Complex Number- Supplement

See If You Need This Video!

1. Which of the following(s) is(are) correct?

    1. $\sqrt{-1} = i$
    2. -1 is a purely real number.
    3. $4i$ is a purely imaginary number.
    4. $2 + i$ is a purely imaginary number.

    A. 1 only.
    B. 2, 3 only.
    C. 1, 2, 3 only.
    D. 2, 3 4 only.
    E. All are correct.

2. Which of the following(s) about $z = 1 - \sqrt{3}i$ is(are) correct?

    1. Re(z)=1
    2. $\mathrm{Im}(z) = -\sqrt{3}i$
    3. $|z| = 2$
    4. $\mathrm{Arg}\,(z) = \pi/3$

    A. 1, 2 only.
    B. 1, 3 only.
    C. 2, 4 only.
    D. 1, 2, 3 only.
    E. 1, 2, 4 only.

3. Which of the following(s) is (are) correct?

   1. $\text{Re}[(a+bi)^*] = -\text{Re}[(a+bi)]$
   2. $\text{Im}[(a+bi)^*] = -\text{Im}[(a+bi)]$
   3. $\text{Arg}\,[(a+bi)^*] = -\text{Arg}\,(a+bi)$
   4. $|(a+bi)^*| = -\,|a+bi|$

   A. 1, 2 only.
   B. 1, 3 only.
   C. 2, 3 only.
   D. 3, 4 only.
   E. All are correct.

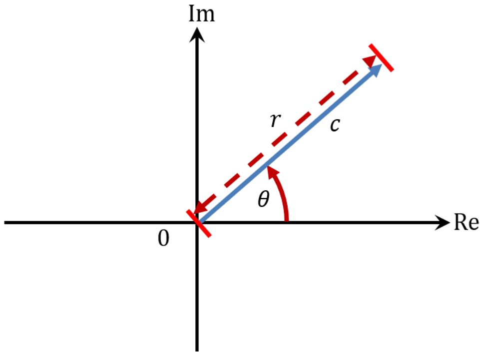
4. Which of the following(s) about A and B is (are) correct?

$$A = a_1 + a_2 i \quad B = b_1 + b_2 i$$

   1. $A + B = (a_1 + b_1) + (a_2 + b_2)\,i$
   2. $AB = (a_1 b_1 + a_2 b_2) + (a_1 b_2 + a_2 b_1)\,i$
   3. $AA^* = |A|^2$
   4. $A/B = \frac{1}{|B|^2} AB^*$

   A. 1 only.
   B. 1, 2 only.
   C. 2, 4 only.
   D. 1, 2, 3 only.
   E. 1, 3, 4 only.

5. Which of the following is the exponential form of c?



A. e^{θ}
B. re^{θ}
C. $e^{i\theta}$
D. $re^{i\theta}$
E. It has no exponential form.

6. Which of the following(s) is (are) correct?

1. $e^{i2\pi} = 1$
2. $e^{i\pi/2} = i$
3. $e^{i\pi} = -1$
4. $e^{i\pi/4} = (1 + i)/\sqrt{2}$

A. 1, 2 only.
B. 1, 3 only.
C. 2, 4 only.
D. 3, 4 only.
E. All are correct.

7. Which of the following equals to AB?

$$A = ae^{i\alpha} \quad B = be^{i\beta}$$

A. $e^{i\alpha\beta}$
B. $abe^{i\alpha\beta}$
C. $e^{i(\alpha+\beta)}$
D. $abe^{i(\alpha+\beta)}$
E. $|A+B|\, e^{i\alpha\beta}$

8. What is the phase a leading b?

$$a = Ae^{i(\omega t+\pi/2)} \quad b = Be^{i\omega t}$$

A. $-\pi/2$
B. 0
C. $\pi/2$
D. π
E. They are of different length, no phase leading can be discussed.

Learn More

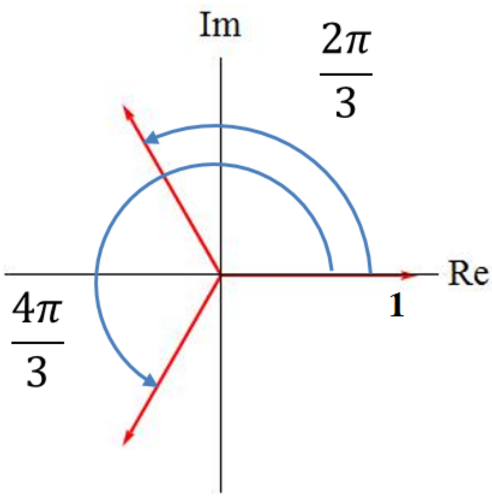
Take a look at the following equation, how many distinct roots are there?

$$z^3 = 1$$

The answer is three of them. Despite the well known $z = 1$, there are two more of them,

$$z = e^{i2\pi/3}$$
$$z = e^{i4\pi/3}$$

And obviously, these three solutions are distinct if we observe them on the argand plane.



In general, there are n distinct roots for a power series of order n. In the simplest case,

$$\begin{aligned} \text{If} \quad & z^n = re^{i\theta} \\ \text{Then} \quad & z = \sqrt[n]{r}e^{i(\theta/n + 2\pi m/n)} \\ \text{Where} \quad & m = 0, 1, 2 ... n-1 \end{aligned}$$

If you take a look at the roots, all of them lies within one period of 2π. Actually, a root with $m = n$ is also a root, but that is duplicate with $m = 0$, and so for all $m < 0$ or $m \geq n$. The 2π range chosen is normally called principal value. Usually it is chosen so that argument of complex number are $-\pi < \text{Arg} \leq \pi$.

And Drill Deeper

We have a few challenges for you. If you really have no idea, take a look at the "Guide" and learn the way of thinking. The way to think may help you in solving problems even in real life. And no solution will be given for this part, just enjoy yourself. :)

Challenge 1.
You may have learnt the following identities, prove them with complex number you have just learnt.

$$\sin(a \pm b) = \sin a \cos b \pm \sin b \cos a$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin b \sin a$$

Challenge 2.
Before we learn complex number, we don't know how to operate logarithm on a negative number. But now we can. Try to find the following logarithms.

$$\ln(-10)$$

Furthermore, try

$$\ln(i)$$

"Guide"

You can always get other ways to solve the problem. This "guide" is just a helping hand, not the law.

Challenge 1.
When you learn new skills or knowledge that is related to concepts that you know well. Try to link them and check if they are consistent.

In this chapter we said sin and cos can be expressed in exponential functions of complex number, how to link the compound angle formula with this form of complex number?

Challenge 2.
Like the last challenge, we would like you to apply the new knowledge (complex) on the old one (logarithm).

When you face a problem that is new, try to break it down and deal with them one by one.

1. Can you break down $\ln(-10)$ into parts? What do you know about logarithm.

2. Do you remember we said "there is no power index x to make $C^x < 0$, so logarithm of negative number is undefined." Anything changed when complex number comes into play?

3. Take a look at the last section, how many results you may achieve for one single logarithm?