Basics of Vector- Supplement Solution Set

See If You Need This Video!

1. Answer: B.

    1. 2 is just a scalar, no direction.
    2. $|3\hat{x}|$ is the magnitude of the vector $3\hat{x}$.
    3. Vectors are normally represented by over-head-arrow or bold-typed.
    4. Result of cross product is a vector
    5. Result of dot product is scalar

    ≪Basic idea of vectors≫
    Episode 1- Cantonese: 0:15 English: 0:13 Putonghua: 0:14
    ≪Cross product (if you get 4. wrong)≫
    Episode 2- Cantonese: 0:09 English: 0:05 Putonghua: 0:10
    ≪Dot product (if you get 5. wrong)≫
    Episode 2- Cantonese: 0:09 English: 0:05 Putonghua: 0:10

2. Answer: A.
    Vectors equals to each other only if they have same direction and magnitude.
    Choice B, $\overrightarrow{A}$ and $\overrightarrow{B}$ have different magnitude. Choice C, D, E, $\overrightarrow{A}$ and $\overrightarrow{B}$ have different directions.
    Even in choice E, $\overrightarrow{A}$ and $\overrightarrow{B}$ are parallel to $\hat{x}$, the have opposite direction.

    ≪Basic idea of vectors≫
    Episode 1- Cantonese: 0:15 English: 0:13 Putonghua: 0:14
    ≪Vectors addition (if you don't know how $\overrightarrow{A} = \overrightarrow{B}$)≫
    Episode 1- Cantonese: 4:10 English: 3:40 Putonghua: 3:26

3. Answer: C.

$$\overrightarrow{A} + \overrightarrow{B} = [3\hat{x} + \hat{y}] + [-5\hat{y}]$$

We can perform vectors addition component-by-component.

$$\begin{aligned}\overrightarrow{A} + \overrightarrow{B} &= (3+0)\,\hat{x} + (1-5)\,\hat{y} \\ &= 3\hat{x} - 4\hat{y}\end{aligned}$$

Vectors addition or subtraction only gives you a vector (even it is a zero vector $\overrightarrow{0}$.

≪Vectors addition≫
Episode 1- Cantonese: 6:27 English: 5:45 Putonghua: 5:52

4. Answer: A
Dot Product can be done by algebra like

$$\overrightarrow{A} \cdot \overrightarrow{B} = A_x B_x + A_y B_y + A_z B_z$$

So we find

$$\begin{aligned}\overrightarrow{A} \cdot \overrightarrow{B} &= (5)\,(0) + (1)\,(-1) \\ &= -1\end{aligned}$$

Dot product always give you a scalar, not a vector. It can be positive or negative or zero.

≪Dot product≫
Episode 2- Cantonese: 0:09 English: 0:05 Putonghua: 0:10

Choice D is NOT performing dot product, it is a wrong answer. However, the writing like $\hat{x}\hat{z}$ is an advanced mathematical idea called dyadic, you may search by yourself if you are interested.

5. Answer: C.

    1. Dot product only gets 0 when two vectors are $\perp$ or one of them with 0 magnitude.
    2. Nothing special, it is correct.
    3. It is not cross product, it is dot product!
       You only get a scalar after dot product.
    4. It is correct, and it is an useful relation.
    5. If you get 4 right, you won't get this if $\overrightarrow{A} \neq \overrightarrow{0}$

    ≪Dot product≫
    Episode 2- Cantonese: 1:10 English: 1:22 Putonghua: 1:23

6. Answer: C.
    Useful relation about cross-product that you may memorized!

$$\begin{aligned}\hat{x} \times \hat{y} &= \hat{z} \\ \hat{y} \times \hat{z} &= \hat{x} \\ \hat{z} \times \hat{x} &= \hat{y}\end{aligned}$$

    And we may separate the component like scalar multiplication.

