal{G}_A(\hat{T}, l) = U\}}. \tag{12}$$

We define a bucket partition $U \in M$ as a *breaching partition*, if any QI-group $G \in U$ contains a tuple t, such that $t[A^{id}] = o$ and $t[A^s] = v$. Observe that, for any $\hat{T} \in S_{o,v}$, if U is not a breaching partition, then $Pr\{\mathcal{G}_A(\hat{T}, l) = U\} = 0$. We divide M into disjoint clusters, such that each cluster is a maximal subset of symmetric bucket partitions in M. Let n be the total number of clusters in M, and M_j ($j \in [1,n]$) be the j-th cluster. Let M'_j be the set of breaching partitions in M_j. We have

$$\frac{\sum_{\hat{T}\in S_{o,v}} Pr\{\mathcal{G}(\hat{T},l)=T^*\}}{\sum_{\hat{T}\in S} Pr\{\mathcal{G}(\hat{T},l)=T^*\}} = \frac{\sum_{j=1}^{n}\sum_{U\in M'_j}\sum_{\hat{T}\in S_{o,v}} Pr\{\mathcal{G}_A(\hat{T},l)=U\}}{\sum_{j=1}^{n}\sum_{U\in M_j}\sum_{\hat{T}\in S} Pr\{\mathcal{G}_A(\hat{T},l)=U\}}. \tag{13}$$

For simplicity, let $p(U,\hat{T})$ denote $Pr\{\mathcal{G}_A(\hat{T},l)=U\}$, and $q(M,S)$ denote $\sum_{U\in M}\sum_{\hat{T}\in S} p(U,\hat{T})$. We will show that $q(M'_j, S_{o,v})/q(M_j,S) \le 1/l$ for any $j \in [1,n]$. This will lead to

$$\begin{aligned}\frac{\sum_{\hat{T}\in S_{o,v}} Pr\{\mathcal{G}(\hat{T},l)=T^*\}}{\sum_{\hat{T}\in S} Pr\{\mathcal{G}(\hat{T},l)=T^*\}} &= \frac{\sum_{j=1}^{n} q(M'_j,S_{o,v})}{\sum_{j=1}^{n} q(M_j,S)} \\ &\le \frac{\sum_{j=1}^{n} q(M_j,S)/l}{\sum_{j=1}^{n} q(M_j,S)} = \frac{1}{l},\end{aligned} \tag{14}$$

which proves the theorem.

Without loss of generality, consider that $j=1$. Let U_k be the k-th ($k \in [1,|M_1|]$) partition in M_1, and $T_k = \bigcup_{B\in U_k} B$. For any microdata $\hat{T}$ different from T_k, we have $p(U_k,\hat{T}) = 0$, since U_k is not a partition of $\hat{T}$. Therefore, $T_k \in S$ should hold, otherwise $p(U_k,\hat{T}) = 0$ for all $\hat{T} \in S$, which contradicts the assumption that $U_k \in M$. Thus, $\sum_{\hat{T}\in S} p(U_k,\hat{T}) = p(U_k,T_k)$. By our assumption on $\mathcal{G}_A$, for any $k_1, k_2 \in [1,|M_1|]$, we have $p(U_{k_1},T_{k_1}) = p(U_{k_2},T_{k_2})$. Hence,

$$q(M_1,S) = \sum_{j=1}^{|M_1|}\sum_{\hat{T}\in S} p(U_j,\hat{T}) = \sum_{j=1}^{|M_1|} p(U_j,T_j) = |M_1|\cdot p(U_k,T_k).$$

Similarly, it can be verified that $q(M'_1,S_{o,v}) = |M'_1|\cdot p(U_k,T_k)$. Therefore, $q(M'_j,S_{o,v})/q(M_j,S) = |M'_1|/|M_1|$.

