Fibre Optical Transmission Beyond Linear Capacity Limit

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Linear Capacity Limits

Demands for Capacity[1]

Via better transmission schemes

- 8 billion mobile broadband subscriptions worldwide by 2022
- Internet of Things
- Video accounts for 55% data on mobile networks, and itÕs growing by 55% each year.
- Ultra High-Definition and 4K TV – data increasing exponentially.
- Major events: 213.6 terabits traffic generated during the 2014 World Cup

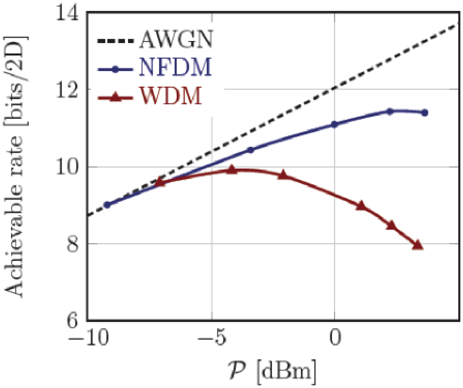
[1] `https://primex.com/capacity-crunch-coming-soon`

Growth in Capacity

- Using coherent detection, wavelength division multiplexing, advanced coding schemes, modulation formats and digital signal processing,
  - ability to mitigate linear transmission impairment (e.g., chromatic and polarization mode dispersion)
  - data rates can now exceed 100G bits/sec, delivering significant benefits
- Methods based on ones developed for linear channels – ignoring the intrinsic fibre nonlinearities
- Approaches break down when fibre nonlinearity becomes significant.
- In linear regime (when signal power is low)
  - Optical fibre acts as a passive medium
  - WDM employs Fourier Transform (FT) such that each wavelength (or its corresponding frequency) is essentially an independent transmission mode.
  - Linear signal dispersion dominates and well compensated separately using conventional linear signal processing techniques.
- Limited by fibre nonlinearity

Growth in Capacity

- In WDM, “transmission modes” are defined via Fourier Transform
  - WDM modes can interfere each other when nonlinearities become serious
- NFT (Nonlinear Fourier Transform), as name suggested, is a nonlinear transformation analogous to FT
  - Modes defined by NFT will not interfere at all
  - Can design modulation in the NFT domain



Growth in Capacity

Via investment in new fibres or infrastructure

- Space Division Multiplexing

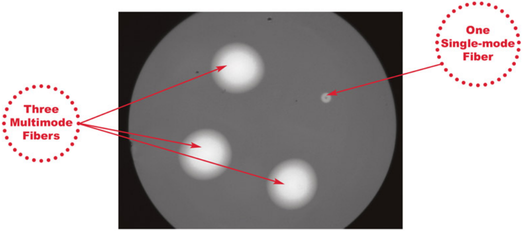


Figure: SDM using Multicore or Multimode Fiber[2]

- New infrastructures – e.g., Pacific Light Cable Network (PLCN) between Los Angeles to Hong Kong (12800km), with capacity 144 Tb/s (6 fibre pair, 240 $\times$ 100Gbps WDM)

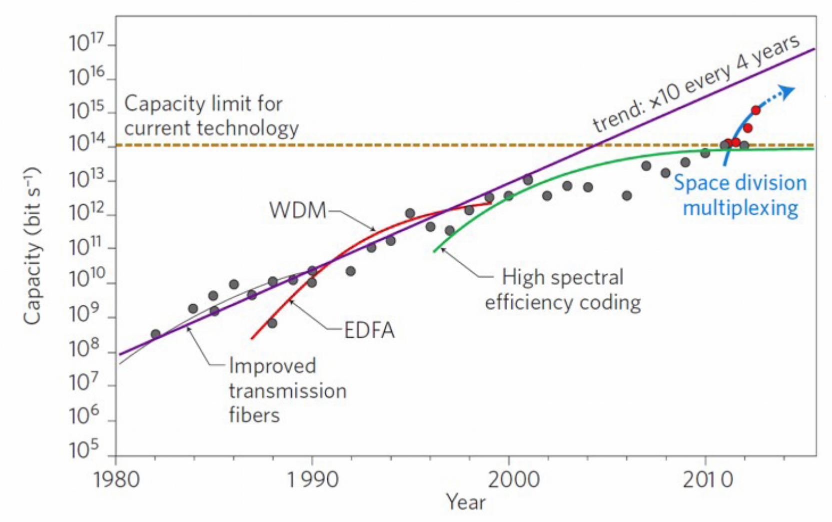
[2] https://www.ofsoptics.com/multicore-optical-fiber/

Space Division Multiplexing SDM

- Space Division Multiplexing (SDM) – promising technique to boost capacity
- transmission capacity increased by using multi-mode or multi-core fibres.
- Implementation requires significant infrastructure modification and investment to replace existing fibres.
- Capacity gain achieved by using multiple modes in the same fibre, rather than by solving the nonlinearity issue.
- Capacity (per mode) will still be below the linear capacity limit.
- In SDM, fibre nonlinearities can even be more severe
- Calls for an alternative and innovative approach to handle fibre nonlinearities.

