THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530B/C: Games & Strategic Thinking 2024-2025 Term 2
Homework Assignment 1
Due Date: 28 February, 2025 (Friday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature _____ Date _____

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2024R2 Games and Strategic Thinking (UGEB2530C)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Write your solutions on A4 white paper or use an iPad or other similar device to present your answers and submit a digital form via Gradescope. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Please be aware that you can only use a ball-point pen to write your answers for any exams.

- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

Please attempt to solve all the problems. Your solutions for problems 1 - 10 are to be submitted.

We strongly recommended that you study Extra Exercises 1 - 10, though you are not required to submit their solutions. Suggested solutions for all the problems will be provided.

1. Evaluate the following matrix products.

   (a) $\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

   (c) $\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{pmatrix}$

   (d) $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix}$

2. The HK Life Insurance Limited have 150 shares of KYD Corp., 100 shares of Info Tech, and 240 shares of ABC in an investment portfolio. The closing prices of these stocks one week were:

Monday :	KYD, \$56;	Info Tech, \$132;	ABC, \$19
Tuesday :	KYD, \$55;	Info Tech, \$133;	ABC, \$19
Wednesday :	KYD, \$55;	Info Tech, \$131;	ABC, \$20
Thursday :	KYD, \$54;	Info Tech, \$130;	ABC, \$22
Friday :	KYD, \$53;	Info Tech, \$128;	ABC, \$21

   Answer the following questions:

   (a) Summarize the closing prices in a matrix, letting each column represent a stock and each row a day.

   (b) Define the number of shares in a matrix accordingly and then determine the value of the HK Life Insurance Limited' portfolio each day by matrix multiplication. Hint: Using rows to represent stocks and columns to represent days is also acceptable.

3. Suppose a dice and 2 coins are tossed together. Let x be the number obtained from the dice and y be the number of heads shown among the coins.

   (a) Fill in the blanks in the following tables:

x	1	2	3	4	5	6
$P(\text{getting } x)$						

y	0	1	2
$P(\text{getting } y)$			

   (b) Using part (a), fill in the blanks in the following table:

z	1	2	3	4	5	6	7	8
$P(x+y=z)$								

   (c) Now, evaluate the expected value of $x + y$.

4. In a game, two players call out one of the numbers 1, 2, or 3 simultaneously. Let S be the sum of the two numbers. If S is even, then player 2 pays S dollars to player 1. If S is odd, then player 1 pays S dollars to player 2.

(a) Write down the game matrix (with player 1's payoffs).

(b) Find the expected payoff of player 1 if player 1 calls out the numbers 1, 2, 3 with probabilities 0.5, 0.3, 0.2 respectively, and player 2 calls out the numbers 1, 2, 3 with probabilities 0.1, 0.4, 0.5 respectively.

(c) Suppose player 2 calls out the numbers 1,2,3 with probabilities 0.1, 0.4, 0.5 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?

5. In a modified rock-paper-scissors game, the loser pays the total number of fingers among the two gestures to the winner. The payoff is 0 if there is a draw.

(a) Write down the game matrix (with player 1's payoffs).
(Use rock, paper, scissors, as the order of strategies.)

(b) Suppose player 1 uses $(0.2, 0.4, 0.4)$ and player 2 uses $(0.3, 0.5, 0.2)$. Find the expected payoff of player 1.

(c) If player 1 uses $(0.2, 0.4, 0.4)$, what is player 2's best strategy?

(d) If player 2 uses $(0.3, 0.5, 0.2)$, what is player 1's best strategy?

(e) By considering equalizing strategies, find a Nash equilibrium and the value of the game.

6. For each of the following payoffs matrices (2-person, zero-sum, simultaneous games), circle all the saddle point(s) (if any).

(a) $\begin{pmatrix} -1 & -4 & 4 & -2 \\ -4 & 4 & -1 & 0 \\ 2 & 3 & -1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 5 & -2 & 0 \\ 0 & -5 & -1 & -3 \\ 1 & 3 & 7 & 4 \\ -2 & 2 & 3 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 3 & 5 & 3 \\ 2 & 1 & -1 & -20 \\ 3 & 3 & 4 & 3 \\ -16 & 0 & 16 & 1 \end{pmatrix}$

7. By eliminating dominated strategies, find the optimal strategies for both players and determine the value of the game.

		Player B				
		I	II	III	IV	V
Player A	I	2	4	3	8	4
	II	5	6	3	7	8
	III	6	7	9	8	7
	IV	4	2	8	4	3

8. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

   (a) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}$

9. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

   (a) $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \end{pmatrix}$

   (b) $\begin{pmatrix} -1 & 6 \\ 0 & 4 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$

10. Solve the 2-person, simultaneous, zero-sum games with the following payoffs matrices.

    (a) $\begin{pmatrix} 1 & 0 & 4 \\ 0 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

    (b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

    (c) $\begin{pmatrix} 2 & -4 & -4 \\ -4 & 6 & -4 \\ -4 & -4 & 16 \end{pmatrix}$

    (d) $\begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{pmatrix}$

Extra exercises

E1. (a) Suppose

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}.$$

Compute AB, BC and ABC.

