Exponential Functions & Logarithm-Supplement

See If You Need This Video!

1. Which of the following(s) is (are)correct?

    1. $2^3 = 8$
    2. $4^{0.25} = 1$
    3. $10^{-2} = 0.01$
    4. $10^2 \times 10^{-5} = 10^{-3}$

    A. 1, 2 only
    B. 1, 3 only
    C. 2, 3 only
    D. 3, 4 only
    E. 1, 3, 4 only

2. Which of the following(s) is (are) correct?

    1. $\log_{10} 1 = \log_2 1$
    2. $\log_{10}(-10) = -1$
    3. $\log_3 9 = 2$
    4. $\log_4 2 = 0.5$

    A. 1, 2 only
    B. 2, 4 only
    C. 3, 4 only
    D. 1, 2, 4 only
    E. 1, 3, 4 only

3. Which of the following(s) is (are) correct?

    1. $\log_2 5 + \log_2 10 = \log_2 50$
    2. $\log_4 5 + \log_{10} 5 = \log_5 40$
    3. $\log_3 10 - \log_3 7 = \log_3 3$

    A. 1 only
    B. 2 only
    C. 1, 2 only
    D. 2, 3 only
    E. All are correct.

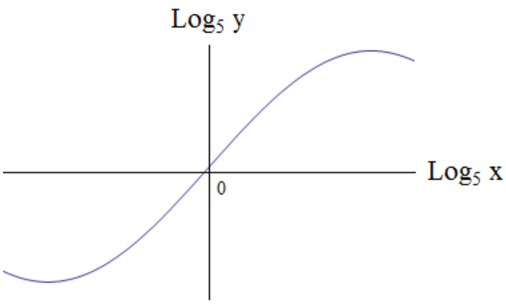
4. Which of the following(s) is (are) correct?

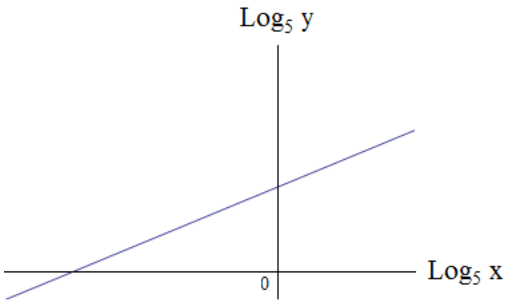
    1. $4 \log_3 2 = \log_3 16$
    2. $-2 \log_{10} 2 = \log_{10} 0.25$
    3. $(\log_{10} 20)(\log_{10} 50) = \log_{10} 70$
    4. $(\log_2 11)(\log_{10} 11) = \log_{20} 11$

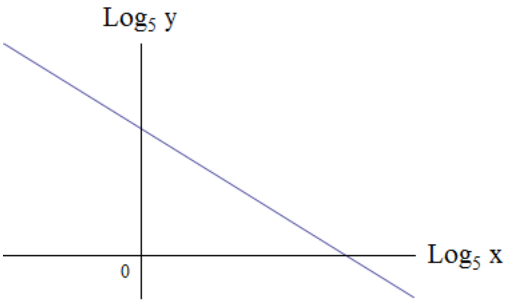
    A. 1 only
    B. 1, 2 only
    C. 2, 3 only
    D. 2, 4 only
    E. 1, 2, 4 only

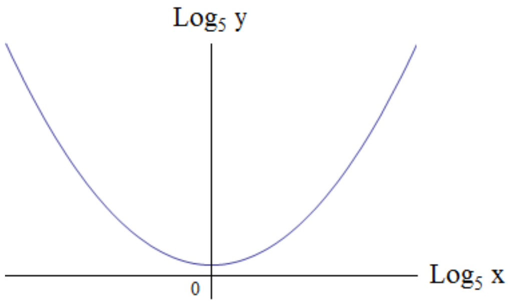
5. Which of the following equals to $\log_4 10$
    A. $\log_{10} 4$
    B. $(\log_3 10) / (\log_3 4)$
    C. $(\log_2 10)^2$
    D. $(\log_2 10) / (\log_4 2)$
    E. $(\log_5 4) / (\log_{10} 4)$

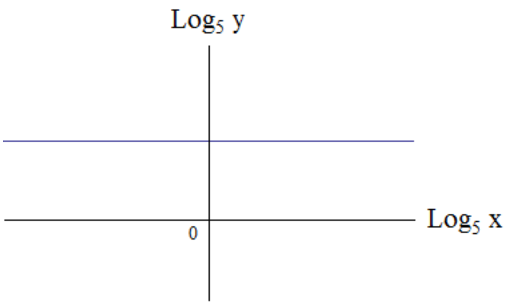
6. Which of the following curve represents $y = 3x^{-2}$?