Growth in Capacity

Reaching Linear Capacity Limit[3]



3 H. Chen and A.M.J. Koonen "Spatial Division Multiplexing," in *Fibre Optic Communication*, Springer, 2017

Challenges

- Without heavy investing of new infrastructures, how can we breakthrough the current transmission capacity limit by overcoming fibre nonlinearity effects

Signal Propagation and Channel Impairments

System model

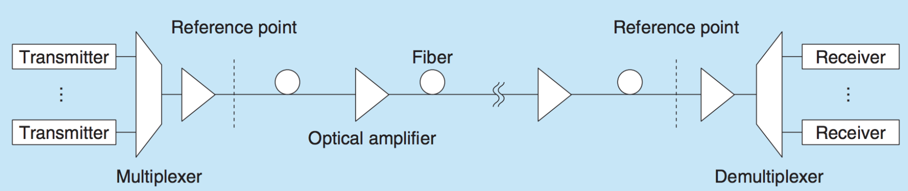


Figure: A WDM System[4]

- Distributed or lumped amplification – to compensate the fibre loss
- Noise will be introduced during amplification

[4] https://www.ntt-review.jp/archive/ntttechnical.php?contents=ntr201101gls.pdf&mode=show_pdf

Propagation Model

Signal propagation across an optical fibre is modelled by the nonlinear Schrödinger equation (NLSE)

$$\frac{\partial A(s,l)}{\partial l} + \frac{j\beta_2}{2}\frac{\partial^2 A(s,l)}{\partial s^2} + \frac{\alpha}{2}A(s,l) = j\gamma|A(s,l)|^2 A(s,l) + N(s,l)$$

- $A(s,l)$ – complex envelope of the signal propagating along the fibre
- β_2 term – linear dispersion
- α term – attenuation coefficient which describes the (linear) loss effect,
- γ term – nonlinear coefficient.
- $N(s,l)$ – optical noise

where

$$\begin{aligned} \beta_2 &= -21.668\, ps^2 km^{-1} \\ \alpha &= 4.605 \times 10^{-5} m^{-1} \\ \gamma &= 1.27 W^{-1} km^{-1} \end{aligned}$$

Standardised Model

- Assume no noise and loss (where the loss is perfectly compensated via ideal distributed Raman amplification)
- Applying variable transformations

$$q = \frac{A}{\sqrt{P}}, \qquad t = \frac{s}{T}, \qquad z = \frac{l}{\mathfrak{L}},$$

where

$$P = \frac{2}{\gamma \mathfrak{L}}, \qquad T = \sqrt{\frac{|\beta_2|\mathfrak{L}}{2}},$$

we obtain the normalised NLSE

$$jq_z(t,z) = q_{tt}(t,z) + 2|q(t,z)|^2 q(t,z)$$

- Let $q(t,0)$ be the channel input and $q(t,z)$ be the signal, after propagating for a distance of z.

Example

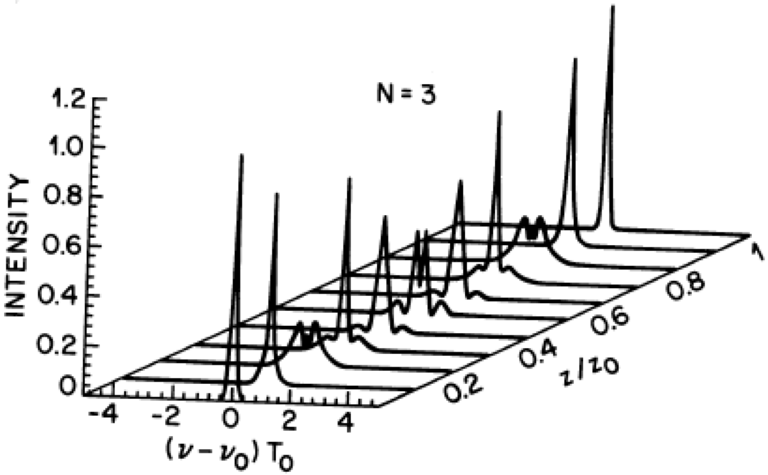
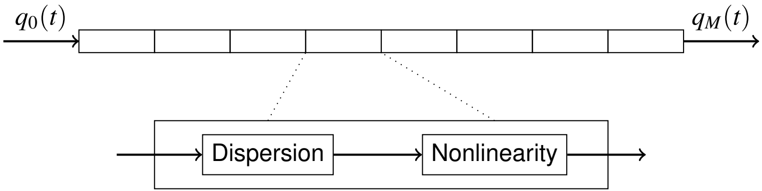


Figure: Propagation of Soliton[5]

[5] Agrawal, *Nonlinear Fibre Optics*

Split Step Fourier Method

To model the propagation of the signals ...