Next, we will derive the value of $|M_1|$. Let U_s be any partition symmetric to U_k, and $T_s = \bigcup_{B\in U_s} B$. Then, T_s and T_k should contain the same set of individuals. Hence, $T_s \in S$. Since U_s and U_k are symmetric, $p(U_s,T_s) = p(U_k,T_k) > 0$ holds. Therefore, U_s is a valid partition. Let $U'_s = \mathcal{G}_B(U_s)$, and $U'_k = \mathcal{G}_B(U_k)$. By our assumption on $\mathcal{G}_B$, U'_s and U'_k are symmetric. Observe that symmetric partitions are isomorphic, and thus, they always lead to the same anonymization. Since U'_k and f decides T^*, U'_s and f should also determine T^*, which indicates that $U_s \in M$. In other words, any bucket symmetric to U_k should be contained in M. Consequently, by the definition of M_1, $|M_1|$ equals the total number of partitions symmetric to U_k.

By Definition 13, we can obtain any partition symmetric to U_k, by substituting any buckets in U_k with their symmetric counterparts. Let B_i be the i-th ($i \in [1,|U_k|]$) bucket in U_k, and α_i be the number of buckets symmetric to B_i. Then, $|M_1| = \prod_{i=1}^{|U_k|}\alpha_i$. Without loss of generality, assume that o appears in B_1. Among all buckets symmetric to B_1, let α'_1 be the number of them that contain a tuple t, with $t[A^{id}] = o$ and $t[A^s] = v$. We have $|M_1| = \alpha'_1 \cdot \prod_{i=2}^{|U_k|}\alpha_i$. Therefore, $|M'_1|/|M_1| = \alpha'_1/\alpha_1$.

Assume B_1 has a signature V with x sensitive values. If $v \notin V$, then $\alpha'_1 = 0$. Consider that $v \in V$. Recall that, we can transform B_1 into any bucket symmetric to B_1, by swapping the sensitive values between different columns of B_1. Totally, there are $x!$ distinct ways to assign x sensitive values to the x columns of B_1. Because each of these assignment corresponds to bucket symmetric to B_1, we have $\alpha_1 = x!$. Next, consider that we assign an A^s value v to the column that o appears. The other $x-1$ sensitive values can be assigned in $(x-1)!$ different manner, i.e., $\alpha'_1 = (x-1)!$. Hence, $\alpha'_1/\alpha_1 = 1/x$. According to the way *Assign* constructs each bucket, we have $x \geq l$. Therefore, $|M'_1|/|M_1| = \alpha'_1/\alpha_1 \leq 1/l$, which completes the proof. □

Proof of Theorem 3. Let S the set of possible microdata instances based on E, and v be an arbitrary sensitive value. Let $S_{o,v}$ be the subset of S, such that each $\hat{T} \in S_{o,v}$ involves o, and sets v as the A^s value of o. By Proposition 1, Theorem 3 holds if and only if

$$\frac{\sum_{\hat{T} \in S_{o,v}} Pr\{Hybrid(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{Hybrid(\hat{T}, l) = T^*\}} \leq \frac{1}{l}. \tag{15}$$

Consider that we apply *Hybrid* on any $\hat{T} \in S$, with the given l value. *Hybrid* first employs *Tailor* to obtain a partition P of $\hat{T}$. We define P as the *essential partition* of $\hat{T}$, and use G_j to denote the j-th ($j \in [1, |P|]$) QI-group in P. Then, *Hybrid* invokes *Ace* to transform each $G_j \in P$ into a set T_j^* of anonymized tuples. We define the ordered set $\{T_1^*, T_2^*, ..., T_{|P|}^*\}$ as a *decomposition* of P. Since *Ace* is a randomized algorithm, there may exist multiple decompositions of P. At last, *Hybrid* returns the union T^* of all T_j^*. We use $\gamma(P, T^*)$ to denote the probability that *Hybrid* transforms P into T^*.

Let Q (Q') be a set that includes the essential partition of any $\hat{T} \in S$ ($\hat{T} \in S_{o,v}$). We divide Q into several clusters, such that each cluster is a maximal set of isomorphic partitions in Q. Let n be the total number of clusters in Q, and C_k ($k \in [1, n]$) be the k-th cluster. Let $C'_k = C_k \cap Q'$. Then, we have

$$\frac{\sum_{\hat{T} \in S_{o,v}} Pr\{Hybrid(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{Hybrid(\hat{T}, l) = T^*\}} = \frac{\sum_{j=1}^{n} \sum_{P \in C'_k} \gamma(P, T^*)}{\sum_{k=1}^{n} \sum_{P \in C_k} \gamma(P, T^*)}. \tag{16}$$

