Limit- Supplement Solution Set

See If You Need This Video!

1. Answer: A.
   When x increases, $1/x$ decreases. And when $x \to \infty$, $1/x$ gets infinitely close to 0.

   ≪Basic idea of taking limit≫
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2. Answer: C
   The function $f(x)$ is piecewise, we can still find the limit as normal. When $x \neq 0$, $f(x) = 1$ and it keeps $= 1$ no matter how x is close to zero. So, the limit is 1.
   You have to be careful $x \to 0$ does not mean $x = 0$, so the limit is not 0.

   ≪Basic idea of taking limit≫
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3. Answer: E
   Just like $\sin x$, $\cos x$ is oscillating at $x \to \infty$, it never get steady to a fixed value. The limit does not exist.

   ≪Basic idea of reason that oscillating function does not exist≫
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4. Answer: D
   The limit of summation of two functions like this equals to the summation of separated limit of them (if both of them exist).

$$\lim_{x \to C} [f(x) + g(x)] = \lim_{x \to C} f(x) + \lim_{x \to C} g(x)$$

   Both $3x$ and x^2 tends to 0 separately in this question.
   We also have similar relation for subtraction, multiplication, division.

   ≪Algebraic limit theorem≫
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5. Answer: A
   The limit of product of two functions like this equals to the product of separated limit of them (if both of them exist).

$$\lim_{x \to C} [f(x) g(x)] = \lim_{x \to C} f(x) \lim_{x \to C} g(x)$$

   In this question,

$$\lim_{x \to 0} 10^x = 1$$
$$\lim_{x \to 0} x^1 0 = 0$$

   Thus, there product is 0.

   ≪Algebraic limit theorem≫
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6. Answer: C
   When $x \to \infty$,

$$\begin{aligned}(2x^2 + \sin x) &\approx 2x^2 \qquad \text{because } |\sin x| \leq 1 \\ (1 + x^2) &\approx x^2\end{aligned}$$

   So we have

$$\lim_{x\to\infty} \frac{2x^2 + \sin x}{1 + x^2} \approx \lim_{x\to\infty} \frac{2x^2}{x^2} = 2$$

   This skill helps you find the limit with complicated expression.
   You can alternatively solve it by L'Hôpital's rule, but that is not necessary.

   ≪Simplification when taking limit≫
   Cantonese: 3:28 English: 4:05 Putonghua: 3:47

7. Answer: E

   1. $\cos x$ oscillates within -1 and 1, limit does not exist.
   2. $\cos x$ oscillates within -1 and 1, it is much smaller than x when $x \to \infty$, the limit of $\lim_{x\to\infty} \frac{\cos x}{x} = 0$
   3. Obviously this tends to 0 when $x \to \infty$
   4. Same as 2

   This question does nothing with L'Hôpital's rule. If you apply it, you will get wrong. Please do not apply L'Hôpital's rule blindly when the function is "ugly". L'Hôpital's rule only works for indeterminate form.

   ≪Algebraic limit theorem≫
   Cantonese: 3:07 English: 3:28 Putonghua: 3:24

8. Answer: A
   It is indeterminate form in the form 0/0, we apply L'Hôpital's rule.

$$\begin{aligned}\lim_{x\to 0}\frac{x^2}{\sin x} &= \lim_{x\to 0}\left(\frac{\mathrm{d}x^2}{\mathrm{d}x}\right) / \left(\frac{\mathrm{d}\sin x}{\mathrm{d}x}\right) \\ &= \lim_{x\to 0}\frac{2x}{\cos x} \\ &= 0\end{aligned}$$

   ≪L'Hôpital's rule≫
   Cantonese: 6:45 English: 6:40 Putonghua: 7:25

9. Answer: D
   It is indeterminate form in the form 0/0, we apply L'Hôpital's rule.
   But always keep in mind that you have to perform the same number of differentiation for both numerator and denominator.

$$\begin{aligned}\lim_{x\to 0}\frac{x^3+x^2}{1-\cos x} &= \lim_{x\to 0}\left(\frac{\mathrm{d}\,(x^3+x^2)}{\mathrm{d}x}\right) / \left(\frac{\mathrm{d}\,(1-\cos x)}{\mathrm{d}x}\right) \\ &= \lim_{x\to 0}\frac{3x^2+2x}{\sin x} \\ &\text{Still in } 0/0 \text{ form} \\ &= \lim_{x\to 0}\left(\frac{\mathrm{d}\,(3x^2+2x)}{\mathrm{d}x}\right) / \left(\frac{\mathrm{d}\,(\sin x)}{\mathrm{d}x}\right) \\ &= \lim_{x\to 0}\frac{6x+2}{\cos x} = 2\end{aligned}$$

   ≪L'Hôpital's rule≫
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10. Answer: E

Again, 0/0 indeterminate form, we need L'Hôpital's rule.

$$\begin{aligned}\lim_{x\to 0}\frac{\sin x}{x-\sin x} &= \lim_{x\to 0}\frac{\mathrm{d}sinx}{\mathrm{d}x}\Big/\frac{\mathrm{d}\,(x-\sin x)}{\mathrm{d}x} \\ &= \lim_{x\to 0}\frac{\cos x}{1-\cos x}\end{aligned}$$

Now it is numerator$\to$ 1 but denominator$\to$ 0, limit does not exist. Please keep in mind that L'Hôpital's rule does not guarantee an existing limit.

≪L'Hôpital's rule≫
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≪Algebraic limit theorem≫
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