(b) Suppose

$$D = \begin{pmatrix} p & 1-p \end{pmatrix}.$$

Find the value of p such that the entries of DB are all equal.

(c) Suppose

$$E = \begin{pmatrix} q \\ 1-q \end{pmatrix}.$$

Find the value of q such that the entries of BE are all equal.

E2. Let

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 3 & -2 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}.$$

Compute the following. Write "undefined" for those which are not defined.

(a) AB

(b) $B^T A^T$ and CAB

(c) $C^T - D^2$

(d) AD

E3. An **unfair** die with six sides is tossed. Let x be the number that comes up. The probability of obtaining different values of x are shown in the following table:

x	$P(\text{getting } x)$
1	0.3
2	0.2
3	0.2
4	0.1
5	0.1
6	0.1

(a) Find the expected value of x.

(b) Suppose the unfair die is tossed with a fair die. Find the probability that the sum of the two numbers shown is 9.

E4. Suppose a family has 5 children and the probability of having a girl is $\frac{1}{2}$. Find the probability that the family has the following children:

(a) Exactly 2 girls and 3 boys

(b) Exactly 3 girls and 2 boys

(c) No girls

(d) No boys

(e) At least 3 boys

(f) No more than 4 girls

E5. Suppose a die with six sides is tossed. Let x be the number that comes up. Evaluate the expected values of $x + 1$ and x^2.

E6. A life insurance company is offering a $100,000 one-year term life insurance policy to Mary, a 55-year-old nonsmoking female in moderately good health. What should be a reasonable premium for this policy?

To determine this, we will let $P denote the break-even, or fair, premium that the life insurance company should charge Mary if it were not aiming to make a profit. This can be calculated by setting the expected value to zero. Using mortality tables, the life insurance company can determine that the probability of someone in Mary's demographic group dying within the next year is 1 in 500, or 0.002.

The second row of the following table provides the value of the two possible outcomes for the life insurance company, and the third row gives the respective probabilities:

Outcome	Mary dies	Mary doesn't die
Payoff	$\$(P - 100,000)$	$\$P$
Probability	0.002	0.998

Setting the expected payoff equal to zero gives $P = 200$. This is the premium the insurance company should charge to break even. However, insurance companies aim to make a profit. A standard gross profit margin in the insurance industry is 20%, which in this case would add $40 to the premium. Therefore, we can conclude that the premium for Mary's policy should be about $240.

Answer the following questions:

(a) Explain how the fair premium of $P = 200$ is obtained.

(b) Find the value of a fair premium for a person with probability of death over the next year estimated to be 3 in 1000.

E7. For each of the following payoffs matrices (2-person, zero-sum, simultaneous games), circle all the saddle point(s) (if any).

(a)

$$\begin{pmatrix} -3 & 5 & -7 & 0 \\ -1 & -3 & -5 & -2 \\ 2 & 4 & -6 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 5 & -1 & -3 & 0 \\ 2 & -2 & 3 & 1 \\ -4 & 0 & -1 & -3 \\ 3 & 1 & 6 & 4 \end{pmatrix}$$

E8. For the following 2-person, zero-sum, simultaneous game, find a mixed Nash equilibrium and the value of the game:

	C1	C2	C3	C4
R1	4	3	2	-1
R2	2	5	0	-2
R3	-1	2	0	7
R4	3	0	1	-2

E9. For each of the following games,

(i) Find all the pure Nash equilibrium(s) if they/it exist(s). If not, find a mixed Nash equilibrium.

(ii) Find the value of the game.

(a)

	C1	C2
R1	3	0
R2	-1	1

(b)

	C1	C2	C3	C4
R1	1	5	-2	-3
R2	-1	-3	3	6

(c)

	C1	C2	C3
R1	-1	3	4
R2	1	1	2
R3	-2	5	5
R4	4	0	1

(d)

	C1	C2	C3	C4
R1	0	-2	3	-4
R2	-3	-2	0	5
R3	2	0	1	-3
R4	1	-1	-2	-4

(e)

	C1	C2	C3	C4	C5
R1	1	3	2	7	4
R2	3	4	1	5	6
R3	6	5	7	6	5
R4	2	0	6	3	1

E10. Solve the zero sum game with game matrix

$$\begin{pmatrix} 5 & 3 & 8 & 1 \\ 2 & 3 & 5 & 10 \\ 7 & 5 & 6 & 2 \\ 6 & 4 & 3 & 1 \end{pmatrix}$$