We will prove that, for any $k \in [1, n]$,

$$\frac{\sum_{P \in C'_k} \gamma(P, T^*)}{\sum_{P \in C_k} \gamma(P, T^*)} \leq \frac{1}{l}. \tag{17}$$

This will establish the Theorem, since it ensures that

$$\begin{aligned} \frac{\sum_{\hat{T} \in S_{o,v}} Pr\{Hybrid(\hat{T}, l) = T^*\}}{\sum_{\hat{T} \in S} Pr\{Hybrid(\hat{T}, l) = T^*\}} &= \frac{\sum_{j=1}^{n} \sum_{P \in C'_k} \gamma(P, T^*)}{\sum_{k=1}^{n} \sum_{P \in C_k} \gamma(P, T^*)} \\ &\leq \frac{\sum_{j=1}^{n} \sum_{P \in C_k} \gamma(P, T^*)/l}{\sum_{k=1}^{n} \sum_{P \in C_k} \gamma(P, T^*)} = \frac{1}{l}. \end{aligned}$$

Without loss of generality, consider that $k = 1$. Let P be an arbitrary partition in C_1, and G_j the j-th QI-group in P. Assume that o is involved in G_1. Further assume that, for any $P' \in C_1$, the j-th ($j \in [1, |P|]$) QI-group in P' is isomorphic to G_j. Then, for any $P' \in C_1$, o should appear in the first QI-group of P'. We split C_1 into sub-clusters, such that for any two partitions in the same sub-cluster, they coincide on all but the first QI-group. Let n' be the number of sub-clusters in

C_1, D_i ($i \in [1, n']$) be the i-th sub-cluster, and $D'_i = D_i \cap Q'$. To prove that Equation 17 holds for $k = 1$, it suffices to show that, for any $i \in [1, n']$,

$$\frac{\sum_{P \in D'_i} \gamma(P, T^*)}{\sum_{P \in D_i} \gamma(P, T^*)} \leq \frac{1}{l}. \tag{18}$$

This is because, once the above inequality is established, we have

$$\begin{aligned} \frac{\sum_{P \in C'_1} \gamma(P, T^*)}{\sum_{P \in C_1} \gamma(P, T^*)} &= \frac{\sum_{i=1}^{n'} \sum_{P \in D'_i} \gamma(P, T^*)}{\sum_{i=1}^{n'} \sum_{P \in D_i} \gamma(P, T^*)} \\ &\leq \frac{\sum_{i=1}^{n'} \sum_{P \in D_i} \gamma(P, T^*)/l}{\sum_{i=1}^{n'} \sum_{P \in D_i} \gamma(P, T^*)} = \frac{1}{l}. \end{aligned} \tag{19}$$

Assume, without loss of generality, that $i = 1$. Let P_x ($x \in [1, |D_1|]$) be x-th partition in D_1, and G_{xm} be the m-th ($m \in [1, |P_x|]$) QI-group in P_x. Let S_d be a set containing any decomposition of any $P_x \in D_1$, and Ω be the subset of S_d, such that each decomposition $W \in \Omega$ leads to T^*, i.e., $\bigcup_{T^*_s \in W} T^*_s = T^*$. Let W_j be the j-th decomposition in Ω, and T^*_{jm} the m-th set of anonymized tuples in W_j. Observe that $|W_j| = |P_x|$ for any $j \in [1, |\Omega|]$ and any $x \in [1, |D_1|]$. By the definition of $\gamma(P_x, T^*)$, we have

$$\gamma(P_x, T^*) = \sum_{j=1}^{|\Omega|} \prod_{m=1}^{|P_x|} Pr\{Ace(G_{xm}, l) = T^*_{jm}\} \tag{20}$$

For simplicity, we denote $\prod_{m=1}^{|P_x|} Pr\{Ace(G_{xm}, l) = T^*_{jm}\}$ as $p(P_x, W_j)$. Then,

$$\frac{\sum_{P \in D'_1} \gamma(P, T^*)}{\sum_{P \in D_1} \gamma(P, T^*)} = \frac{\sum_{P_x \in D'_1} \sum_{j=1}^{|\Omega|} p(P_x, W_j)}{\sum_{P_x \in D_1} \sum_{j=1}^{|\Omega|} p(P_x, W_j)}. \tag{21}$$