Effects of Linear Dispersion

Original NLSE:

$$\frac{\partial q(t,z)}{\partial l} + \frac{j\beta_2}{2}\frac{\partial^2 q(t,z)}{\partial t^2} = j\gamma|q(t,z)|^2 q(t,z)$$

- Ignoring Fibre Nonlinearity

$$\frac{\partial q(t,z)}{\partial l} = -\frac{j\beta_2}{2}\frac{\partial^2 q(t,z)}{\partial t^2}$$

- The DE can be solved analytically (in Fourier Frequency Domains)

$$Q(\omega, z+h) = Q(\omega, z)e^{j\frac{\beta_2}{2}\omega^2 h}$$

Effects of Nonlinearity

Original NLSE:

$$\frac{\partial q(t,z)}{\partial l} + \frac{j\beta_2}{2}\frac{\partial^2 q(t,z)}{\partial t^2} = j\gamma |q(t,z)|^2 q(t,z)$$

- Ignoring Linear Dispersion

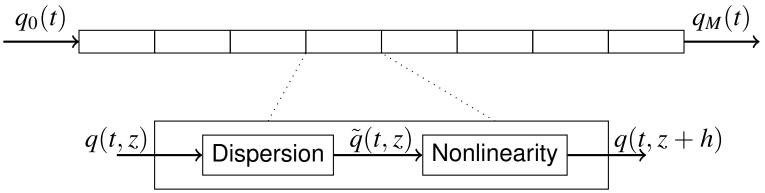
$$\frac{\partial q(t,z)}{\partial l} = j\gamma |q(t,z)|^2 q(t,z)$$

- The DE can be solved analytically (in Time Domain)

$$q(t,z+h) = q(t,z+h)e^{j\gamma |q(t,z)|^2 h}$$

  - The form illustrates why the signal distortion is nonlinear

Combining Both Effects



$$\tilde{Q}(\omega, z) = Q(\omega, z)e^{j\frac{\beta_2}{2}\omega^2 h}$$

$$q(t, z+h) = \tilde{q}(t, z)e^{j\gamma|\tilde{q}(t,z)|^2 h}$$

Digital Back-Propagation

- Signal distortion caused by fibre nonlinearity cannot be perfectly compensated by linear signal processing
- One approach to compensate distortion is based on digital back-propagation
- Numerically invert the channel – similar to zero-forcing equalisation
  - via split-step Fourier method with a negative step size – signal propagates back along the fibre
  - Compensation for nonlinear and dispersion, neglecting any noise (e.g., added due to amplification during propagation or at receiver)
- Commonly performed on a single channel (by filtering out other channels)

Nonlinear Shannon Limit

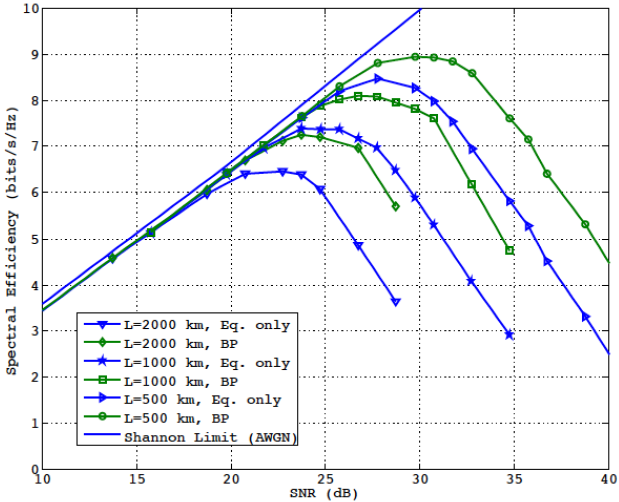


Figure: Nonlinear Shannon Limits[6]

[6] B. P. Smith and F. R. Kschischang, J. Lightwave Techn., vol. 30, pp. 2047 – 2053, 2012

