Completion Delay of Random Linear Network Coding in Wireless Broadcast Networks

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Outline

- Review of Random Linear Network Coding (RLNC)
- RLNC in **Classical Wireless Broadcasts**
  - Completion Delay of conventional RLNC
  - Circular-shift RLNC
- RLNC in **Full-duplex Relay (Broadcast) Networks**
  - Perfect RLNC with Buffer
  - Perfect RLNC without Buffer
  - General FBPF RLNC

[1] Su R, Sun Q T, Zhang Z. Delay-complexity trade-off of random linear network coding in wireless broadcast, *IEEE ICC & IEEE Trans. Commun.*, 2020.

[2] Su R, Sun Q T, Zhang Z, et al. Completion delay of random linear network coding in full-duplex relay networks, *IEEE ISIT*, 2021 *& IEEE Trans. Commun.*, 2022.

[3] Su R, Sun Q T, Li X, et al. On the buffer size of perfect RLNC in full-duplex relay networks, *IEEE Trans. Veh. Technol.* 2023.

Random linear network coding (RLNC)

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RLNC:

Coding coefficients are randomly selected from finite field $\mathbb{F}$.

- Distributed;
- Can run w/t feedback, network topology info.

The theorem of RLNC. When $|\mathbb{F}| > r$, the probability for a *randomly constructed* $\mathbb{F}$-linear code to achieve the multicast capacity is at least $(1 - r / |\mathbb{F}|)^{|E|}$.

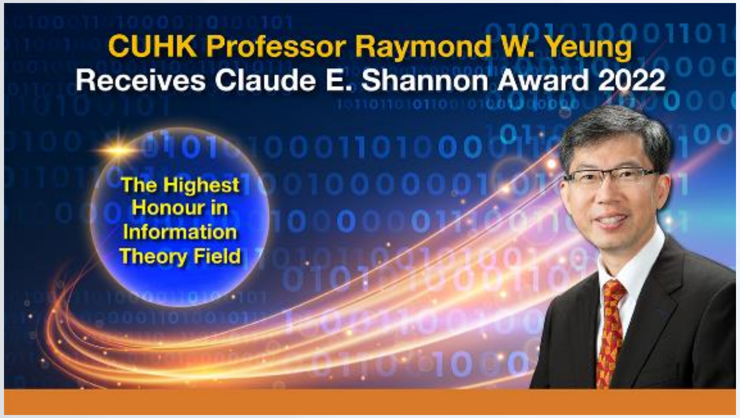
Ho T, Médard M, Koetter R, et al. A random linear network coding approach to multicast. *IEEE Trans. Inf. Theory*, 2006. // ITSoc-ComSoc joint paper award.

Random linear network coding (RLNC)

RLNC: a key concept for NC technique deployment.

- **BATched Sparse (BATS)** code: Fountain codes + RLNC
  - — Low encoding/decoding complexity
  - — Constant computational complexity & constant buffer requirement
  - — Small coefficient overhead
  - — High transmission rate

Yang S, Yeung R W. Batched sparse codes. *IEEE Trans. Inf. Theory*, 2014, 60(9): 5322-5346.



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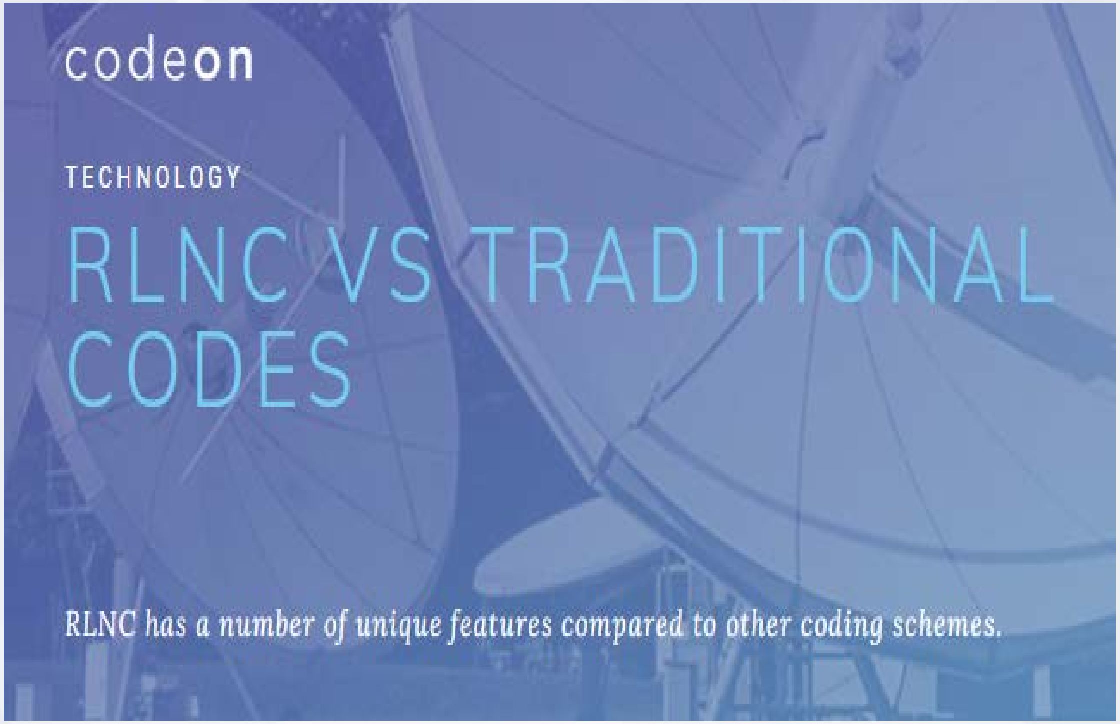

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Advantages of RLNC

Code Capabilities	RLNC	Rateless Codes	Block Codes	Characteristics/Benefits
Erasure correction	✓	✓	✓	Corrects for missing or corrupted data packets
Code is carried within each packet	✓	✗	✗	Eliminates tracking overhead
Completely distributed operation	✓	✗	✗	Enables stateless management
De-code using both unencoded and coded packets	✓	✗	✗	No forklift upgrade; adds implementation tunability
Able to generate valid codes from coded or unencoded packets	✓	✗	✗	Gradual implementation; no forklift
Composability without decoding (adding incremental redundancy)	✓	✗	✗	Enables addition of redundancy when and where needed
Encode data in a sliding window	✓	✗	✗	Flexible integration with protocols for greater efficiency

Source: https://www.codeontechnologies.com/en/home/

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Classical Wireless Broadcasts

- Crowded WiFi
- Live content distribution
- Content distribution networks (CDNs)
- IPTV
- Stadium wireless access
- DOCSIS
- Software defined networking (SDN)
- Network function virtualization (NFV)
- Satellite broadcasting