A. 

B. 

C. 

D. 

E. 

7. Which of the followings is (are) correct?

   1. ln is a logarithm with base e.
   2. e is an irrational number.
   3. $e > 1$
   4. e is called natural number.

   A. 1 only
   B. 2, 3 only
   C. 1, 2, 3 only
   D. 1, 2, 4 only
   E. All are correct.

Learn More

Actually, the decaying example used in our video is not completed. Here is a more detailed scenario that shows how special e is. You need the idea of taking limit for this discussion, watch that chapter first if you need. The story is rather long, but interesting. Just suit yourself if you are busy.

For example, we have N_0 radioactive particles A that decays to B.

$$A \xrightarrow{\text{decay}} B$$

For every one unit of time, say one second, there is a fraction x of A decays to B. So after one second, the number of A is

$$N = N_0 (1 - x)$$

However, this expression means that particles only have a chance to decay at integral second ($t = 1s, 2s, 3s...$) and no decay happens at time between. In real case, each particle takes a chance to decay at every single moment.

Let's refine the argument, we now break the $1s$ period into two halves. For each half second, there are $x/2$ of them decayed. Now for one second, each of them has 2 chances to decay. After $1s$

$$N = N_0 (1 - x/2)^2$$

And we can break $1s$ into infinite number of them for perfection. After $1s$,

$$N = N_0 \lim_{n\to\infty} (1 - x/n)^n$$

It means each particle has a chance x/n to decay in each $1/n$ second period. When n is infinitely large, it is what happening in physical world. And now the factor e^{-x} comes, because

$$e^{-x} = \lim_{n\to\infty} (1 - x/n)^n$$

After time t, number of A is

$$N(t) = N_0 e^{-xt}$$

It seems not the same equation you learnt in high school, right? That is because x is used instead of half life. Consider the idea of "half life" T you may have learnt, it means

$$N(t = T) = 0.5N_0$$

So let's convert x to something T related,

$$\begin{aligned} 0.5N_0 &= N(t = T) \\ &= N_0 e^{-xT} \\ -\ln 2 &= -xT \\ x &= \frac{\ln 2}{T} \end{aligned}$$

Finally, this is what you may feel familiar

$$\begin{aligned} N(t) = N_0 e^{-xt} &= N_0 e^{-(t/T)\ln 2} \\ &= N_0 2^{-t/T} \end{aligned}$$

Okay, back to the e^{-x}. When we cut the period of $1s$ into infinite number of them, it means that the number of decay at each moment is dropping because the number of particles keeps dropping. So the rate is directly connected to the number at that time. That's why when a decay or grow mechanism is continuous in time, natural exponential function e^x or e^{-x} comes into play. Thanks for reading.

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
Some relations involving logarithm can be proved by yourself! Prove the followings

$$\log_a A + \log_a B = \log_a AB$$
$$\log_a A = \frac{\log_b A}{\log_b a}$$

Challenge 2.
Try to sketch the following functions y against x.

1. $y = \ln x$
2. $y = e^x$
3. $y = 1 - e^{-x}$

Sketching is different from plotting, you don't have to be really accurate and precise, just guess the shape and draw it out is okay.

”Guide”

You can always get other ways to solve the problem. This ”guide” is just a helping hand, not the law.

Challenge 1.
You may try to start from both right and left sides, logarithm it self is hard to be manipulated, why don't you start with exponential function?
You may make use of identity more than once repeatedly.

$$a^{\log_a b} = b$$

This a or b can be anything. You may take a look at the identities you are aiming at, then select what to put into the identity.

Challenge 2.
When you have to guess the shape of a curve, try to do it step by step, the followings are my suggestion. Please modify it to suit yourself.

1. Take a look at the function, guess its value at extreme values ($\to 0$, $\to \infty$ or other points that you think they are special).

2. Take a look at the functional form, is it increasing or decreasing at some area? Is it concave or convex? Is there any extremum?

3. Join the points in 1 according to the guess in 2.

To be more direct, build up a database in your mind of the shapes of common functions like exponential, power series, reciprocal, logarithm, sinusoidal etc. Then combine them into the function you are taking care of.