Transmission Mode Decoupling

Motivation – ISI free transmission

- In a typical QAM, the transmitted signal is

$$x(t) = \sum_k u_k p(t - kT)$$

  such that the set of time-shifted pulses

$$\{p(t - kT) : \; k = \ldots, -2, -1, 0, 1, 2, \ldots\}$$

  is orthonormal
- Each pulse $p(t - kT)$ (for choices of k) essentially defines a channel which do not interfere with each other

Motivation – OFDM Analogy

- Consider wireless signal transmit to the receiver over a distance d
- Despite the multpath, there is a channel transfer function such that

$$y(t,d) = y(t,0) * h(t,d)$$

  where the input signal is $x(t) = y(t,0)$.
- Invoke Fourier Transform on $y(t,0)$ and $y(t,d)$

$$Y(f,d) = Y(f,0)H(f,d)$$

- Channel is “diagonalised” into multiple independent channels (each indexed by a different frequencu)
- In the presence of noise,

$$y(t,d) = y(t,0) * h(t,d) + n(t)$$
$$Y(f,d) = Y(f,0)H(f,d) + N(f)$$

- Question: Can we achieve the same when the channel input-output relations are characterised by NLSE.

Nonlinear Fourier Transform
NFT

Nonlinear Fourier Transform

- Analogy: Fourier Transform:

$$q(t) \mapsto Q(f)$$

Here: $t \in \mathbb{R}$ is time and $f \in \mathbb{R}$ is frequency

- NFT of a signal $q(t)$ is composed of its spectrum (eigenvalues) such that
  - continuous spectrum: $\lambda \in \mathbb{R}$
  - discrete spectrum: $\{\lambda_1, \ldots, \lambda_N\} \subset \mathbb{C}^+$

  such that for each eigenvalue (in continuous or discrete spectrum), it is associated with a spectral amplitude

$$q(t) \mapsto (Q^{(c)}(\lambda), Q^{(d)}(\lambda_k))$$

for $\lambda \in \mathbb{R}$ and $\Lambda_{\text{dis}} \triangleq \{\lambda_1, \ldots, \lambda_N\} \subset \mathbb{C}^+$

- To simplify our notation, let $\Lambda = \mathbb{R} \cup \{\lambda_1, \ldots, \lambda_N\}$

$$Q(\lambda) = \begin{cases} Q^{(c)}(\lambda) & \text{if } \lambda \in \mathbb{R} \\ Q^{(d)}(\lambda_k) & \text{if } \lambda \in \Lambda_{\text{dis}} \end{cases}$$

for all $\lambda \in \Lambda$.

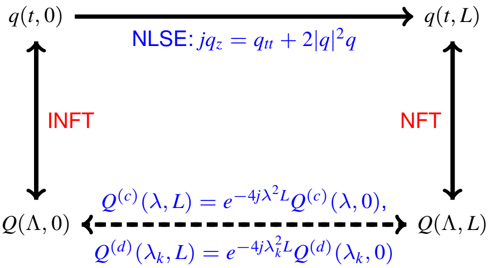
- Nonlinear Fourier Transform

$$q(t) \mapsto Q(\lambda) : \ \lambda \in \Lambda$$

Nonlinear Fourier Transform

- $q(t,z)$ is the signal propagating along the fibre.
- $q(t,z=0)$ is the input to the fibre when $z=0$
- $q(t,z=L)$ is the input to the fibre when $z=L$

$$q(t,0) \mapsto Q(\lambda \quad |q(t,0)) \triangleq Q(\lambda,0),\ \lambda \in \Lambda(0)$$
$$q(t,L) \mapsto Q(\lambda \quad |q(t,L)) \triangleq Q(\lambda,L),\ \lambda \in \Lambda(L)$$



(Yousefi and Kschischang)

- Invariance of Spectrum:

$$\Lambda(0) = \Lambda(L) \triangleq \Lambda$$

- Channel Diagonalisation

$$Q(\lambda,L) = e^{-4j\lambda^2 L} Q(\lambda,0)$$

Nonlinear Fourier Transform

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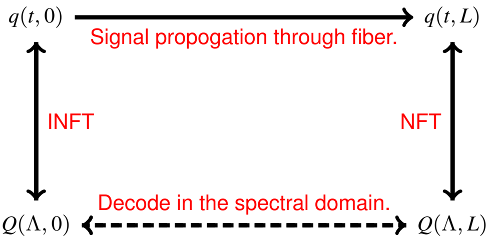

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Computing NFT (1)

- Computation of $Q(\lambda)$ requires several steps:
  1. Compute scattering data
  $$a(\lambda), b(\lambda)$$
  2. The set of discrete spectrum is roots of $a(\lambda) = 0$, i.e.,
  $$a(\lambda_k) = 0, \forall k = 1, \ldots, N$$
  3. For $\lambda \in \mathbb{R}$,
  $$Q(\lambda) \triangleq \frac{b(\lambda)}{a(\lambda)}$$
  4. for λ in the discrete spectrum (hence $a(\lambda_k) = 0$),
  $$Q(\lambda) \triangleq \frac{b(\lambda)}{a'(\lambda)}$$

