Differentiation- Supplement

See If You Need This Video!

1. Which of the following corresponds to $\left.\frac{\mathrm{d}x^5}{\mathrm{d}x}\right|_{x=2}$?

   A. $\lim\limits_{\Delta x \to 0} \frac{(x+\Delta x)^5 - x^5}{2-\Delta x}$
   B. $\lim\limits_{\Delta x \to 0} \frac{(2+\Delta x)^5 - 2^5}{\Delta x}$
   C. $\lim\limits_{\Delta x \to 0} \frac{(2+\Delta x)^5 - 2^5}{2-\Delta x}$
   D. $\lim\limits_{\Delta x \to 0} \frac{\Delta x^5 - 2^5}{\Delta x}$
   E. $\lim\limits_{\Delta x \to 0} \frac{(2+\Delta x)^5 - \Delta x^5}{\Delta x}$

2. Which of the following(s) is(are) correct about derivative?

   1. It is the slope of a function at a point.
   2. It is the slope of tangent line touching a function at a point.
   3. It is the slope of normal line crossing a function at a point.
   4. One derivative is correspondent to only one function.

   A. 1 only
   B. 4 only
   C. 1, 2 only
   D. 1, 3 only
   E. 2, 4 only
   F. None of them is correct.

3. Which of the following(s) is(are) correct?

    1. $\frac{\mathrm{d}x^n}{\mathrm{d}x} = nx^n$

    2. $\frac{\mathrm{d}\cos x}{\mathrm{d}x} = \sin x$

    3. $\frac{\mathrm{d}\sin x}{\mathrm{d}x} = \cos x$

    4. $\frac{\mathrm{d}50}{\mathrm{d}x} = 0$

   A. 1, 2 only
   B. 2, 3 only
   C. 3, 4 only
   D. 2, 3, 4 only
   E. None of them is correct.

4. Which of the following(s) is(are) correct?

    1. $\frac{\mathrm{d}\ln x}{\mathrm{d}x} = \frac{1}{x}$

    2. $\frac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$

    3. $\frac{\mathrm{d}\left(1/x^2\right)}{\mathrm{d}x} = -2/x^3$

   A. 1 only
   B. 1, 2 only
   C. 1, 3 only
   D. 2, 3 only
   E. All are correct.

5. Find $\frac{\mathrm{d}\left(x^2+\cos x\right)}{\mathrm{d}x}$.
   A. $x^2 + \sin x$
   B. $x^2 - \sin x$
   C. $2x + \sin x$
   D. $2x - \sin x$
   E. $2x + \cos x$

6. Find $\frac{\mathrm{d}\left(x^2 \ln x\right)}{\mathrm{d}x}$.
   A. x
   B. $2x$
   C. $2x \ln x$
   D. $x + 2x \ln x$
   E. $2x + 1/x$

7. Find $\frac{\mathrm{d}e^{-t \ln 2/T}}{\mathrm{d}t}$.
   A. $e^{-t \ln 2/T}$
   B. $-e^{-t \ln 2/T}$
   C. $-\frac{\ln 2}{T} e^{-t \ln 2/T}$
   D. $\frac{-1}{T} e^{-t \ln 2/T}$
   E. $\frac{\ln 2}{T} e^{-t \ln 2/T}$

8. Find 2nd $x-$ derivative of $f\left(x\right)$, $\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2}$

$$\text{Where } f\left(x\right) = xe^{-x}$$

   A. xe^{-x}
   B. $e^{-x} - xe^{-x}$
   C. $e^{-x} + xe^{-x}$
   D. $-2e^{-x} + xe^{-x}$
   E. $-2e^{-x} - xe^{-x}$

9. Find $\frac{\mathrm{d}[\sin(\omega x)/(x-1)]}{\mathrm{d}x}$.
   A. $\omega \cos\left(\omega x\right) / \left(x-1\right) + \sin\left(\omega x\right) / \left(x-1\right)^2$
   B. $\omega \cos\left(\omega x\right) / \left(x-1\right) - \sin\left(\omega x\right) / \left(x-1\right)^2$
   C. $\cos\left(\omega x\right) / \left(x-1\right) - \omega \sin\left(\omega x\right) / \left(x-1\right)^2$
   D. $\omega \cos\left(\omega x\right) / \left(x-1\right) + \sin\left(\omega x\right) / \left(x-1\right)$

10. Find $\frac{\mathrm{d}\tan x}{\mathrm{d}x}$
    A. $\sec x$
    B. $\sec^2 x$
    C. $1 + \tan x$
    D. $\tan^2 x$
    E. $\cos^2 x$

Learn More

The coming sections involve ideas of vector. Please come back after watching related chapters if you need.
In physics, calculation often involves vectors. Like kinematics,

$$\frac{\mathrm{d}\overrightarrow{s}(t)}{\mathrm{d}t} = \overrightarrow{v}(t)$$
$$\frac{\mathrm{d}\overrightarrow{v}(t)}{\mathrm{d}t} = \overrightarrow{a}(t)$$

Linearity, product and chain rules are all applicable.
For examples,

$$\overrightarrow{s}(t) = 3t\hat{x} - 5t^2\hat{y}$$

Then by linearity, we separate $\hat{x}$ and $\hat{y}$ terms and differentiate,

$$\overrightarrow{v}(t) = 3\hat{x} - 10t\hat{y}$$
$$\overrightarrow{a}(t) = -10\hat{y}$$