Source: https://www.codeontechnologies.com/en/home/

RLNC in Classical Wireless Broadcasts

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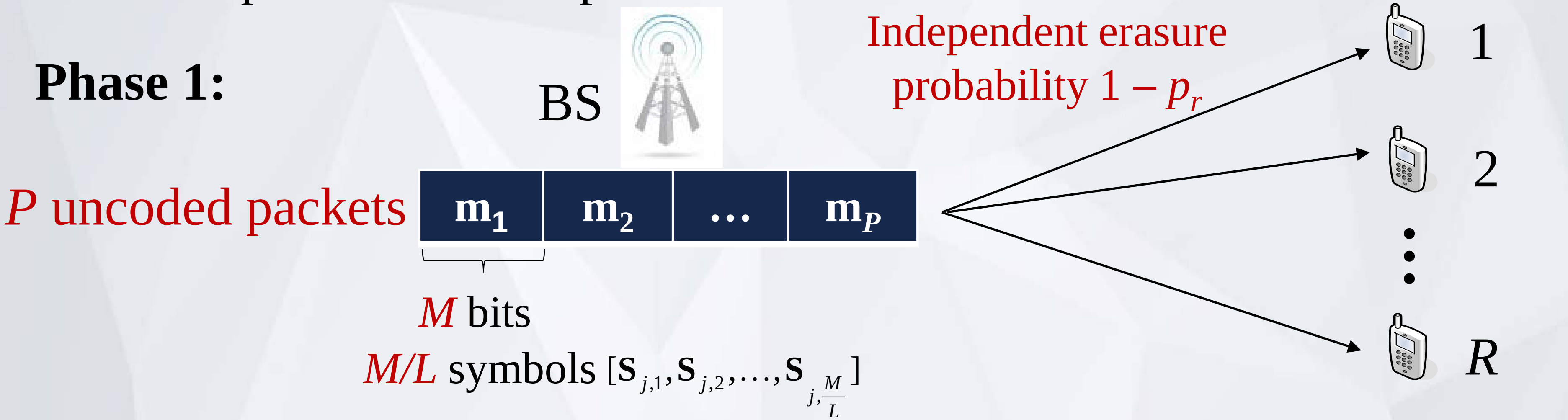
- A. Eryilmaz, A. E. Ozdaglar, M. Medard, et al. On the delay and throughput gains of coding in unreliable networks. *IEEE Trans. Inf. Theory*, 2008.
- D. E. Lucani, M. Medard, M. Stojanovic. On coding for delay – network coding for time-division duplexing, *IEEE Trans. Inf. Theory*, 2012.
- B. T. Swapna, A. Eryilmaz, N. B. Shroff. Throughput-delay analysis of random linear network coding for wireless broadcasting, *IEEE Trans. Inf. Theory*, 2013.
- A. Tassi, F. Chiti, R. Fantacci, et al. An energy-efficient resource allocation scheme for RLNC-based heterogeneous multicast communications, *IEEE Commun. Lett.*, 2014.
- J. Huang, H. Gharavi, H. Yan, et al. Network coding in relay-based device-to-device communications, *IEEE Network*, 2017.
- I. Chatzigeorgious, A. Tassi. Decoding delay performance of random linear network coding for broadcast, *IEEE Trans. Veh. Technol.*, 2017.
- E. Tsimbalo, A. Tassi, R. J. Piechocki. Reliability of multicast under random linear network coding, *IEEE Trans. Commun.*, vol. 66, no. 6, 2018.
- D. E. Lucani, F. H. P. Fitzek, M. Reisslein, et al. DSEP Fulcrum: dynamic sparsity and expansion packets for fulcrum network coding, *IEEE Access*, 2020.

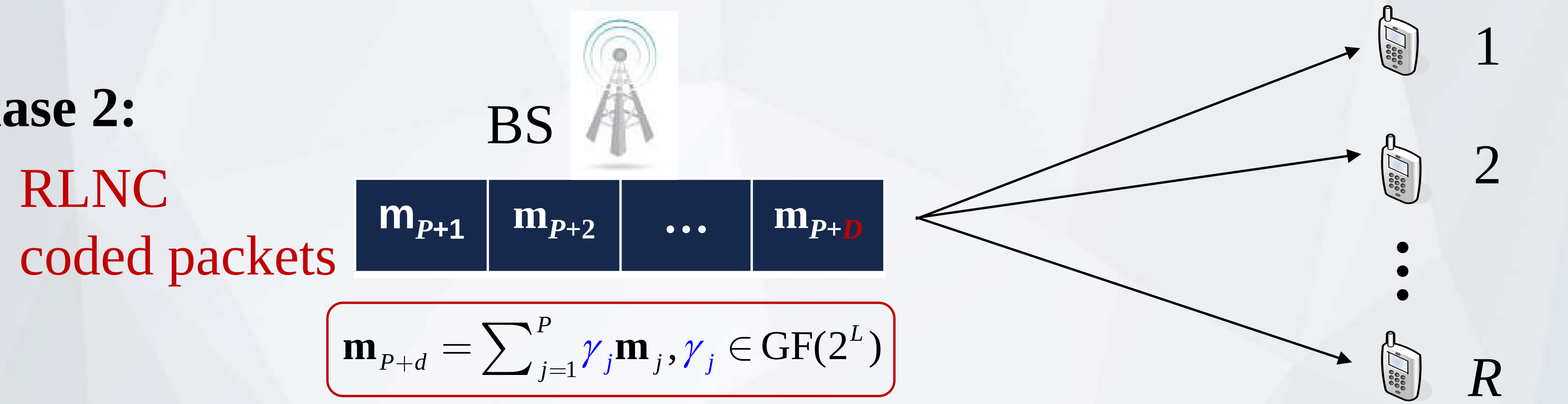
Classical Wireless Broadcast — System Model

BS attempts to deliver P packets to a set of R receivers.

Phase 1:



Phase 2:



$$\mathbf{m}_{P+d}=\sum_{j=1}^{P}\gamma_j\mathbf{m}_j,\ \gamma_j\in \mathrm{GF}(2^L)$$

- D : system completion delay, $D=\max\{D_1,D_2,\ldots,D_R\}$;
- D_r : completion delay at single receiver r.
- No feedback

Strength & Weakness of RLNC

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TABLE I
PERFORMANCE OF RANDOM, OPPORTUNISTIC, AND INSTANTLY DECODABLE NETWORK CODING ACCORDING TO VARIOUS CRITERIA

Criterion \ Scheme	Random Network Coding	Opportunistic Network Coding	Instantly Decodable Network Coding
Throughput	Optimal	Sub-optimal	Sub-optimal
Delay	Huge delay	Moderate depending on the scheme	Moderate depending on the scheme
Complexity	Large field size	Depends on the scheme	Binary field
Encoding	Mix using random independent coefficients	Mix using diversity of lost and received packets	Mix using binary XOR
Decoding	Complexity cubical with the number of packets	Moderate depending on the scheme	Simple binary XOR
Progressive Decoding	Decoding is performed after getting the whole frame	Depend on the scheme but usually better than RNC	Instantaneous decoding
Overhead	Moderate depending on the scheme	Moderate depending on the scheme	Minimal
Buffer Size	As large as the frame size	Moderate depending on the scheme	No need for buffer
Feedback Load	Minimal feedback and can run even without feedback	More or less heavy depending on the scheme	Performance heavily depends on feedback
Broadcast Efficiency	Optimal	Sub-optimal	Sub-optimal
Multicast Efficiency	Inefficient	Depends on the scheme	Depends on the scheme

Douik A, Sorour S, Al-Naffouri T Y, et al. Instantly decodable network coding: from centralized to device-to-device communications, *IEEE Commun. Surveys & Tutorials*, 2017.

Two Benchmark RLNC Schemes

Field size	Completion delay	Decoding complexity
↑	↓ 😁	↑ 😢
↓	↑ 😢	↓ 😁

➢ Perfect RLNC

- Assume arbitrary P packets generated by the source are linearly independent.
- Optimal in terms of completion delay.
- High computation complexity caused by large finite fields.
- $\mathbb{E}[P+D_r]=P/p_r, \quad \mathbb{E}[D_r]=P/p_r-P=\frac{1-p_r}{p_r}P$

➢ GF(2)-RLNC

- Optimal in terms of computation complexity.
- High completion delay.
- $\mathbb{E}[D_r^{\mathrm{GF}(2)}]\le(P+2)/p_r$

RLNC in Wireless Broadcast

Field size	Completion delay	Decoding complexity
↑	↓ 😁	↑ 😢
↓	↑ 😢	↓ 😁

Our goal:

1. Theoretically analyze the system completion delay performance of RLNC.
2. Design an RLNC scheme with a better completion delay vs decoding complexity tradeoff.

Completion Delay Analysis

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Proposition. For GF(q)-RLNC scheme,

$$\mathbb{E}[D] = \sum_{d\geq 0}\left(1-\prod_{1\leq r\leq R}\Pr(D_r \leq d)\right)$$ // D_r: completion delay at receiver r

$$\Pr(D_r = d) = \sum_{u=\max\{0,P-d\}}^{P-1}\binom{P}{u} p_r^{\,u}(1-p_r)^{P-u}\Pr(D_r = d \mid U_r = u)$$ // u : # received uncoded packets

$$\Pr\left(D_r = d \mid U_r = u\right) = \begin{cases} 0 \text{ for } u = P, u < P-d \\ \sum_{\mathbf{a}\in\mathcal{A}_{P-u,d}} \prod_{j=1}^{P-u}\left(1-p'_{r,u+j-1}\right)^{a_j-1} p'_{r,u+j-1} \text{ otherwise} \end{cases}$$

Proposition. For the perfect RLNC scheme,

$$\mathbb{E}[D] = \sum_{d\geq 0}\left(1-\prod_{1\leq r\leq R}\Pr(D_r \leq d)\right) = \sum_{d\geq 0}\left(1-\prod_{1\leq r\leq R} I_{p_r}(P, d+1)\right)$$

// $I_{p_r}(P,d+1) = \sum_{j=0}^{d}\binom{P+j-1}{P-1} p_r^{\,P}(1-p_r)^j$ is *incomplete beta function.*

Completion Delay Analysis

Theorem. For GF(2)-RLNC, $\lim_{P\to\infty} \mathbb{E}[D^{\mathrm{GF}(2)}]/P = \lim_{P\to\infty} \mathbb{E}[D^{\mathrm{perf}}]/P$.