Computing NFT (2)

- Suppose $q(t)$ is time limited, $q(t) = 0$ when $t < 0$ or $t > T$
- Step 1: Solve

$$v'(t) = \begin{pmatrix} -j\lambda & q(t) \\ -q^*(t) & j\lambda \end{pmatrix} v(t)$$

  where

$$v(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix}$$

  and subject to boundary condition

$$v(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Step 2:

$$a(\lambda) = v_1(T)e^{j\lambda T}$$
$$b(\lambda) = v_2(T)e^{-j\lambda T}$$

Solitons

- A signal is called a soliton if

$$b(\lambda) = 0, \quad \forall \lambda \in \mathbb{R}$$

- To characterise a soliton, it suffices to specify
  - Discrete spectrum

$$\Lambda_{\text{dis}} = \{\lambda_1, \ldots, \lambda_N\}$$

  - Corresponding spectral amplitudes

$$Q(\lambda), \text{ for } \lambda \in \Lambda_{\text{dis}}$$

- Example: Satsuma-Yajima function

$$q(t) = A \,\text{sech}(t)$$

where $A \geq 0$. Then

$$Q(\lambda) = -\frac{\Gamma(-j\lambda + \frac{1}{2} + A)\Gamma(-j\lambda + \frac{1}{2} - A)\sin(\pi A)}{\Gamma^2(-j\lambda + \frac{1}{2})\cosh(\pi\lambda)}$$

Furthermore, it is a soliton if A is a positive integer. In that case,

$$\Lambda_{\text{dis}} = \{0.5j, 1.5j, \ldots, (A - 0.5)j\}$$

Another Example: Rectangular Pulse (Yousefi and Kschischang)

Let

$$q(t) = \begin{cases} A & \text{if } t \in [T_1, T_2] \\ 0 & \text{otherwise.} \end{cases}$$

Then

$$Q(\lambda) = \frac{A^*}{j\lambda} e^{-2j\lambda T_2} \left(1 - \frac{D}{j\lambda} \cot(DT_2 - DT_1)\right)^{-1}$$

where $D = \sqrt{\lambda^2 + |A|^2}$

Transmission using NFT

- Modulating spectral amplitudes:
  - Transmitted signals are solitons with fixed discrete spectrum Λ_{dis}
  - Modulation in the spectral amplitudes

$$Q(\lambda), \quad \lambda \in \Lambda_{\text{dis}}$$

- Modulating the discrete spectrum
  - By choose the size $|\Lambda_{\text{dis}}|$ and the elements in the set

Propagation

Recall

$$Q(\lambda, L) = e^{-4j\lambda^2 L} Q(\lambda, 0)$$
$$|Q(\lambda, L)| = |Q(\lambda, 0)|, \quad \text{when } \lambda \in \mathbb{R}$$

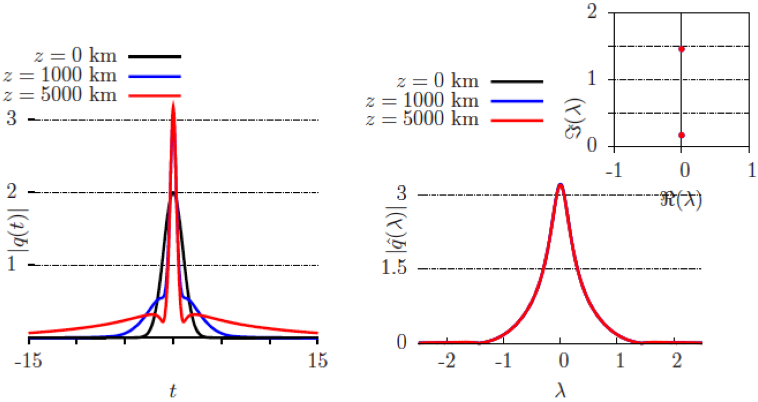


Figure: Propagation of Gaussian Pulse[7]

[7] Yousefi and Kschischang

Properties of NFT

$$\text{NFT}: \quad q(t) \mapsto Q(\lambda)$$

- (Constant Phase Change)

$$e^{j\phi} q(t) \mapsto e^{j\phi} Q(\lambda)$$

- (Time Shift)

$$q(t-\tau) \mapsto e^{-2j\lambda\tau} Q(\lambda)$$

- (Frequency Shift)

$$q(t) e^{-2j\omega t} \mapsto Q(\lambda - \omega)$$

- (Parseval Identity)

$$E = \frac{1}{\pi} \int \log(1 + |Q(\lambda)|^2) d\lambda + 4 \sum_{k=1}^{N} \text{Im}(\lambda_k)$$