To prove that Equation 18 is valid when $i = 1$, we will show that

$$\frac{\sum_{P_x \in D'_1} p(P_x, W_j)}{\sum_{P_x \in D_1} p(P_x, W_j)} \leq \frac{1}{l}, \tag{22}$$

for any $j \in [1, |\Omega|]$. In particular, the above inequality ensures that

$$\begin{aligned} \frac{\sum_{P \in D'_1} \gamma(P, T^*)}{\sum_{P \in D_i} \gamma(P, T^*)} &= \frac{\sum_{P_x \in D'_1} \sum_{j=1}^{|\Omega|} p(P_x, W_j)}{\sum_{P_x \in D_1} \sum_{j=1}^{|\Omega|} p(P_x, W_j)} \\ &\quad \text{(by Equation 21)} \\ &\leq \frac{\sum_{j=1}^{|\Omega|} \sum_{P_x \in D_1} p(P_x, W_j)/l}{\sum_{j=1}^{|\Omega|} \sum_{P_x \in D_1} p(P_x, W_j)} = \frac{1}{l} \end{aligned} \tag{23}$$

Let $q(G_{xm}, T^*_{jm})$ denote $Pr\{Ace(G_{xm}, l) = T^*_{jm}\}$. Recall that any two partitions in D_1 coincide on all but the first QI-group. Therefore, given any $m \in [2, |P_x|]$ and any $j \in [1, |\Omega|]$, the value of

$q(G_{xm}, T^*_{jm})$ is fixed for all $P_x \in D_1$. Let r_j denote $\prod_{k=2}^{|P_x|} q(G_{xm}, T^*_{jm})$. Then,

$$\begin{aligned} p(P_x, W_j) &= \prod_{m=1}^{|P_x|} Pr\{Ace(G_{xm}, l) = T^*_{jm}\} \\ &= \prod_{m=1}^{|P_x|} q(G_{xm}, T^*_{jm}) \\ &= r_j \cdot q(G_{x1}, T^*_{j1}). \end{aligned} \tag{24}$$

Therefore, for any $j \in [1, |\Omega|]$,

$$\begin{aligned} \frac{\sum_{P_x \in D'_1} p(P_x, W_j)}{\sum_{P_x \in D_1} p(P_x, W_j)} &= \frac{\sum_{P_x \in D'_1} \left(r_j \cdot q(G_{x1}, T^*_{j1})\right)}{\sum_{P \in D_1} \left(r_j \cdot q(G_{x1}, T^*_{j1})\right)} \\ &= \frac{\sum_{P_x \in D'_1} q(G_{x1}, T^*_{j1})}{\sum_{P_x \in D_1} q(G_{x1}, T^*_{j1})}. \end{aligned} \tag{25}$$

Consequently, to prove Equation 22, it suffices to show that

$$\frac{\sum_{P_x \in D'_1} q(G_{x1}, T^*_{j1})}{\sum_{P_x \in D_1} q(G_{x1}, T^*_{j1})} \le \frac{1}{l}, \tag{26}$$

for any $j \in [1, |\Omega|]$.

Let S_1 be a set containing all G_{x1} ($x \in [1, |D_1|]$), and S'_1 the maximal subset of S_1, such that each $G \in S'_1$ contains a tuple t with $t[A^{id}] = o$ and $t[A^s] = v$. Then,

$$\frac{\sum_{P_x \in D'_1} q(G_{x1}, T^*_{j1})}{\sum_{P_x \in D_1} q(G_{x1}, T^*_{j1})} = \frac{\sum_{G \in S'_1} q(G, T^*_{j1})}{\sum_{G \in S_1} q(G, T^*_{j1})}. \tag{27}$$

