Basics of Vector- Supplement

See If You Need This Video!

1. Which of the following(s) is(are) vectors?

    1. 2
    2. $|3\hat{x}|$
    3. $\overrightarrow{B}$
    4. $2\hat{x} \times \hat{z}$
    5. $\hat{x} \cdot \hat{x}$

    A. 3 only
    B. 3, 4 only
    C. 2, 3, 5 only
    D. 2, 3, 4, 5 only
    E. All are vectors.

2. Which of the following pair satisfies $\overrightarrow{A} = \overrightarrow{B}$

	$\overrightarrow{A}$	$\overrightarrow{B}$
A.	$2\hat{x}$	$3\hat{x} - \hat{x}$
B.	$2\hat{y}$	$\hat{y}$
C.	$3\hat{z}$	$3\hat{y}$
D.	$\hat{x} + \hat{y}$	$\hat{y} + \hat{z}$
E.	$\hat{x}$	$-\hat{x}$

3. Find $\overrightarrow{A} + \overrightarrow{B}$

$$\overrightarrow{A} = 3\hat{x} + \hat{y}, \quad \overrightarrow{B} = -5\hat{y}$$

A. $-2\hat{x} + \hat{y}$
B. -2
C. $3\hat{x} - 4\hat{y}$
D. $-\hat{z}$
E. No head and tail is shown, not doable.

4. Perform the following dot product $\overrightarrow{A} \cdot \overrightarrow{B}$

$$\overrightarrow{A} = 5\hat{x} + \hat{z}, \quad \overrightarrow{B} = -\hat{z}$$

A. -1
B. $5\hat{x} - 1$
C. $-5\hat{y}$
D. $-5\hat{x}\hat{z} - \hat{z}\hat{z}$
E. 1

5. Which of the following(s) about dot product is(are) correct?

1. $\hat{x} \cdot \hat{y} = 0$
2. $\hat{x} \cdot \hat{x} = 1$
3. $\hat{y} \cdot \hat{z} = \hat{x}$
4. $\overrightarrow{A} \cdot \overrightarrow{A} = |A|^2$
5. $\overrightarrow{A} \cdot -\overrightarrow{A} = |A|^2$

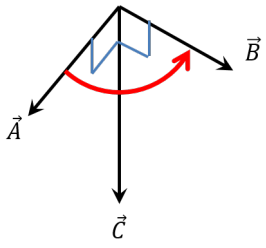
A. 2 only
B. 4, 5 only
C. 2, 4 only
D. 1, 2, 3 only
E. 2, 4, 5 only

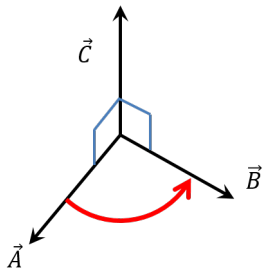
6. Perform the following cross product $\overrightarrow{A} \times \overrightarrow{B}$

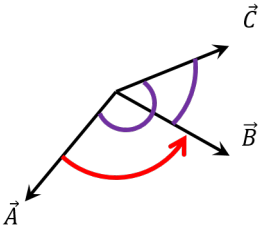
$$\overrightarrow{A} = 2\hat{y} - \hat{z}, \quad \overrightarrow{B} = \hat{x}$$

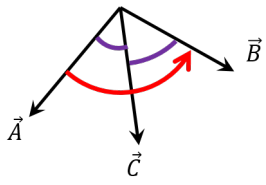
A. $-\hat{y} + 2\hat{z}$
B. $2\hat{x}\hat{y} - \hat{x}\hat{z}$
C. $-\hat{y} - 2\hat{z}$
D. $\hat{x} + 2\hat{y} - \hat{z}$
E. 0

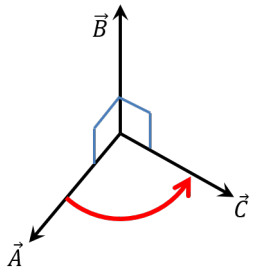
7. Which of the following is describing the $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{C}$?

A.


B.


C.


D.


E.


8. Which of the following(s) about cross product is (are) correct?

   1. $\hat{y} \times \hat{x} = \hat{z}$
   2. $\hat{x} \times \hat{x} = 0$
   3. $\hat{y} \times \hat{z} = 1$
   4. $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{B} \times \overrightarrow{A}$
   5. $\overrightarrow{A} \times \overrightarrow{A} = \overrightarrow{0}$

   A. 5 only
   B. 1, 4 only
   C. 2, 3 only
   D. 4, 5 only
   E. 2, 5 only

9. Which of the following(s) is (are) correct?

   1. $\hat{x} \times (\hat{x} \times \hat{z}) = \overrightarrow{0}$
   2. $\hat{z} \cdot (\hat{x} \times \hat{y}) = 0$
   3. $\hat{z} \cdot \hat{x} \parallel \hat{y}$
   4. If $\overrightarrow{A} = \overrightarrow{B} \neq \overrightarrow{0}$, $\overrightarrow{A} \cdot \overrightarrow{B} = 0$

   A. 1 only
   B. 1, 3 only
   C. 2, 3 only
   D. 2, 4 only
   E. None of them is correct.

10. Which of the following finds the x-component of an unknown vector $\overrightarrow{A}$?
    A. $\overrightarrow{A} + \hat{x}$
    B. $\overrightarrow{A} \cdot \hat{x}$
    C. $\overrightarrow{A} \times \hat{x}$
    D. $\left|\overrightarrow{A}\right|$
    E. $\overrightarrow{A} - \hat{x}$