- (Time Dilation)

$$q\left(\frac{t}{a}\right) \mapsto |a| Q(a\lambda)$$

Properties of NFT

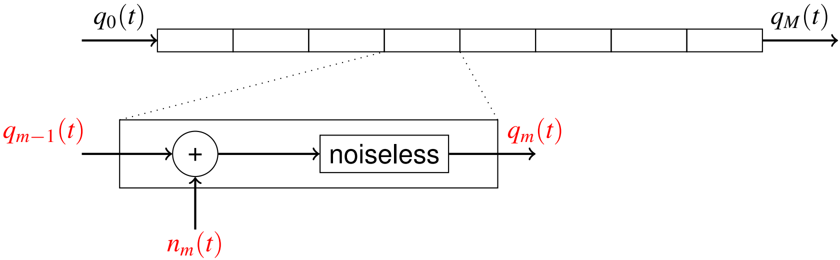
- (Fourier Transform) If $||q(t)||_1$ is very small, then

$$Q(\lambda) = -\int q^*(t) e^{-2j\lambda t} dt$$

and has null discrete spectrum.

When Noise is Present

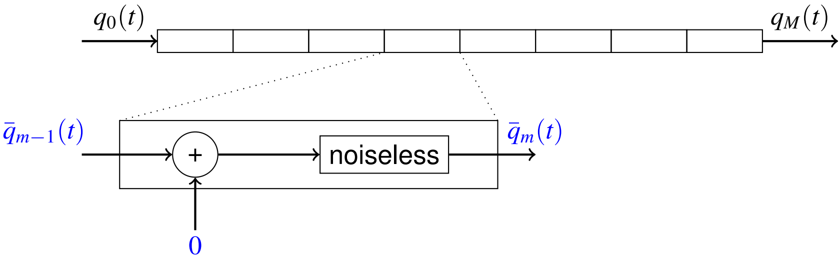
Propagation Model



- $q_m(t)$ – signal after propagating m segments

What is the effect of noise on the NFT of the signal of $q_m(t)$?

Propagation Model



- $q_m(t)$ – signal after propagating m segments
- $\bar{q}_m(t)$ – signal after propagating m segments without additive noise

What is the effect of noise on the NFT of the signal of $q_m(t)$?

Noise Analysis

As illustration, consider only the perturbation of the discrete spectrum

- $\Lambda_{\text{dis},m}$ – discrete eigenvalues of $q_m(t)$ (signal after propagating m segments).
- Due to noises, $\Lambda_{\text{dis},m}$ is no longer invariant
- The perturbation $\Lambda_{\text{dis},m} - \Lambda_{\text{dis},m-1}$ from one segment to another segment depending on
  - Noise $n_m(t)$ added in the m segment
  - The signal $q_{m-1}(t)$

The goal is to model perturbations of the discrete eigenvalues

- The distortion of $q_m(t)$ is affected by the dispersion, fibre nonlinearity, and noises introduced in segments $0, 1, \ldots m-1$.
- Strictly speaking, these effects interact with each other. How can we model the interaction?

Simplified Model

- Note that

$$\Lambda_{\text{dis},M} - \Lambda_{\text{dis},0} = \sum_{m=1}^{M} (\Lambda_{\text{dis},m} - \Lambda_{\text{dis},m-1}).$$

- Suffice to model perturbation $\Lambda_{\text{dis},m} - \Lambda_{\text{dis},m-1}$ in each segment
- Recall that $\Lambda_{\text{dis},m} - \Lambda_{\text{dis},m-1}$ depends on
  - noise $n_m(t)$ added in the segment
  - $q_{m-1}(t)$ – signal after propagating m segments which further depends on 1) all noises added in previous segments $n_0(t), \ldots, n_{m-1}(t)$, and 2) distortion accumulated due to fibre nonlinearity and dispersion in m segments

Simplified Model

- Let
  - $\bar{q}_m(t)$ is a deterministic signal and discrete eigenvalues unchanged
  - $\hat{q}_m(t) = \bar{q}_m(t) + n_m(t)$
  - $\hat{\Lambda}_{\text{dis},m}$ be its set of discrete eigenvalues.