// $D^{\mathrm{GF}(2)}$: the completion delay of GF(2)-RLNC.
// D^{perf} : the completion delay of perfect RLNC.

Circular-shift RLNC — Motivation

Issues for conventional RLNC over large GF(2^L):

- The larger GF(2^L) is, the lower probability random $\gamma_j = 0$.
- Heavy large field multiplications lead to high decoding complexity.

$$\mathbf{m}_{P+d} = \sum_{j=1}^{P} \gamma_j \mathbf{m}_j, \gamma_j \in \mathrm{GF}(2^L)$$

Design motivation:

- Using sparse encoding vectors to alleviate the decoding complexity.
- Adopt vector RLNC & choose circular-shifts as linear operation.

[1] Tang H, Sun Q T, Li Z, et al. Circular-shift linear network coding. *IEEE Trans. Inf. Theory*, 2019.

[2] Sun Q T, Tang H, Li Z, et al. Circular-shift linear network codes with arbitrary odd block lengths. *IEEE Trans. Commun.*, 2019.

[3] Tang H, Sun Q T, Yang X, et al. On encoding and decoding of circular-shift linear network codes, *IEEE Commun. Letters*, 2019.

[4] Sun Q T, Yang X, Long K, et al. On vector linear solvability of multicast networks. *IEEE Trans. Commun.*, 2016.

Circular-shift RLNC — Scheme Description

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- Let L be an even integer such that $L + 1$ is a prime with a primitive root 2. $2^L \bmod (L+1) = 1$

- Matrix coding coefficients Γ_j ($L \times L$) are randomly and independently selected from

$$\mathcal{C} = \{\mathbf{0}, \mathbf{G}\mathbf{C}_{L+1}\mathbf{H}, \mathbf{G}\mathbf{C}_{L+1}^2\mathbf{H}, \cdots, \mathbf{G}\mathbf{C}_{L+1}^{L+1}\mathbf{H}\}$$

where $\mathbf{C}_{L+1} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_L \\ 1 & \mathbf{0} \end{bmatrix}$, $\mathbf{G} = [\mathbf{I}_L\ \mathbf{1}]$, $\mathbf{H} = [\mathbf{I}_L\ \mathbf{0}]^{\mathrm{T}}$.

$(L+1)\times(L+1)$ $\quad L\times(L+1)$ $\quad (L+1)\times L$

- For the case $L = 4$,

$$\mathbf{C}_{L+1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Circular-shift RLNC — Scheme Description

- **Example.** Assume M = 8bits, L = 4. Given two packets

$$\mathbf{m}_1 = [(0\ 1\ 1\ 1), (1\ 1\ 1\ 0)] \text{ and } \mathbf{m}_2 = [(0\ 1\ 1\ 1), (1\ 1\ 1\ 0)],$$

and two coding coefficients

$$\boldsymbol{\Gamma}_1 = \mathbf{G}\mathbf{C}_{L+1}\mathbf{H}, \boldsymbol{\Gamma}_2 = \mathbf{G}\mathbf{C}_{L+1}^2\mathbf{H}$$

$$\begin{aligned} & \mathbf{m}_1 \circ \boldsymbol{\Gamma}_1 + \mathbf{m}_2 \circ \boldsymbol{\Gamma}_2 \\ &= [(0\ 1\ 1\ 1\ 1)\mathbf{C}_{L+1}\mathbf{H}, (1\ 1\ 1\ 0\ 1)\mathbf{C}_{L+1}\mathbf{H}] + \\ &\quad [(0\ 1\ 1\ 1\ 1)\mathbf{C}^2_{L+1}\mathbf{H}, (1\ 1\ 1\ 0\ 1)\ \mathbf{C}^2_{L+1}\mathbf{H}] \\ &= [(1\ 0\ 1\ 1\ 1)\mathbf{H}, (1\ 1\ 1\ 1\ 0)\mathbf{H}] + [(1\ 1\ 0\ 1\ 1)\mathbf{H}, (0\ 1\ 1\ 1\ 1)\mathbf{H}] \\ &= [(0\ 1\ 1\ 0), (1\ 0\ 0\ 0)] \end{aligned}$$

For a binary row vector, multiplying C-S matrices cost no decoding complexity.

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Circular-shift RLNC — Scheme Description

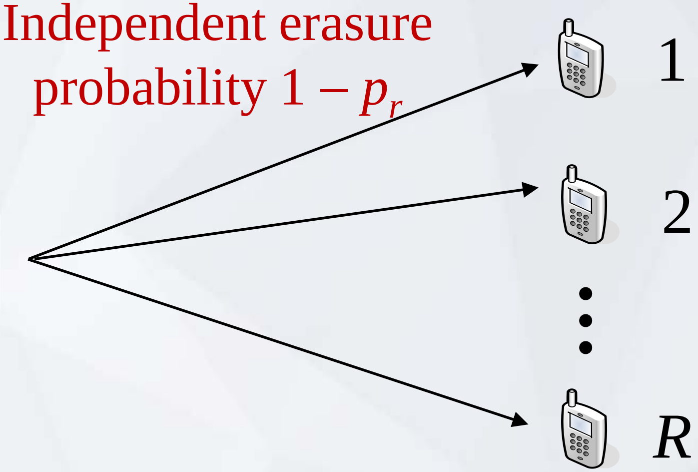
Phase 1:

Independent erasure probability $1 - p_r$

BS

P uncoded packets

$\mathbf{m}_1$	$\mathbf{m}_2$	…	$\mathbf{m}_P$



M bits

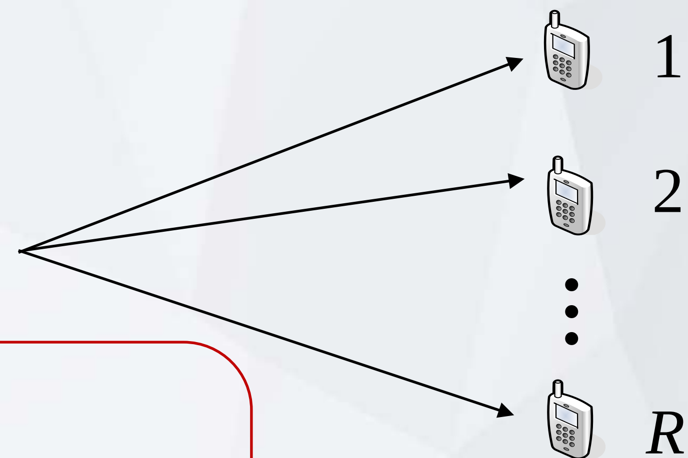
M/L symbols $[\mathbf{s}_{j,1}, \mathbf{s}_{j,2}, \ldots, \mathbf{s}_{j,\frac{M}{L}}]$

Phase 2:

BS

C-S RLNC coded packets

$\mathbf{m}_{P+1}$	$\mathbf{m}_{P+2}$	…	$\mathbf{m}_{P+D}$



$$\mathbf{m}_{P+d} = \sum_{j=1}^{P} \mathbf{\Gamma}_j \circ \mathbf{m}_j$$

$$\mathbf{\Gamma} \circ \mathbf{m}_j = [\mathbf{s}_{j,1}\mathbf{\Gamma}, \mathbf{s}_{j,2}\mathbf{\Gamma}, \cdots, \mathbf{s}_{j,M/L}\mathbf{\Gamma}]$$

symbol-wise multiplications

// $\mathbf{\Gamma}$: $L \times L$ random GF(2)-matrix coefficients.