Think carefully, $\hat{x}$, $\hat{y}$, $\overrightarrow{s}(t)$ are all vectors. These unit vectors ($\hat{x}, \hat{y}, \hat{z}$) are not anything special. We have to differentiate them just like $\hat{s}(t)$. The complete steps are actually:

$$\begin{aligned}\overrightarrow{v}(t) &= \frac{\mathrm{d}(3t\hat{x})}{\mathrm{d}t} + \frac{\mathrm{d}(-5t^2\hat{y})}{\mathrm{d}t} \\ &= \frac{\mathrm{d}(3t)}{\mathrm{d}t}\hat{x} + 3t\frac{\mathrm{d}\hat{x}}{\mathrm{d}t} + \frac{\mathrm{d}(-5t^2)}{\mathrm{d}t}\hat{y} - 5t^2\frac{\mathrm{d}\hat{y}}{\mathrm{d}t} \\ &= 3\hat{x} - 10t\hat{y}\end{aligned}$$

However, $\hat{x}$ and $\hat{y}$ are independent of time (do not change with time t). So

$$\frac{\mathrm{d}\hat{x}}{\mathrm{d}t} = \overrightarrow{0}\ ,\ \frac{\mathrm{d}\hat{y}}{\mathrm{d}t} = \overrightarrow{0}$$

In other words, you have to deal with the differentiation of unit vectors if the basis you are using changes with time t. A related problem is given in the next section, please take a look if you are interested.

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
Find

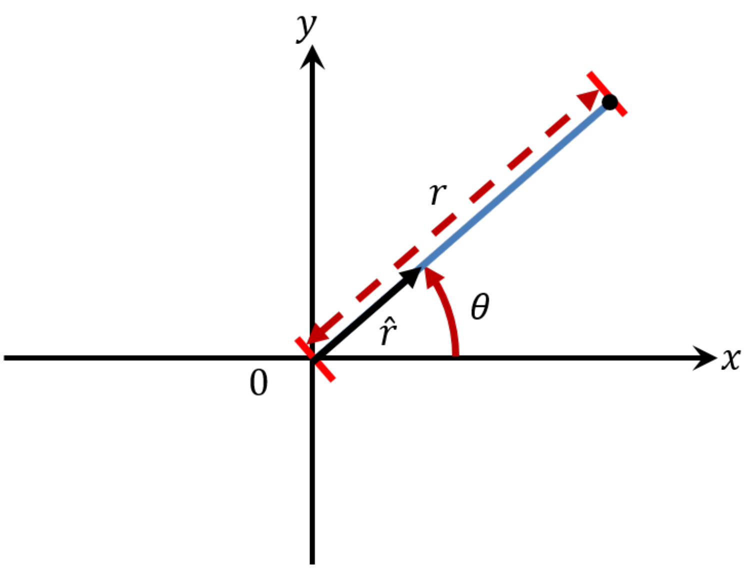
$$\frac{\mathrm{d}\sin^{-1}x}{\mathrm{d}x}$$

(This $\sin^{-1}x$ is arc-sin, not $1/\sin x$.)

Challenge 2.
Following the last section, take polar coordinates (r, θ) as an example.

$$\hat{r} = \cos\theta\hat{x} + \sin\theta\hat{y}$$
$$\hat{\theta} = -\sin\theta\hat{x} + \cos\theta\hat{y}$$

Before we discuss differentiation, please make sure you know what do they mean. r is the distance from origin. θ is the angle anticlockwise from x-axis to line joining origin and that point. $\hat{r}$, r, θ are shown in the figure, draw $\hat{\theta}$ by yourself.



For

$$\overrightarrow{r} = r\hat{r}$$

Prove that

$$\frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}t}\hat{r} + r\frac{\mathrm{d}\theta}{\mathrm{d}t}\hat{\theta}$$

If you find it too easy, further prove

$$\frac{\mathrm{d}^2\overrightarrow{r}}{\mathrm{d}t^2} = \left[\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right]\hat{r} + \left[2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t} + r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}\right]\hat{\theta}$$

"Guide"

You can always get other ways to solve the problem. This "guide" is just a helping hand, not the law.

Challenge 1.
Sometimes, you may define some variables or functions to help you solving a problem.
We don't know much about arcsine, but we know something about sine. Why don't we start with defining a function $u(x)$? Such that

$$u(x) = \sin^{-1} x$$
$$x = \sin u$$

Now, what we have to find is $\frac{\mathrm{d}u}{\mathrm{d}x}$. How to get some "d" from this?

Challenge 2.
You will see these two relations when you study rotational motion in $2D$ space. But we do nothing with physics here.
As we said in the last section, in general

$$\frac{\mathrm{d}\hat{r}}{\mathrm{d}t} \neq \overrightarrow{0} \ , \ \frac{\mathrm{d}\hat{\theta}}{\mathrm{d}t} \neq \overrightarrow{0}$$

You may find them separately before you try to prove the relations.
The rest is just applying product rule, don't be frightened of the vector sign.
It will take you $\sim$ 10 mins. to prove them if you work step-by-step, carefully.

If you really want to know the physical meaning of the relations, search online "2D rotational kinematics" or so. You may have to spend a couple of hours if you have just learnt vector and differentiation.