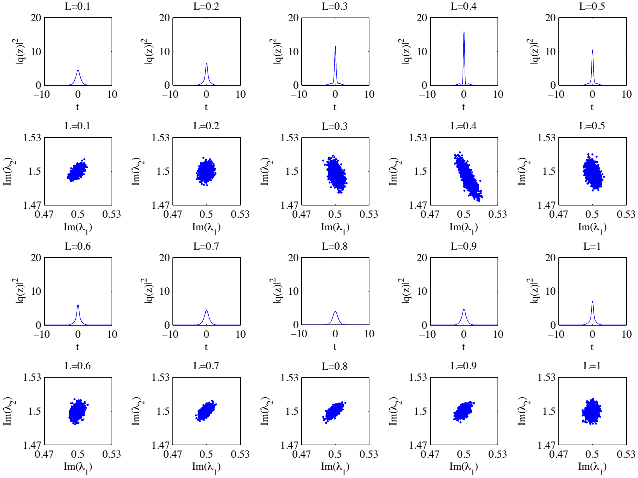
Theorem (Simplified Model)

$$\Lambda_{dis,M} = \Lambda_{dis,0} + \sum_{m=1}^{M} (\Lambda_{dis,m} - \Lambda_{dis,m-1})$$

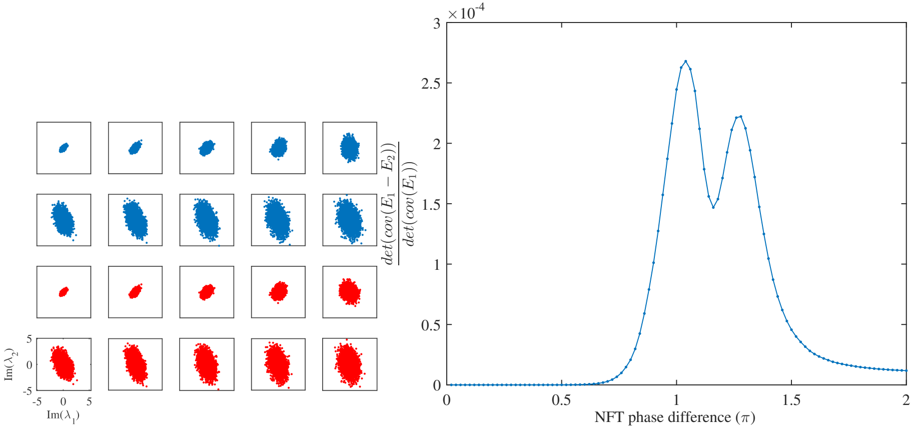
$$\approx \Lambda_{dis,0} + \sum_{m=1}^{M} (\hat{\Lambda}_{dis,m} - \Lambda_{dis,m-1}).$$

Numerical Example

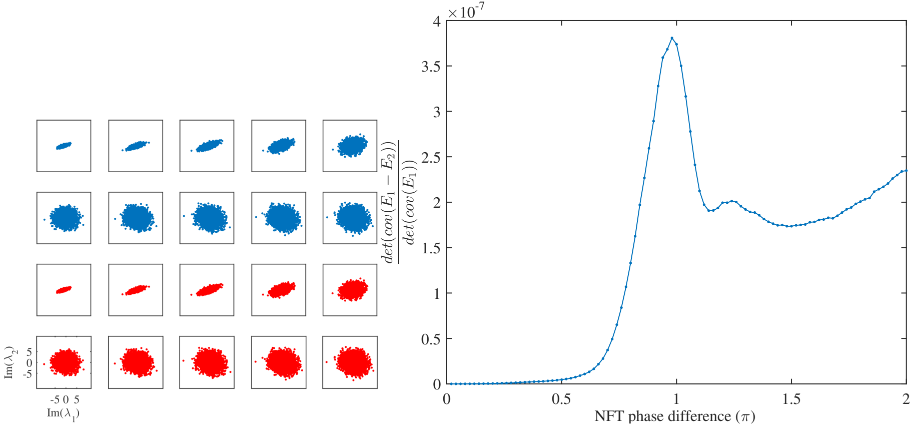
- Consider a 2-soliton input signal, $q_0(t) = 2sech(t)$, which has two discrete eigenvalues at $0.5j$ and $1.5j$.
- Propagation distance up to 1500 km which consisting of 30 loops, where every 3 loops correspond to a 0.1 normalized length.