Circular-shift RLNC

- Random coefficients $\mathbf{\Gamma}_j$ follows the distribution

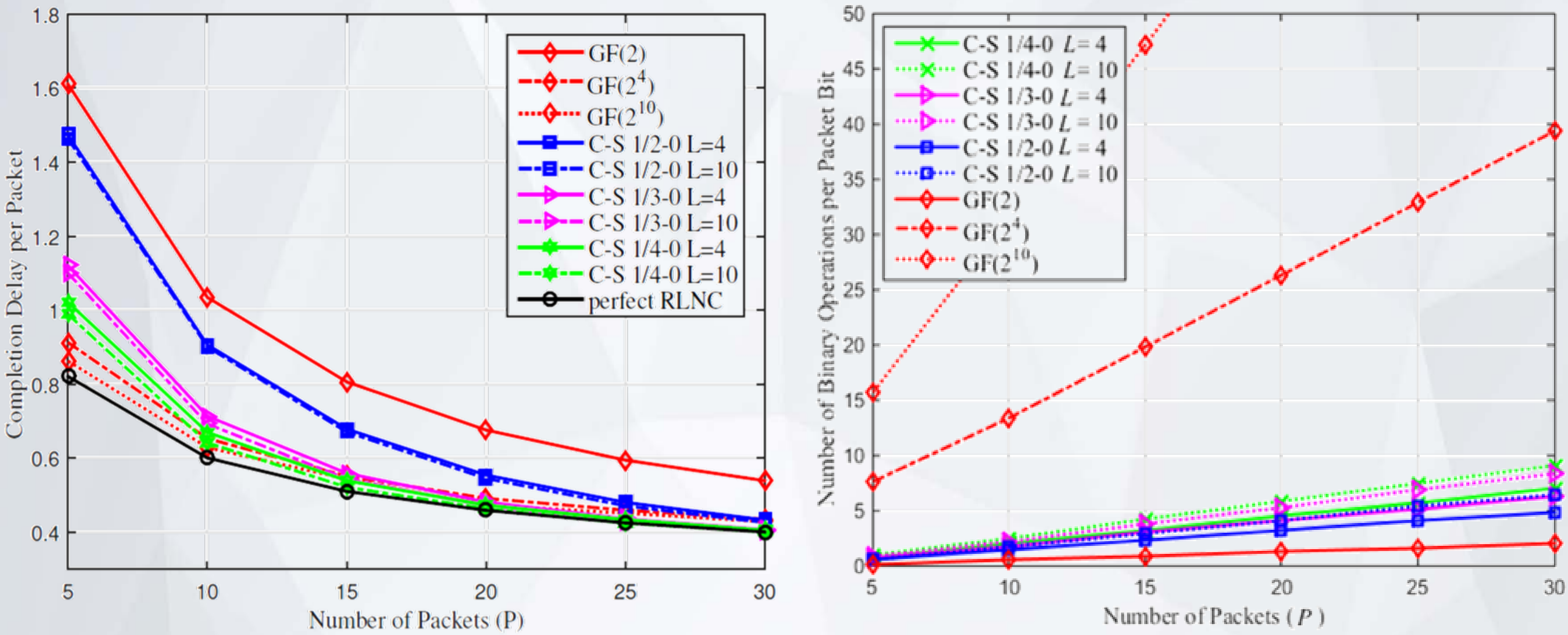
$$\Pr(\mathbf{\Gamma}_j = \mathbf{\Gamma}) = \begin{cases} p_z, \mathbf{\Gamma} = \mathbf{0} \\ \dfrac{1-p_z}{L+1}, \mathbf{\Gamma} \in \mathcal{C} \setminus \{\mathbf{0}\} \end{cases}$$

// p_z is a particular parameter to control the probability of **0** to occur.

Theorem. For circular-shift RLNC with $p_z \geq 1/(L+2)$, GF(q)-RLNC with $p_z \leq 1/q$, $\mathbb{E}[D^{\text{circ}}] \leq \mathbb{E}[D^{\text{GF}(q)}]$, $\lim_{P\to\infty} \mathbb{E}[D^{\text{circ}}]/P = \lim_{P\to\infty} \mathbb{E}[D^{\text{perf}}]/P$.

Numerical Analysis

- Setting: $R = 60$, $p_r \sim \mathrm{U}(0.8, 0.9)$



- Decoding complexity ⟶ # binary operations required in the decoding process.
- C-S RLNC performs well when $p_z = 1/4$.
- C-S RLNC has comparative completion delay but a much lower decoding complexity.

Numerical Analysis

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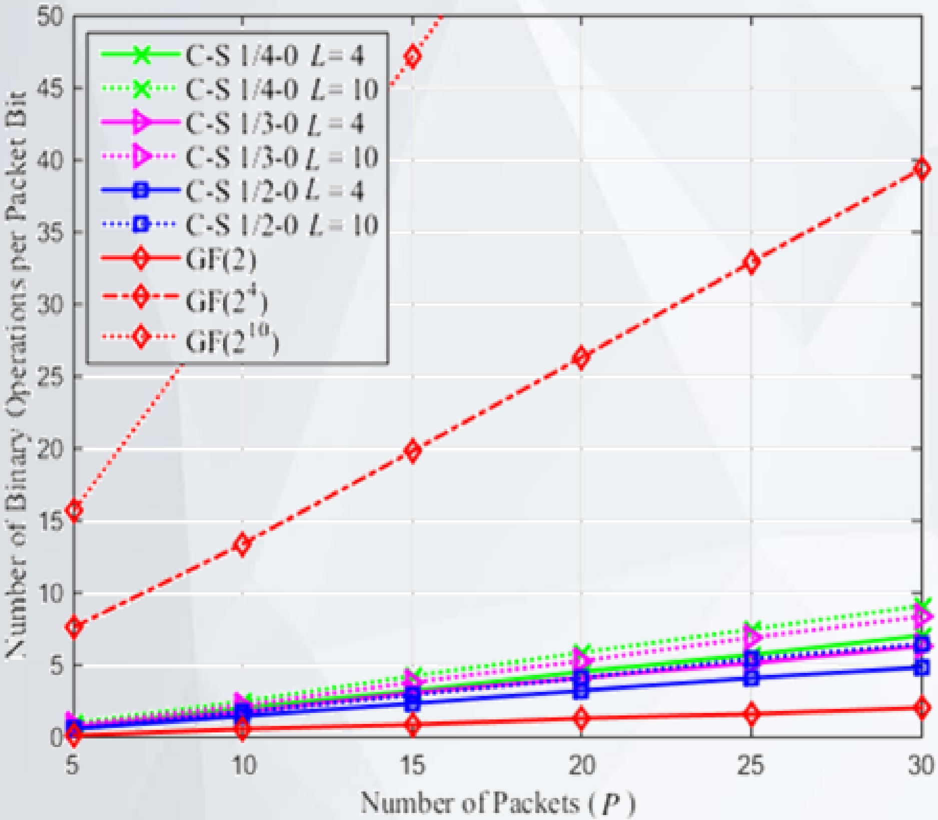
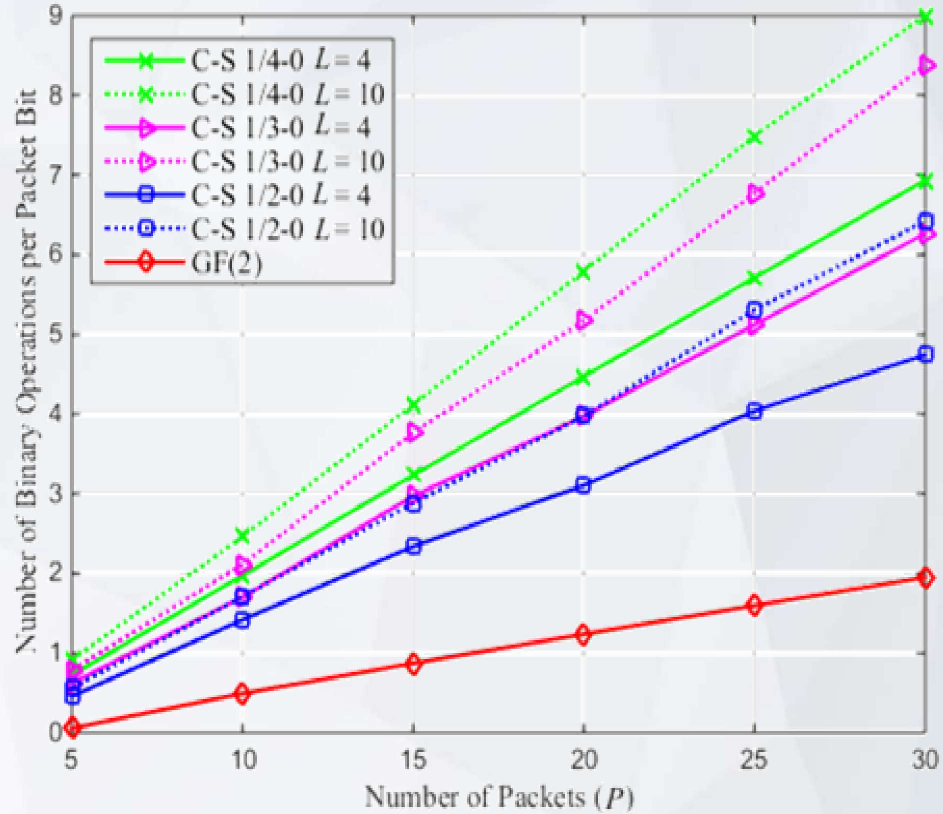
- Setting: $R = 60$, $p_r \sim \mathrm{U}(0.8, 0.9)$





