Matrix- Supplement

See If You Need This Video!

Consider the following matrix for Q.$1-2$.

1. How many rows and columns are there in the matrix?

$$\mathbf{M} = \begin{pmatrix} 7 & 8 & 9 & 0 \\ a & i & j & k \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

	Row	Column
A.	3	4
B.	4	3
C.	12	1
D.	1	12
E.	4	4

2. Which of the following if M_{23}?
   A. 9
   B. i
   C. j
   D. 2
   E. 7

3. Calculate

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} + \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{pmatrix}$$

A.

$$\begin{pmatrix} 1 & 2 & 3 & 4 & a & b & c & d \\ 5 & 6 & 7 & 8 & e & f & g & h \\ 9 & 10 & 11 & 12 & i & j & k & l \end{pmatrix}$$

B.

$$\begin{pmatrix} a & b & c & d & 1 & 2 & 3 & 4 \\ e & f & g & h & 5 & 6 & 7 & 8 \\ i & j & k & l & 9 & 10 & 11 & 12 \end{pmatrix}$$

C.

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ a & b & c & d \\ e & f & g & h \\ i & j & k & l \end{pmatrix}$$

D.

$$\begin{pmatrix} 1+a & 2+b & 3+c & 4+d \\ 5+e & 6+f & 7+g & 8+h \\ 9+i & 10+j & 11+k & 12+l \end{pmatrix}$$

E. They cannot be summed.

Consider $\mathbf{A}$ and $\mathbf{B}$ for Q.4 − 6.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ a & b \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 3 & 4 \\ c & d \end{pmatrix}$$

4. Calculate $\mathbf{BA}$

A.

$$\begin{pmatrix} 3+2c & 4+2d \\ 3a+bc & 4a+bd \end{pmatrix}$$

B.

$$\begin{pmatrix} 3+4a & 6+4b \\ c+ad & 2c+bd \end{pmatrix}$$

C.

$$\begin{pmatrix} 3 & 8 \\ ac & bd \end{pmatrix}$$

D.

$$\begin{pmatrix} 6c & 8d \\ 3abc & 4abd \end{pmatrix}$$

E.

$$\begin{pmatrix} 12a & 24b \\ acd & 2bcd \end{pmatrix}$$

5. If $\mathbf{0}$ is a 2×2 zero matrix. Which of the following is (are) true?

   1. $\mathbf{AB} = \mathbf{0}$
   2. $\mathbf{A0} = \mathbf{0}$
   3. $\mathbf{B0} = \mathbf{0}$
   4. $\mathbf{B} + \mathbf{0} = \mathbf{B}$

   A. 1 only.
   B. 2 only.
   C. 2, 3 only.
   D. 1, 2, 3 only.
   E. 2, 3, 4 only.

6. If $\mathbf{I}$ is a 2×2 identity matrix. Which of the following is (are) true?

   1. $\mathbf{BI} = \mathbf{I}$
   2. $\mathbf{BB}^{-1} = \mathbf{I}$
   3. $\mathbf{BI} = \mathbf{A}$
   4. $\mathbf{IB} = \mathbf{B}$
   5. $\mathbf{BI} = \mathbf{B}$

   A. 1 only.
   B. 2 only.
   C. 1, 3 only.
   D. 2, 5 only.
   E. 2, 4, 5 only.

Consider the following $\mathbf{P}$ for Q.7 – 8.

$$\mathbf{P} = \begin{pmatrix} -5 & -6 \\ -7 & -8 \end{pmatrix}$$

7. Find $|\mathbf{P}|$.
   A. -82
   B. -2
   C. 2
   D. 82
   E. $\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

8. Which of the following is $\mathbf{P}^{-1}$?
   A. $\begin{pmatrix} -2.5 & 3 \\ 3.5 & -4 \end{pmatrix}$
   B. $\begin{pmatrix} 2.5 & -3 \\ -3.5 & 4 \end{pmatrix}$
   C. $\begin{pmatrix} -4 & 3 \\ 3.5 & -2.5 \end{pmatrix}$
   D. $\begin{pmatrix} 4 & -3 \\ -3.5 & 2.5 \end{pmatrix}$
   E. $\mathbf{P}$ is a singular matrix.