Validation $2sech(t)$ pulses



Validation square pulses



Exploiting Correlation

Nonlinear Frequency Keying

Consider a simple "Nonlinear Frequency Keying" scheme:

- Let

$$\mathcal{K}_1 = \{\lambda_{1,1} \ldots, \lambda_{1,K_1}\}$$
$$\mathcal{K}_2 = \{\lambda_{2,1} \ldots, \lambda_{1,K_2}\}$$

- For any integers $1 \leq r \leq K_1$ and $1 \leq s \leq K_2$, the transmitted signal is a soliton with two discrete eigenvalues $\lambda_{1,r}$ and $\lambda_{2,s}$.
- Receiver aims to determine the discrete spectrum of the transmitted signal
- One decoding approach:
  - Let $y(t) = q(t, L)$ be the received signal
  - Compute the scattering function $a(\lambda)$ for the received signal
  - Numerically solve for λ such that $a(\lambda) = 0$
  - Ideally, we can decode two roots, say μ_1 and μ_2
  - Minium Distance Decoding:

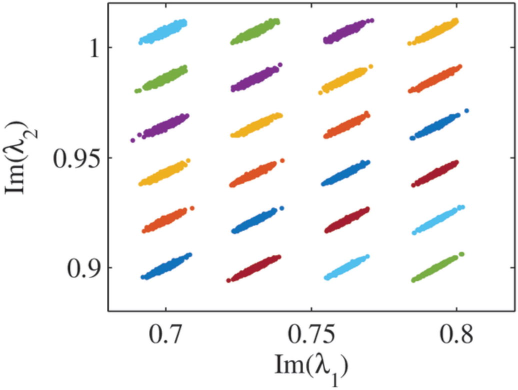
$$\hat{\lambda}_{1,r} = \arg \min_{1,\ldots,K_1} ||\mu_1 - \lambda_{1,r}||$$

$$\hat{\lambda}_{2,s} = \arg \min_{1,\ldots,K_2} ||\mu_2 - \lambda_{2,s}||$$

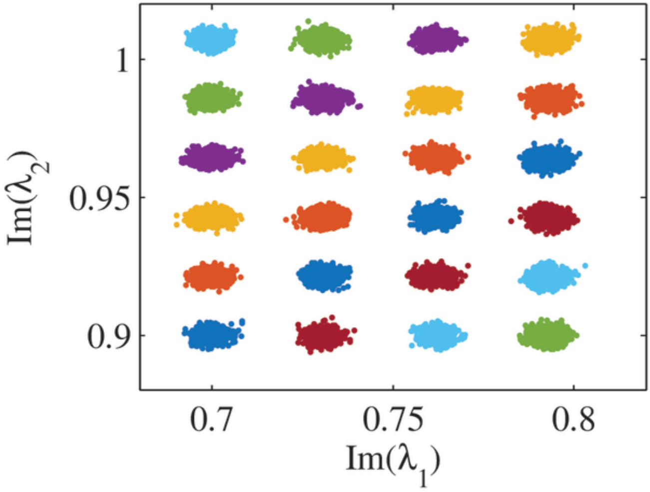
Minimum Distance Decoding is suboptimal

- Essentially assume that the probability $\Pr(\mu_1, \mu_2)$ is a monotonic decreasing function of $||\mu_1 - \lambda_{1,r}||$ and $||\mu_2 - \lambda_{2,s}||$.
  distribution of $(\mu_1, \mu_2|\lambda_{1,r}, \lambda_{2,s})$ is of the from

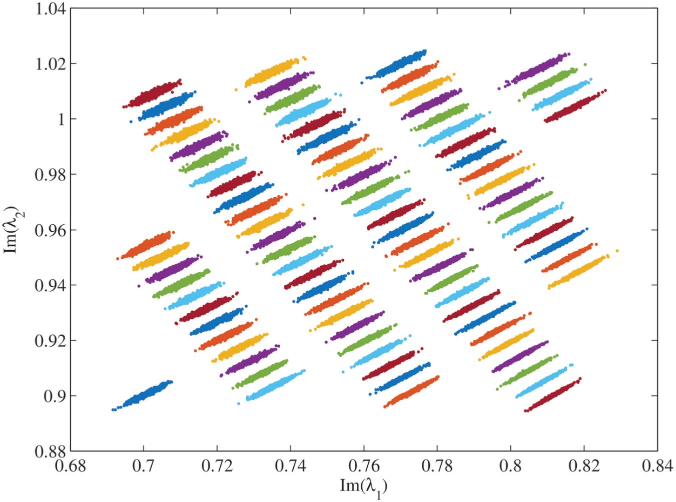
$$\Pr(\mu_1, \mu_2|\lambda_{1,r}, \lambda_{2,s}) \uparrow \text{ if } ||\mu_1 - \lambda_{1,r}||, ||\mu_2 - \lambda_{2,s}|| \downarrow$$



Ignoring correlation



Ignore Correlation



Conclusions

Summary

- Basic of how NFT decompose fibre optical channel into parallel channels
- A model for characterising perturbation of NFT functions, in the presence of noises
- Demonstrate the perturbation correlation due to noises

Challenge

- Complexity
- To WDM or not to WDM