Learn More

There are a few identities that are useful for solving complicated cases.

$$\begin{aligned}\overrightarrow{A}\cdot\left(\overrightarrow{B}\times\overrightarrow{C}\right) &= \overrightarrow{C}\cdot\left(\overrightarrow{A}\times\overrightarrow{B}\right)\\ &= \overrightarrow{B}\cdot\left(\overrightarrow{C}\times\overrightarrow{A}\right)\end{aligned}$$

It keeps the order that loops $A\to B\to C\to A$...

$$\overrightarrow{A}\times\left(\overrightarrow{B}\times\overrightarrow{C}\right)=\left(\overrightarrow{A}\cdot\overrightarrow{C}\right)\overrightarrow{B}-\left(\overrightarrow{A}\cdot\overrightarrow{B}\right)\overrightarrow{C}$$

Think about reversing the order to flip the sign.

$$\left(\overrightarrow{B}\times\overrightarrow{C}\right)\times\overrightarrow{A}=-\left(\overrightarrow{A}\cdot\overrightarrow{C}\right)\overrightarrow{B}+\left(\overrightarrow{A}\cdot\overrightarrow{B}\right)\overrightarrow{C}$$

Now if you write $(A,B,C)\to(C,A,B)$ you have

$$\left(\overrightarrow{A}\times\overrightarrow{B}\right)\times\overrightarrow{C}=\left(\overrightarrow{C}\cdot\overrightarrow{A}\right)\overrightarrow{B}-\left(\overrightarrow{C}\cdot\overrightarrow{B}\right)\overrightarrow{A}$$

Now we have $\overrightarrow{A}\times\left(\overrightarrow{B}\times\overrightarrow{C}\right)\neq\left(\overrightarrow{A}\times\overrightarrow{B}\right)\times\overrightarrow{C}$, impressed? Think about it carefully, and reason how it happens by yourself. (No typo in these few lines, it won't be my fault:))

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

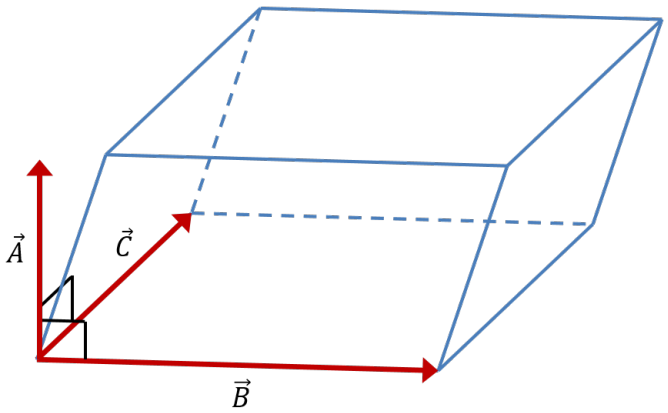
Challenge 1.
Try to prove

$$\begin{aligned}\vec{A} \cdot \left(\vec{B} \times \vec{C}\right) &= \vec{C} \cdot \left(\vec{A} \times \vec{B}\right) \\ &= \vec{B} \cdot \left(\vec{C} \times \vec{A}\right)\end{aligned}$$

and

$$\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \left(\vec{A} \cdot \vec{C}\right) \vec{B} - \left(\vec{A} \cdot \vec{B}\right) \vec{C}$$

Challenge 2.
Show that the volume of the parallelogram shown is $\vec{A} \cdot \left(\vec{B} \times \vec{C}\right)$



"Guide"

You can always get other ways to solve the problem. This "guide" is just a helping hand, not the law.

Challenge 1.

$$\overrightarrow{A} \cdot \left(\overrightarrow{B} \times \overrightarrow{C} \right) = \overrightarrow{C} \cdot \left(\overrightarrow{A} \times \overrightarrow{B} \right)$$

Two ways to prove it:

1. Sit down and spend 10 mins. to expand it, write it carefully and clearly. Sometimes you spend less time on using silly ordinary way than thinking of a smart short-cut.

2. Make use of the result from challenge 2, who labels the vectors $\overrightarrow{A}, \overrightarrow{B}, \overrightarrow{C}$? Free yourself from the symbols and think about the true meaning of the equation.

$$\overrightarrow{A} \times \left(\overrightarrow{B} \times \overleftarrow{C} \right) = \left(\overrightarrow{A} \cdot \overrightarrow{C} \right) \overrightarrow{B} - \left(\overrightarrow{A} \cdot \overrightarrow{B} \right) \overrightarrow{C}$$

Just work it it out step-by-step.

Challenge 2.
Make sure you know what is the goal, otherwise you are just thinking without focus. You have to link two things together, the volume and $\overrightarrow{A} \cdot \left(\overrightarrow{B} \times \overrightarrow{C} \right)$.

1. You have to know what is the volume (at least how to calculate it) before you link them together, so what is the volume? Base area times height?

2. Break the equation into steps, what is $\overrightarrow{B} \times \overrightarrow{C}$? Then what is this $\overrightarrow{B} \times \overrightarrow{C}$ dot $\overrightarrow{A}$?

3. Take a look at this $\overrightarrow{A} \cdot \left(\overrightarrow{B} \times \overrightarrow{C} \right)$, how does this thing link to volume based on the last 2 steps?