By the definition of D_1, all QI-groups in S_1 are isomorphic. Therefore, all QI-groups in S_1 have the same projection on the identifier and QI attributes. Denote this projection as E. If we regard each QI-group $G_{x1} \in S_1$ as a tiny microdata table, then E can be deemed as an external source for G_{x1}. Let S_2 be the set of all possible instances based on E. Let S'_2 be the set of instances in S_2 that contain a tuple t, with $t[A^{id}] = o$ and $t[A^s] = v$. By Theorem 2, given E as the external source, any T^*_{j1} ($j \in [1, |\Omega|]$) ensures that the disclosure risk of o is at most $1/l$, i.e.,

$$\frac{\sum_{G \in S'_2} q(G, T^*_{j1})}{\sum_{G \in S_2} q(G, T^*_{j1})} \le \frac{1}{l}. \tag{28}$$

By Equations 27 and 28, we can establish Equation 26 by showing that

$$\frac{\sum_{G \in S'_1} q(G, T^*_{j1})}{\sum_{G \in S_1} q(G, T^*_{j1})} = \frac{\sum_{G \in S'_2} q(G, T^*_{j1})}{\sum_{G \in S_2} q(G, T^*_{j1})}. \tag{29}$$

For this purpose, it suffices to prove that $q(G, T^*_{j1}) = 0$, for any $G \in (S_1 - S_2) \cup (S_2 - S_1)$ and any $G \in (S'_1 - S'_2) \cup (S'_2 - S'_1)$.

Since S_2 contains all microdata instances based on E, we have $G_{x1} \in S_1$ for any $x \in [1, |D_1|]$. Therefore, $S_1 \subseteq S_2$, which indicates that $S'_1 \subseteq S'_2$. Hence, $S_1 - S_2 = S'_1 - S'_2 = \emptyset$. Now consider any $P_x \in D_1$ ($x \in [1, |D_1|]$). Assume that we construct a partition P'_x from P_x, by replacing G_{x1} with any of its isomorphic counterparts. Then, P'_x should be isomorphic to P_x. By Lemma 3, P'_x is an essential partition of some $\hat{T} \in S$, i.e., $P'_x \in Q$. Since P'_x and P_x are isomorphic, and coincide on all but the first QI-group, $P'_x \in D_1$ holds. In other words, for any G isomorphic to G_{x1}, there exists a partition $P_i \in D_1$ ($i \in [1, |D_1|]$), such that $G = G_{i1}$. Hence, S_1 contains any QI-group isomorphic to G_{x1}.

Recall that, any T^*_{j1} ($j \in [1, |\Omega|]$) is a anonymization of a certain QI-group in S_1. Since all QI-groups in S_1 are isomorphic, they contain the same multi-set of sensitive values. This indicates that any T^*_{j1} and any G_{x1} ($x \in [1, |D_1|]$) have an identical multi-set of sensitive values. Let G' be a QI-group, such that $G' \in S_2 - S_1$. Then, G' and G_{x1} are not isomorphic, but involve the same set of individuals. Therefore, G' and G_{x1} must contain distinct multi-sets of A^s values. Hence, for any $j \in [1, |\Omega|]$, the multi-sets of sensitive values in G' and T^*_j are different, i.e., G' cannot be anonymized to T^*_j. Therefore, $q(G, T^*_{j1}) = 0$, for any $G \in S_1 - S_2$. Similarly, it can be shown that $q(G, T^*_{j1}) = 0$, for any $G \in S'_2 - S'_1$. Thus, Equation 29 is valid. In turn, this establishes Equations 29, 26, 22, 18, and 17. Hence, the theorem is proved. □

Appendix II: Privacy attack on *Mask*

Next, we exemplify an attack against the *Mask* algorithm [36], which is designed under the credibility model [36]. Figure 16 illustrates the pseudo-code of *Mask*. The algorithm takes as input a microdata table T, two positive integers k and l, and a subset V of the A^s values. It aims to ensure that, for any individual o and any sensitive value $v \in V$, the adversary would have at most $1/l$ posterior belief in the event that "o appears in T and has a sensitive value v". We will explain the details of *Mask* using an example.

Example 7 Suppose that we apply *Mask* on the microdata T_9 in Table 15, by setting $k = l = 2$ and $V = \{dyspepsia\}$. *Mask* first generates a k-anonymous partition P of T_9, using any of the existing k-anonymity algorithms (Line 1 in Figure 16). Assume that P contains three QI-groups, namely, {Ann, Bob}, {Cate, Don}, and {Ed, Fred, Gill}. Next, *Mask* divides P into two disjoint subsets P_1 and P_2 (Lines 2-6). In particular, P_1 contains all the QI-groups G in P, such that at least one sensitive value in V appears more than $|G|/l$ times in G. Meanwhile, $P_2 = P - P_1$. In our example, P_1 contains only one QI-group $G' = \{\text{Ann, Bob}\}$.