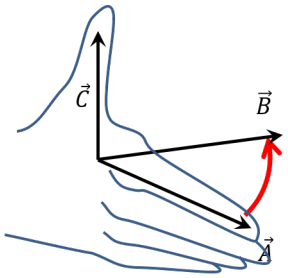
$$\begin{aligned}\overrightarrow{A} \times \overrightarrow{B} &= (2\hat{y} - \hat{z}) \times \hat{x} \\ &= 2\hat{y} \times \hat{x} + (-\hat{z}) \times \hat{x} \\ &= -2\hat{z} - \hat{y}\end{aligned}$$

    Please be noted that when order of cross product inverted, it gets opposite sign.

$$\overrightarrow{A} \times \overrightarrow{B} = -\overrightarrow{B} \times \overrightarrow{A}$$

    ≪Cross product≫
    Episode 2- Cantonese: 5:29 English: 4:56 Putonghua: 5:24

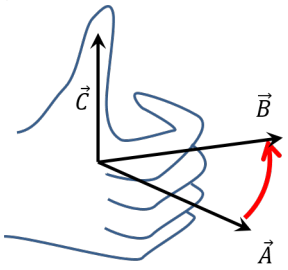
7. Answer: B.
It tests if you know right hand-rule for cross product.
Take out your right hand and tried like this (This $\overrightarrow{A}$ is different from that $\overrightarrow{A}$ in the question, but you are clever enough to change the view point):



Right Hand

1) Align your fingers despite thumb along $\overrightarrow{A}$

↓



Right Hand

2) Sweep them towards $\overrightarrow{B}$ through smallest angle

↓

3) You get the cross product with direction along your thumb.

NOT left hand, so not choice A.
Be careful of which vector is crossing which vector to give which vector, it cant be choice E.
The cross product $\overrightarrow{C} \perp \overrightarrow{A}$ and $\overrightarrow{C} \perp \overrightarrow{B}$, choice C and D are wrong.

≪Cross product≫
Episode 2- Cantonese: 3:57 English: 3:26 Putonghua: 3:54

8. Answer: A.
   Useful relation about cross-product that you may memorized!

$$\hat{x} \times \hat{y} = \hat{z}$$
$$\hat{y} \times \hat{z} = \hat{x}$$
$$\hat{z} \times \hat{x} = \hat{y}$$

   1. $\hat{y} \times \hat{x} = -\hat{z}$ NOT $\hat{z}$.
   2. Although $\hat{x} \times \hat{x}$ is zero vector $\overrightarrow{0}$, but it is a vector not scalar 0. :p
   3. Cross product only gives you a vector even if it is a zero vector $\overrightarrow{0}$.
   4. $\overrightarrow{A} \times \overrightarrow{B} = -\overrightarrow{B} \times \overrightarrow{A}$, when the order of cross product inverted, sign flips.
   5. When two vectors have same direction, their cross product is always zero.

   ≪Cross product≫
   Episode 2- Cantonese: 5:29 English: 4:56 Putonghua: 5:24

9. Answer: E.

   1. $\hat{x} \times \hat{z} = -\hat{y}$ then $\hat{x} \times (-\hat{y}) = -\hat{z}$.

   2. $\hat{x} \times \hat{y} = \hat{z}$ then $\hat{z} \cdot \hat{z} = 1$.

   3. $\hat{z} \cdot \hat{x} = 0$, dot product gives scalar only, no direction. So a scalar can't parallel to a vector.

   4. If $\overrightarrow{A} = \overrightarrow{B} \neq \overrightarrow{0}$, $\overrightarrow{A} \cdot \overrightarrow{A} = \left|\overrightarrow{A}\right|^2 \neq 0$

   ≪Dot product≫
   Episode 2- Cantonese: 1:10 English: 1:22 Putonghua: 1:23
   ≪Cross product≫
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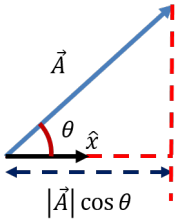
10. Answer: B.



Dot Product $\overrightarrow{A} \cdot \hat{x} = \left|\overrightarrow{A}\right| \cos\theta$, where θ is the angle between $\overrightarrow{A}$ and $\hat{x}$. As $\hat{x}$ has length 1, we have the x-component of $\overrightarrow{A}$.

≪Dot product≫
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