Consider the following $\mathbf{M}$ for Q.9 − 10.

$$\mathbf{M} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

9. Which of the following is $|\mathbf{M}|$?
   A. $a\begin{vmatrix} e & f \\ h & i \end{vmatrix} + b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
   B. $-a\begin{vmatrix} e & f \\ h & i \end{vmatrix} + b\begin{vmatrix} d & f \\ g & i \end{vmatrix} - c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
   C. $a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$
   D. $a\begin{vmatrix} b & c \\ e & f \end{vmatrix} - b\begin{vmatrix} a & c \\ d & f \end{vmatrix} + c\begin{vmatrix} a & b \\ d & e \end{vmatrix}$
   E. $a\begin{vmatrix} b & c \\ e & f \end{vmatrix} + b\begin{vmatrix} a & c \\ d & f \end{vmatrix} + c\begin{vmatrix} a & b \\ d & e \end{vmatrix}$

10. Which of the following is $\left(\mathbf{M}^{-1}\right)_{23}$
   A. f
   B. $\frac{-1}{|\mathbf{M}|}\begin{vmatrix} a & c \\ d & f \end{vmatrix}$
   C. $\frac{-1}{|\mathbf{M}|}\begin{vmatrix} a & b \\ g & h \end{vmatrix}$
   D. $\frac{1}{|\mathbf{M}|}\begin{vmatrix} a & c \\ d & f \end{vmatrix}$
   E. $\frac{1}{|\mathbf{M}|}\begin{vmatrix} a & b \\ g & h \end{vmatrix}$

Learn More

There are still many terms and properties that are not mentioned in the video. Some of them are listed below.

1. $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$
   You can calculate $(\mathbf{BC})$ or $(\mathbf{AB})$ first.
   But don't change the order. In general,
   $\mathbf{A}(\mathbf{BC}) \neq \mathbf{B}(\mathbf{AC})$

2. $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$
   This is called distributive.

3. $|\mathbf{AB}| = |\mathbf{A}|\,|\mathbf{B}|$
   Impressive, the product of two determinants equals to the determinant of the product of respective matrices.

4. $\left|\mathbf{M}^{-1}\right| = 1/|\mathbf{M}|$
   The inverse really looks like "one over" in many sense, but still, we have no division in matrix.

5. For a $n \times n$ matrix ,$\mathbf{M}$.
   $|c\mathbf{M}| = c^n |\mathbf{M}|$
   c is a number , not a matrix.

6. If you exchange the positions of two rows or two columns, the determinant gets a flip in sign.

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = -\begin{vmatrix} b & a & c & d \\ f & e & g & h \\ j & i & k & l \\ n & m & o & p \end{vmatrix} = \begin{vmatrix} f & e & g & h \\ j & i & k & l \\ b & a & c & d \\ n & m & o & p \end{vmatrix}$$

7. For a square matrix $\mathbf{M}$, take 2×2 as an example. If you find c, a constant that makes

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c \begin{pmatrix} x \\ y \end{pmatrix}$$

   c is called eigenvalue, the $(x\ y)^T$,2×1 matrix is called eigen vector.
   THIS IS A VERY IMPORTANT MATH. CONCEPT FOR PHYSICS, but it is too hard in this level, search online if you are interested.

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
Given the following equations:

$$\begin{aligned} x+y+z&=1 \\ w+2x-z&=2 \\ 3w+5x+10y-z&=0 \\ 2w-x+y&=1 \end{aligned}$$

Please construct a square matrix such that?

$$\begin{pmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{pmatrix} \begin{pmatrix} w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$$

If you want to solve for the set (x, y, z, w), please search "Cramer's rule" online.

Challenge 2.
The method to find the determinant and the inverse matrix of a 3×3 matrix can actually be applied to inverse matrix of $n \times n$. For $n \geq 2$.

For example,

$$\mathbf{M} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix}$$

Try to fill in all the "?":

$$|\mathbf{M}| = ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} + ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} + ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} + ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix}$$

$$\left(M^{-1}\right)_{24} = ?$$

Challenge 2.
Prove that the inverse matrix of a square matrix is unique.
Which means if

$$\mathbf{AM} = \mathbf{MA} = \mathbf{I}$$
$$\mathbf{BM} = \mathbf{MB} = \mathbf{I}$$

Then

$$\mathbf{A} = \mathbf{B}$$

"Guide"

You can always get other ways to solve the problem. This "guide" is just a helping hand, not the law.

Challenge 1.
Just try to perform the matrix product, you will get four linear equations. Some of the matrix elements might be zero.

Challenge 2.
Try to extend your knowledge with the following hints.

$$|\mathbf{M}| = ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} - b \times \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} + ? \times \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix}$$

$$\left(M^{-1}\right)_{41} = \frac{-1}{|\mathbf{M}|} \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

Challenge 3.
Try combing the two requirements, it should not be difficult.