

Differentiation- Supplement Solution Set

See If You Need This Video!

1. Answer: B.

Definition of differentiation

$$\left. \frac{df(x)}{dx} \right|_{x=x_0} = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

«Basic idea of differentiation»

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2. Answer: C.

1. This is true and an important idea in physics.
2. It is true.
3. It is false.
4. Consider

$$\frac{d(f(x) + C)}{dx}$$

The results are the same for all constant C .

«Basic idea of differentiation»

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3. Answer: C.

1. $\frac{dx^n}{dx} = nx^{n-1}$

2. $\frac{d \cos x}{dx} = -\sin x$, don't forget the minus sign.

3. $\frac{d \sin x}{dx} = \cos x$

4. Differentiating a constant is 0, or you may consider $\frac{dx^0}{dx} = 0$.

«Derivative of x^n »

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«Derivative of sinusoidal functions»

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4. Answer: E.

1. $\frac{d \ln x}{dx} = \frac{1}{x}$

2. $\frac{de^x}{dx} = e^x$

3. $\frac{dx^{-2}}{dx} = -2x^{-3}$

«Derivative of x^n »

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«Derivative of exponential function»

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«Derivative of natural logarithm»

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5. Answer: D.

By linearity of differentiation

$$\frac{d[f(x) + g(x)]}{dx} = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

So

$$\frac{d[x^2 + \cos x]}{dx} = 2x - \sin x$$

«Linearity of differentiation»

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«Derivative of x^n »

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«Derivative of sinusoidal functions»

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6. Answer: D.

By product rule

$$\frac{d[f(x)g(x)]}{dx} = g(x) \frac{df(x)}{dx} + f(x) \frac{dg(x)}{dx}$$

So

$$\begin{aligned} \frac{d[x^2 \ln x]}{dx} &= \ln x \frac{dx^2}{dx} + x^2 \frac{d \ln x}{dx} \\ &= 2x \ln x + x \end{aligned}$$

«Product rule of differentiation»

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«Derivative of x^n »

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7. Answer: C.

By chain rule

$$\frac{df(g(x))}{dx} = \frac{dg(x)}{dx} \frac{df(g(x))}{dg(x)}$$

So

$$\begin{aligned} \frac{de^{-t \ln 2/T}}{dt} &= \frac{d(-t \ln 2/T)}{dt} \frac{de^{-t \ln 2/T}}{d(-t \ln 2/T)} \\ &= -\frac{\ln 2}{T} e^{-t \ln 2/T} \end{aligned}$$

«Chain rule»

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«Derivative of exponential function»

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«Derivative of x^n »

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8. Answer: D.

With product rule and chain rule

$$\begin{aligned} \frac{d(xe^{-x})}{dx} &= e^{-x} - xe^{-x} \\ \frac{d^2(xe^{-x})}{dx^2} &= \frac{d}{dx} \left(\frac{d(xe^{-x})}{dx} \right) \\ &= \frac{d(e^{-x} - xe^{-x})}{dx} \\ &= -2e^{-x} + xe^{-x} \end{aligned}$$

«Chain rule of differentiation»

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«Product rule of differentiation»

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9. Answer: B.

With product rule and chain rule

$$\begin{aligned}\frac{d[\sin \omega x / (x-1)]}{dx} &= \frac{1}{x-1} \frac{d \sin (\omega x)}{dx} + \sin (\omega x) \frac{d(x-1)^{-1}}{dx} \\ &= \frac{1}{x-1} \omega \cos (\omega x) - \sin (\omega x) \frac{1}{(x-1)^2}\end{aligned}$$

«Chain rule of differentiation»

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«Product rule of differentiation»

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10. Answer: B.

This result can be memorized for convenience.

$$\begin{aligned}\frac{d \tan x}{dx} &= \frac{d}{dx} \left(\frac{\sin x}{\cos x} \right) \\ &= \frac{1}{\cos x} \frac{d \sin x}{dx} + \sin x \frac{d(\cos x)^{-1}}{dx} \\ &= \frac{\cos x}{\cos x} + \sin x (-1) \left(\frac{-\sin x}{\cos^2 x} \right) \\ &= 1 + \frac{\sin^2 x}{\cos^2 x} \\ &= (\cos x)^{-2} = \sec^2 x\end{aligned}$$

«Product rule of differentiation»

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