A better trade-off: For the case $L = 4$, $p_z = 1/4$, when $P \geq 15$, # decoding operations of C-S RLNC is about **3 times** # decoding operations of **GF(2)-RLNC**, while its completion delay is within **5%** higher than **perfect RLNC**.

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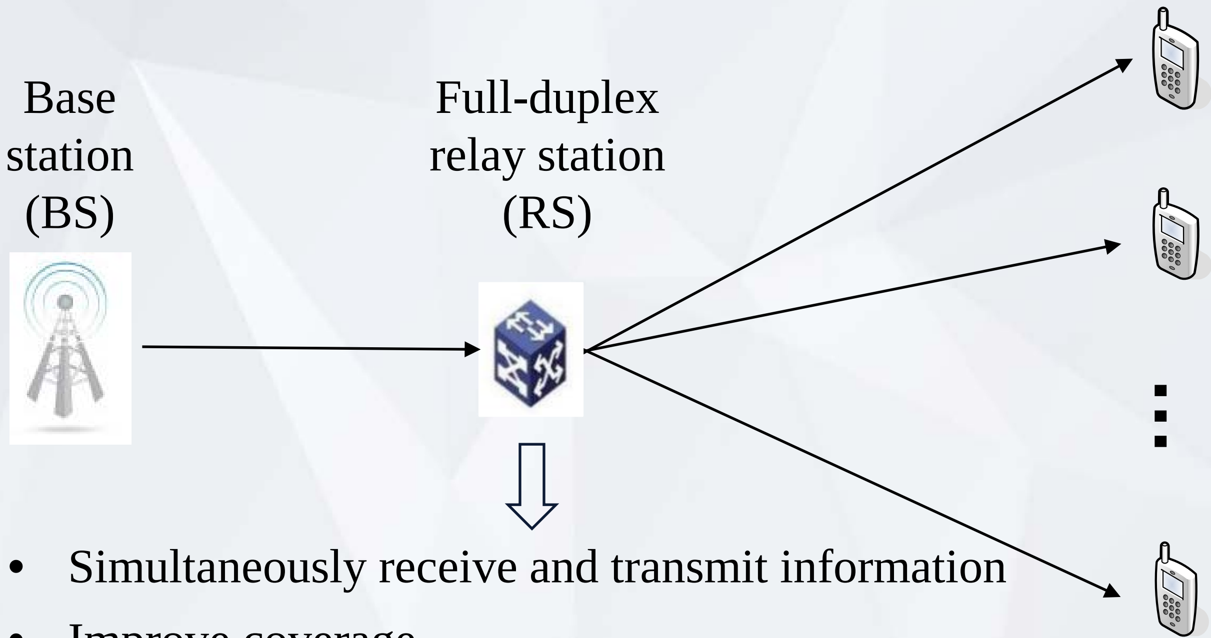
[3] Su R, Sun Q T, Li X, et al. On the buffer size of perfect RLNC in full-duplex relay networks, *IEEE Trans. Veh. Technol.* 2023.

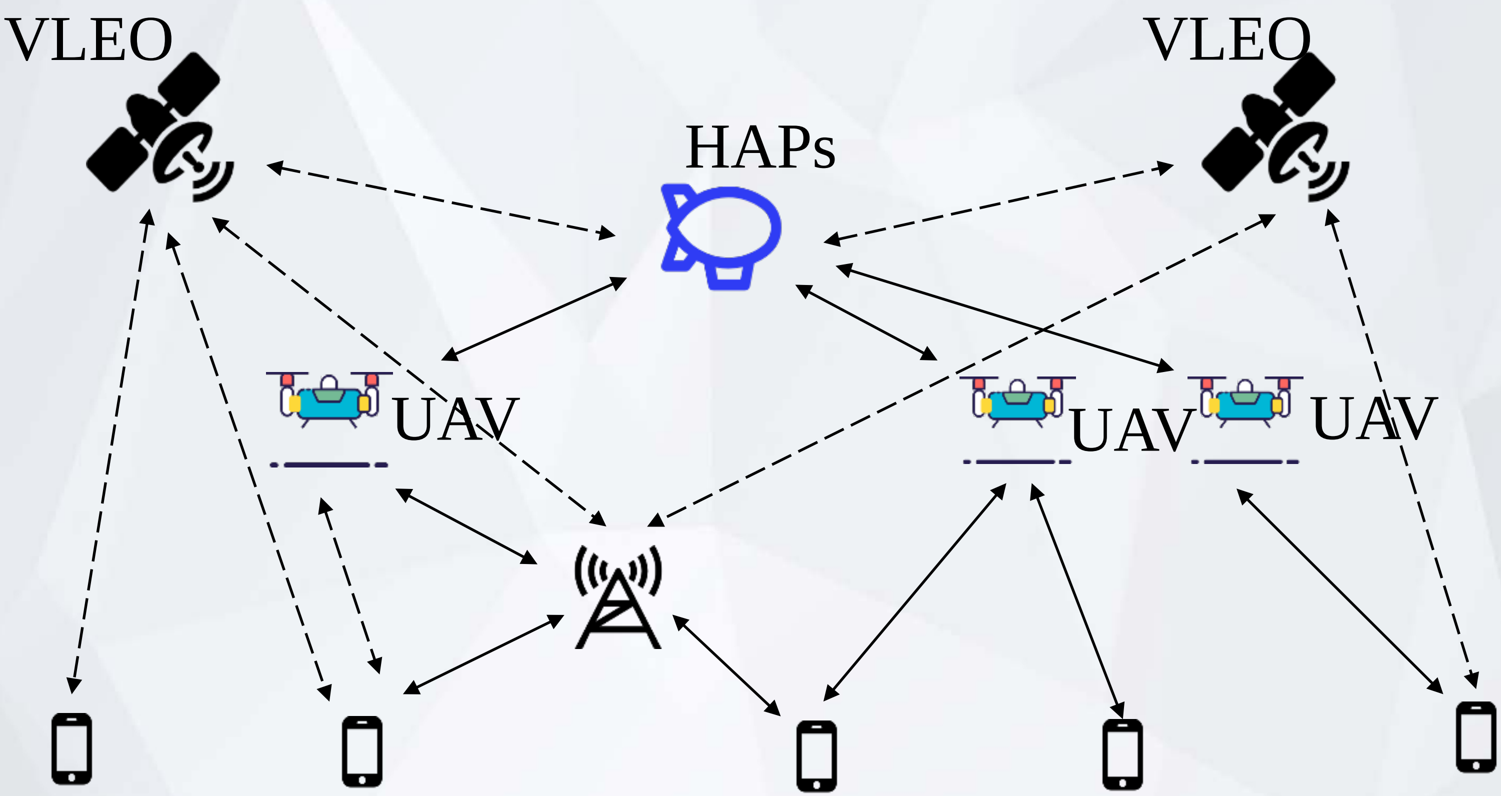
Full-duplex Relay Network — Background



- Simultaneously receive and transmit information
- Improve coverage
- Improve throughput