After that, *Mask* randomly chooses a QI-group G^+ from P_2, and then modifies the sensitive values in G', so that G' and G^+ have the same sensitive value distribution (Lines 7-9). Assume that $G^+ = \{\text{Cate, Don}\}$. Then, G' will be modified in a way, such that 50% of tuples in G' would have a sensitive value *flu*, and the other 50% would have *dyspepsia*. Table 16 illustrates a possible result of the modification. Finally, *Mask* returns the anonymization decided by the modified partition and an anonymization function (say, the MBR function), as illustrated in Table 17. □

As T^*_{10} (in Table 17) is produced by *Mask* with $l = 2$, under the credibility model, an adversary has at most 1/2 posterior belief in the event that "Ann has dyspepsia in the microdata". In the

Algorithm *Mask* (T, k, l, V)

```
generate a k-anonymous partition P of T
P1 = P2 = ∅
for each QI-group G ∈ P
    if one of the sensitive value in V appears more than |G|/l times in G
        insert G into P1
    else insert G into P2
for each QI-group G' ∈ P1
    randomly choose a QI-group G+ ∈ P2
    modify the sensitive values in G', such that the distribution of sensitive values in
    G' becomes the same as that in G+
return the anonymization decided by P1 ∪ P2 and an anonymization function
```

Figure 16: The *Mask* algorithm

Name	**Age**	**Disease**
Ann	21	dyspepsia
Bob	27	dyspepsia
Cate	32	dyspepsia
Don	32	flu
Ed	54	flu
Fred	60	flu
Gill	60	flu

Table 15: Microdata T_9

Name	**Age**	**Disease**
Ann	21	flu
Bob	27	dyspepsia
Cate	32	dyspepsia
Don	32	flu
Ed	54	flu
Fred	60	flu
Gill	60	flu

Table 16: Partition P'

Age	**Disease**
[21, 27]	flu
[21, 27]	dyspepsia
32	dyspepsia
32	flu
[54, 60]	flu
[54, 60]	flu
[54, 60]	flu

Table 17: Generalization T_{10}^*

following, however, we will show that the posterior belief of the adversary can be boosted to 5/8, if s/he has (i) the details of *Mask*, (ii) the parameters k, l, and V with which T_{10}^* is computed, and (iii) an external source that contains only the seven individuals in T_9.

Upon observing T_{10}^*, the adversary knows that T_{10}^* is generated from a partition P with three QI-groups $G_1 = \{\text{Ann, Bob}\}$, $G_2 = \{\text{Cate, Don}\}$, and $G_3 = \{\text{Ed, Fred, Gill}\}$. In addition, the adversary can infer that all sensitive values in G_3 must have not been modified by *Mask*. Otherwise, the distribution of sensitive values in G_3 must be adopted from another QI-group in Table 16, which is impossible since neither G_1 nor G_2 has the same sensitive value distribution as G_3. On the other hand, the sensitive values in G_1 and G_2 may or may not have been modified by *Mask*. This leads to three different cases:

1. *Both G_1 and G_2 have been modified.* This case is impossible; otherwise, the distributions of sensitive values in G_1 and G_2 should have been transformed to the same as in G_3, which is the only QI-group in P that satisfies 2-diversity.

2. *Either G_1 or G_2 has been modified.* In this case, one of G_1 and G_2 should contain two *dyspepsia* before modification (since *dyspepsia* is the only value in V), while the other one should have one *flu* and one *dyspepsia*. This results in 4 possible microdata instances, 3 of which assign *dyspepsia* to Ann.

3. *Neither G_1 nor G_2 has been modified.* This leads to 4 possible microdata instances, 2 of which associate Ann with *dyspepsia*.

In summary, from the adversary's perspective, there exist 8 possible microdata instances that can be generalized into T^*_{10}, among which 5 instances associate Ann with *dyspepsia*. Therefore, the adversary has 5/8 posterior belief in the event that "Ann has dyspepsia".