THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3280 Introductory Probability 2024-2025 Term 2
Homework Assignment 1
Due Date: 6 February, 2024 (Thursday)

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

____________________ ____________________
Signature Date

General Regulations

- All assignments must be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.
- Late assignments will receive a grade of 0.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise, we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily readable). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of getting good marks on this homework. Neatness and organization are also essential.

1. A die is tossed twice and the number of dots facing up in each toss is counted and noted in the order of occurrence.

   (a) Find the sample space.

   (b) Find the set A corresponding to the event *number of dots in first toss is not less than number of dots in second toss.*

   (c) Find the set B corresponding to the event *number of dots in first toss is 6.*

   (d) Does A imply B or does B imply A?

   (e) Find $A \cap B^c$ and describe this event in words.

   (f) Let C correspond to the event *number of dots in dice differs by 2.* Find $A \cap C$.

2. Suppose that A and B are mutually exclusive events for which $P(A) = .35$ and $P(B) = .54$. What is the probability that

   (a) either A or B occurs?

   (b) A occurs but B does not?

   (c) both A and B occur?

3. A total of 28 percent of Hong Kong males smoke cigarettes, 8 percent smoke cigars, and 6 percent smoke both cigars and cigarettes.

   (a) What percentage of males smokes neither cigars nor cigarettes?

   (b) What percentage smokes cigars but not cigarettes?

4. Five people, designated as Aba, Bono, Coco, Dan, and Emma, are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that

   (a) there is exactly one person between Aba and Bono?

   (b) there are exactly two people between Aba and Bono?

   (c) there are exactly three people between Aba and Bono?

5. At a certain university, every year 8 to 12 students are granted University Best Student Awards. This year among the nominated faculty are Elsa, Anna, and Olaf. Let A, B, and C denote the events, respectively, that these professors will be given awards. In terms of A, B, and C, find an expression for the event that the award goes to

   (a) only Elsa;

   (b) at least one of the three;

   (c) none of the three;

   (d) exactly two of them;

   (e) exactly one of them;

   (f) Elsa or Anna but not both.

6. Let A and B be two events. Show that

   (a) $\min\{1, P(A) + P(B)\} \geq P(A \cup B) \geq \max\{P(A), P(B)\}$.

   (b) $\min\{P(A), P(B)\} \geq P(A \cap B) \geq \max\{0, P(A) + P(B) - 1\}$.

7. An elementary school is offering three language classes: one in Spanish, one in French, and one in German. These classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class, and 16 in the German class. There are 12 students who are taking both Spanish and French, 4 who are taking both Spanish and German, and 6 who are taking both French and German. In addition, there are 2 students taking all three classes.

   (a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes?

   (b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class?

   (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class?

8. An urn contains M white and N black balls. If a random sample of size r is chosen, what is the probability that it contains exactly k white balls?

9. Assume that A and B are independent events. Show that the following pairs of events are also independent:

   (a) A and B^c;

   (b) A^c and B;

   (c) A^c and B^c.

10. How many numbers below 100 are divisible by 2, 3,or 5?