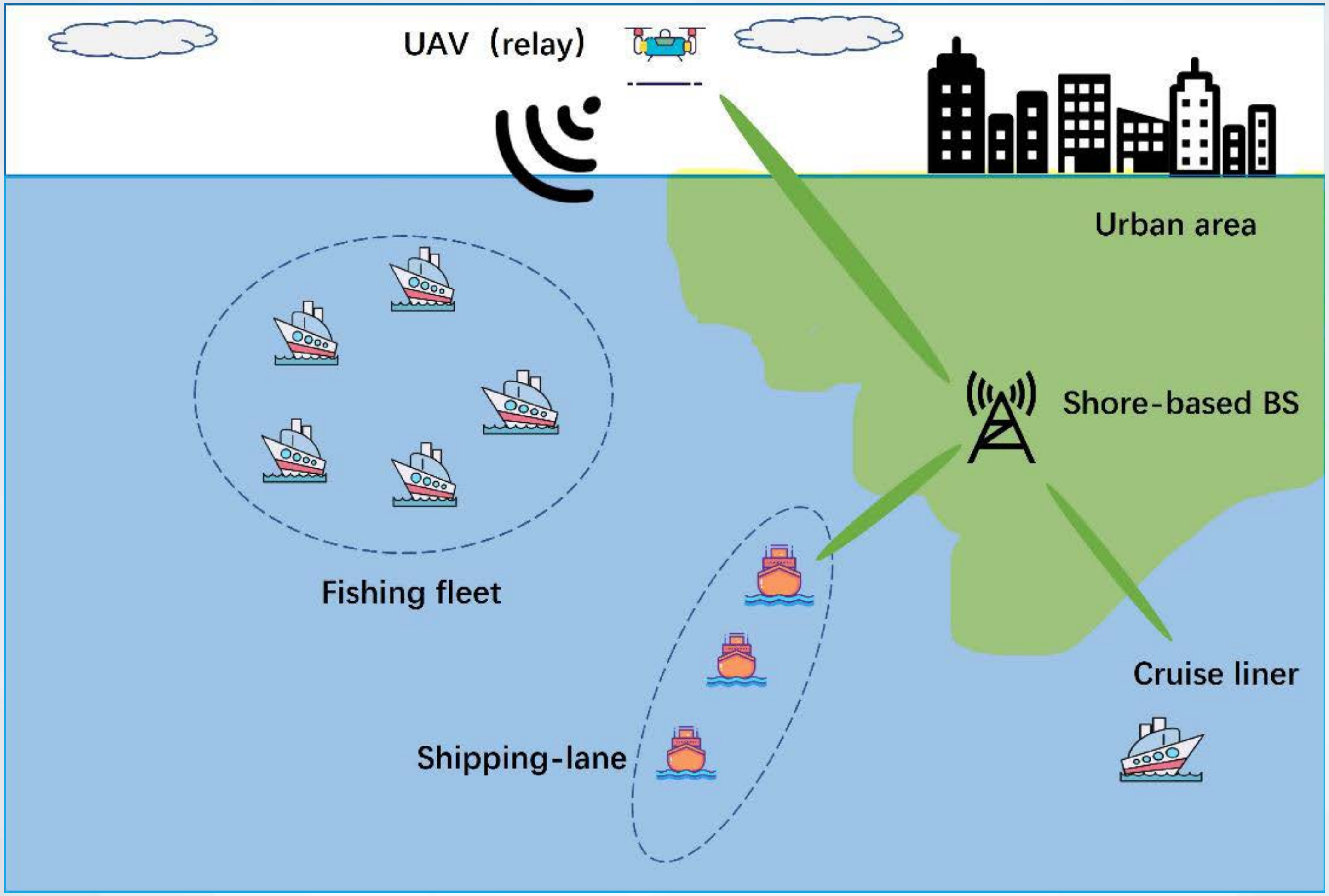
IEEE 802.16j and **3GPP LTE-Advanced** have proposed *two-hop relay networks* for the sake of simplicity and explicitness of system design.

- Evolved multimedia broadcast/multicast services (eMBMS)
- Digital video broadcasting (DVB-T/H)
- Integrated 6G network with UAV、HAPs and VLEO satellites
- Wideband coastal communications



Integrated 6G network with UAV、HAPs and VLEO satellites

Full-duplex Relay Network — Background

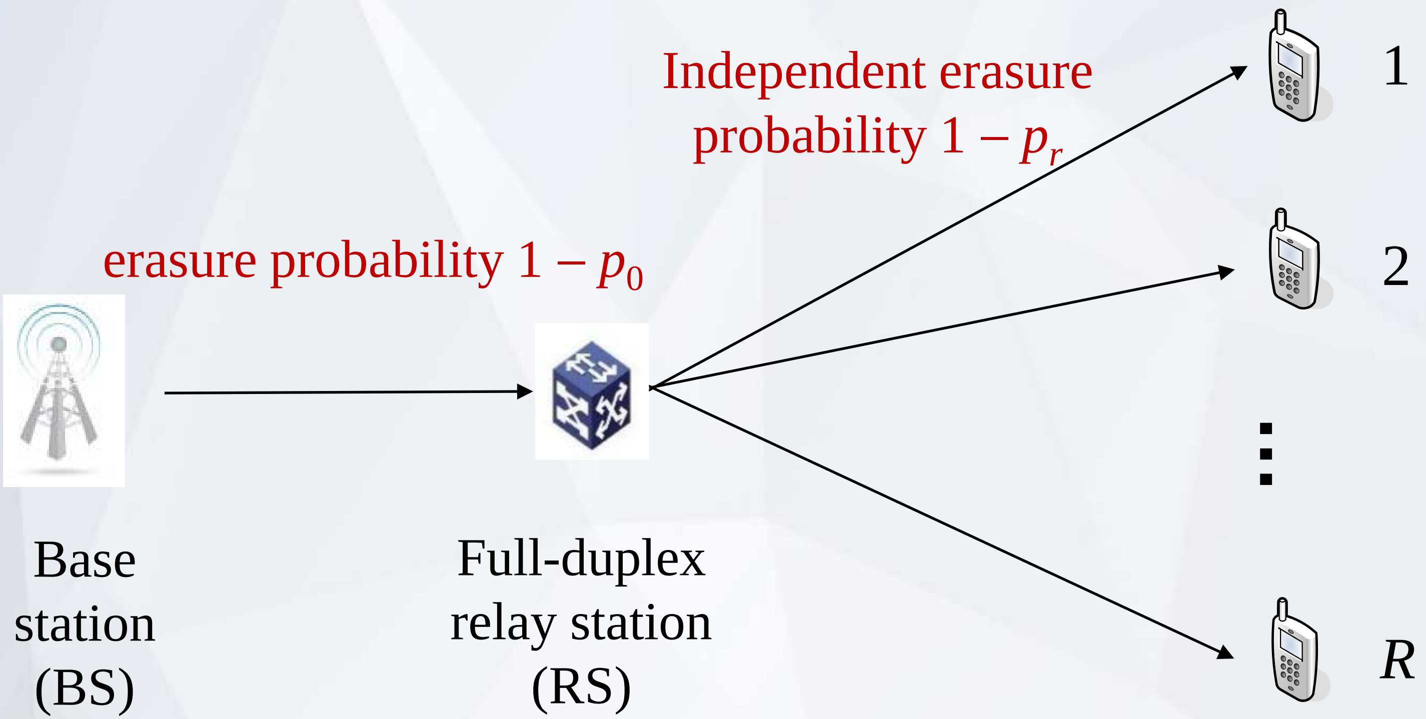


System architecture for wideband coastal communications.

Li Y, Wang J, Zhang S, et al. Efficient coastal communications with sparse network coding. *IEEE Network*, 2018, 32(4): 122-128.

Full-duplex Relay Network — System Model

BS attempts to deliver P packets to a set of R receivers.



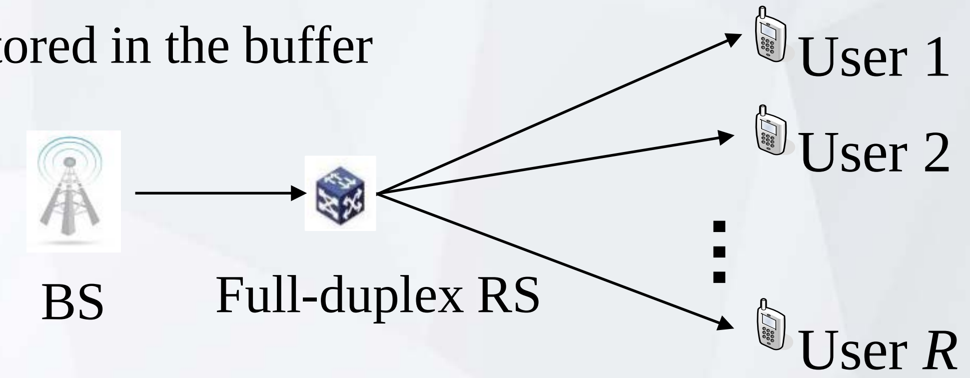
- When $p_0 = 1$, degenerate to classical wireless broadcasts.
- Completion delay D = # packets BS transmits before every receiver can recover all original packets.

A Known Scheme: FBPF RLNC Scheme

Chen C, Meng Z, Baek S J, et al. Low-complexity coded transmission without CSI for full-duplex relay networks. *IEEE GLOBECOM*, 2020.

➢ FBPF scheme:

- **F**ewest **B**roadcast **P**ackets **F**irst
- A **perfect** RLNC scheme
- Unlimited buffer
- No coding
- RS selects and broadcast the packet that has been broadcast the fewest # times
- All packets received at the RS are stored in the buffer



FBPF does not shed light on the best completion delay performance perfect RLNC can achieve.

Our goal to investigate the fundamental performance limit of RLNC in full-duplex relay broadcast networks:

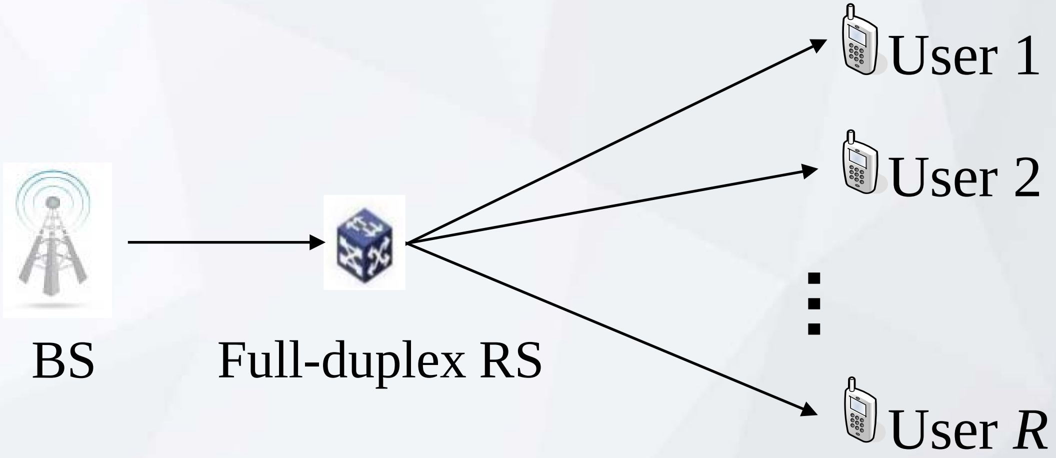
- The best performance gain (*RS can do everything*)

Perfect RLNC with Buffer

➢ Perfect RLNC with buffer

- No coding constraints at RS. Buffer size is *P*.
- *P* original packets can be recovered from any *P* packets generated by the BS.
- No matter the RS receives a packet or not, it broadcasts a random linear combination of all the packets stored in the buffer.
- # linearly independent packets obtained at a receiver is always no larger than # packets buffered at the RS.

Perfect RLNC with buffer attains the best completion delay performance among all RLNC schemes.

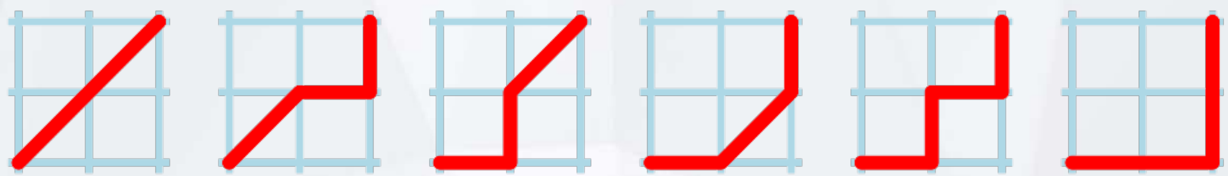


Perfect RLNC with Buffer — Completion Delay

➢ Perfect RLNC with buffer, single receiver case

Theorem. $$\mathbb{E}[D_{P,r}] = \frac{P}{p_0} + \frac{P}{p_r} - 1 + \sum_{i=0}^{P-2}\sum_{j=0}^{i} \frac{(P-i-1)T_{i,j}(p_0 p_r)^i}{(p_0 p_r - p_0 - p_r)^{i+j+1}}.$$

$$T_{i,j} = \frac{1}{j+1}\binom{i+j}{i}\binom{i}{j}$$ // # Schroeder paths from (0, 0) to (i, i) with $j \rightarrow$ or $j \uparrow$, and $i-j \nearrow$



$$\frac{1}{1}\binom{2}{2}\binom{2}{0} = 1 \qquad \frac{1}{2}\binom{3}{2}\binom{2}{1} = 3 \qquad \frac{1}{2}\binom{4}{2}\binom{2}{2} = 2$$

// A Schroeder paths of size i is a lattice path from (0, 0) to (i, i) that never passes below the line $y = x$ and uses only "North" steps, "East" steps and "Northeast" steps.

Corollary: $$\mathbb{E}[D_{P+1,r}] - \mathbb{E}[D_{P,r}] = \frac{1}{p_0} + \frac{1}{p_r} + \sum_{i=0}^{P-1} B(i).$$

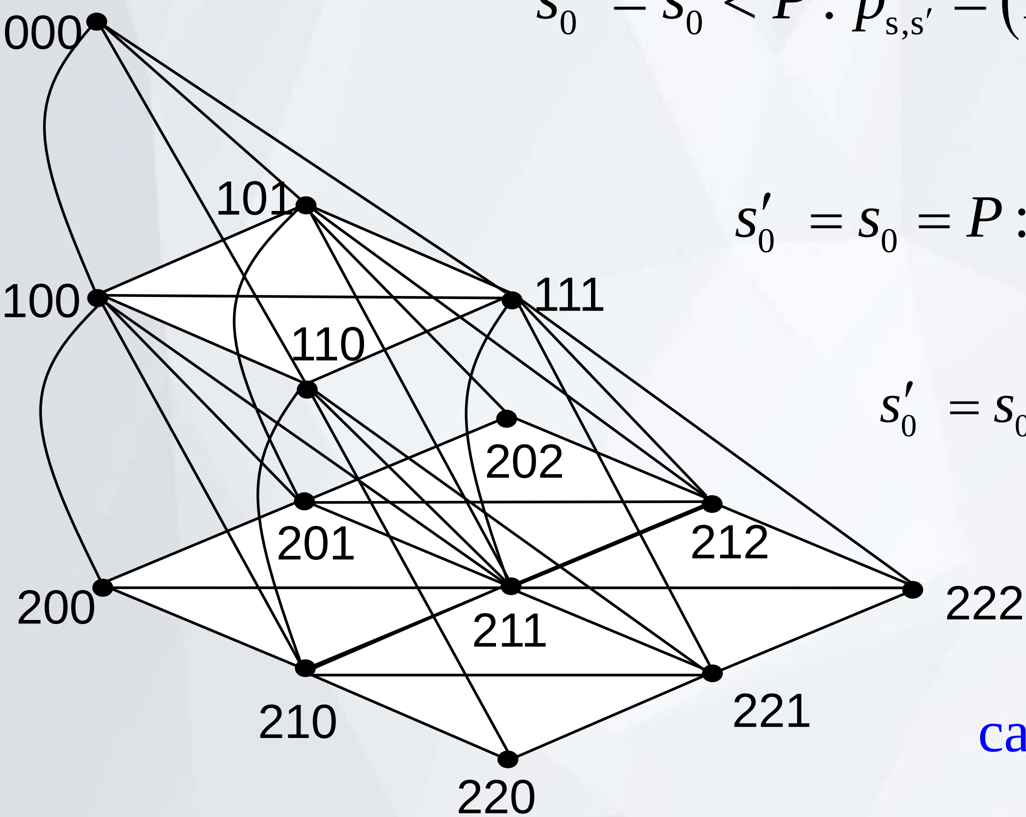
Corollary: $$B(i) = -\frac{p_0 p_r}{\Delta}\left(B(i-1) + \sum_{j=0}^{i-1} B(j)B(i-j-1)\right), B(0) = -\frac{1}{p_0 + p_r - p_0 p_r}.$$

➢ Perfect RLNC with buffer, multiple-receiver case

Approach 1. $\mathbb{E}[D_P] = (1,0,\ldots,0)(\mathbf{I}-\mathbf{P})^{-1}\mathbf{1}$

- Model transmission as a Markov chain with $\sum_{s_0=0}^{P}(s_0+1)^R$ states.

For case $P = R = 2$



$$s_0' = s_0 < P : p_{s,s'} = (1-p_0)\left(\prod_{r\in\mathcal{R}'} p_r\right)\left(\prod_{r\in\mathcal{R}\setminus\mathcal{R}'}(1-p_r)\right)$$

$$s_0' = s_0 = P : p_{s,s'} = \left(\prod_{r\in\mathcal{R}'} p_r\right)\left(\prod_{r\in\mathcal{R}\setminus\mathcal{R}'}(1-p_r)\right)$$

$$s_0' = s_0 + 1 : p_{s,s'} = p_0\left(\prod_{r\in\mathcal{R}'} p_r\right)\left(\prod_{r\notin\mathcal{R}'}(1-p_r)\right)$$

$\sum_{s_0=0}^{P}(s_0+1)^R$ can be very large with increasing *P*.

Approach 2. Deduce an approximation/lower bound for $\mathbb{E}[D_P]$.

Lemma. When $p_0 < 1, \Pr\left(S_{P+1} > T_{P,r}\right) = \frac{1}{1-p_0} + \frac{p_0}{1-p_0}\sum_{i=0}^{P-1} B(i).$

// $S_{P+1} > T_{P,r}$ means upon the reception of the $(P+1)^{\text{st}}$ packet at the RS, receiver r has only received fewer than P packets.

Corollary. For the case $R = 2$ and $P \geq 2$,

$$\mathbb{E}[D_P] \geq \max\left\{\mathbb{E}[D_{P,1}], \mathbb{E}[D_{P,2}], \mathbb{E}[\hat{D}_P] + \tilde{D}_P\right\}$$

$$\mathbb{E}[D_{P+1,r}] - \mathbb{E}[D_{P,r}] = \frac{1}{p_0} + \frac{1}{p_r} + \sum_{i=0}^{P-1} B(i).$$

$$B(i) = -\frac{p_0 p_r}{\Delta}\left(B(i-1) + \sum_{j=0}^{i-1} B(j)B(i-j-1)\right)$$

$$\mathbb{E}[\hat{D}_P] = P + \sum_{d\geq 0}\left(1 - \prod_{1\leq r\leq R}\sum_{j=0}^{d}\binom{P+j-1}{P-1} p_r^P (1-p_r)^j\right)$$

$$\tilde{D}_P = \left(\frac{1}{p_0} - 1\right)\left(1 + \sum_{j=1}^{P-1}\Pr(S_{j+1} > T_{j,1})\Pr(S_{j+1} > T_{j,2})\right)$$

$$\Pr(S_{P+1} > T_{P,r}) = \frac{1}{1-p_0} + \frac{p_0}{1-p_0}\sum_{i=0}^{P-1} B(i).$$

Generalize the approximation from $R = 2$ to $R \geq 2$:

$$\mathbb{E}[D_P] \gtrsim \max\{\max_{1\leq r\leq R} \mathbb{E}[D_{P,r}], \mathbb{E}[\hat{D}_P] + \tilde{D}_P\}$$

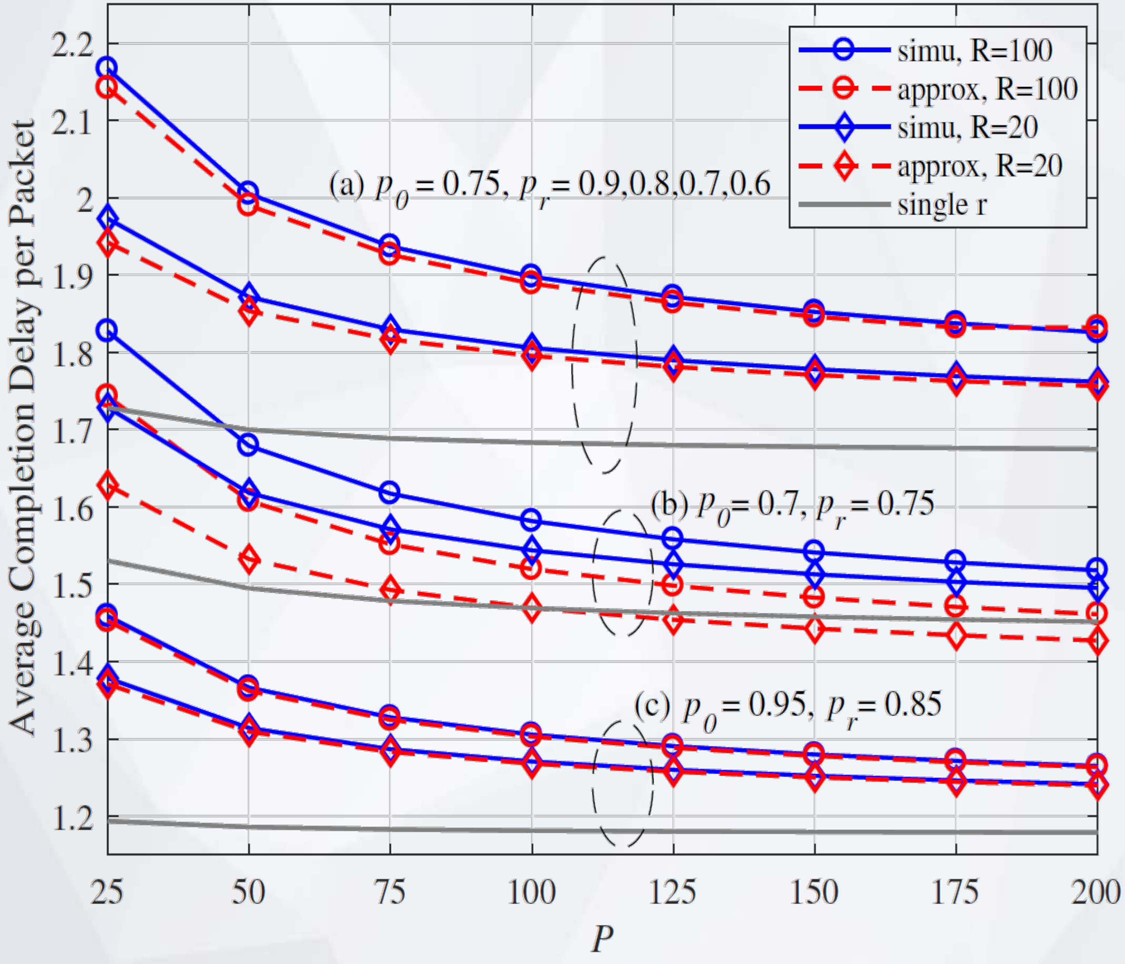
Numerical validation and analysis

- **Setting :** R **= 20 or 100**

$$\mathbb{E}[D_P] \gtrsim \max\{\max_{1\le r\le R} \mathbb{E}[D_{P,r}], \mathbb{E}[\hat{D}_P] + \tilde{D}_P\}$$

The expected completion delay of the single receiver with the worst channel condition

The expected system completion delay stems from classical wireless broadcast



Perfect RLNC without buffer

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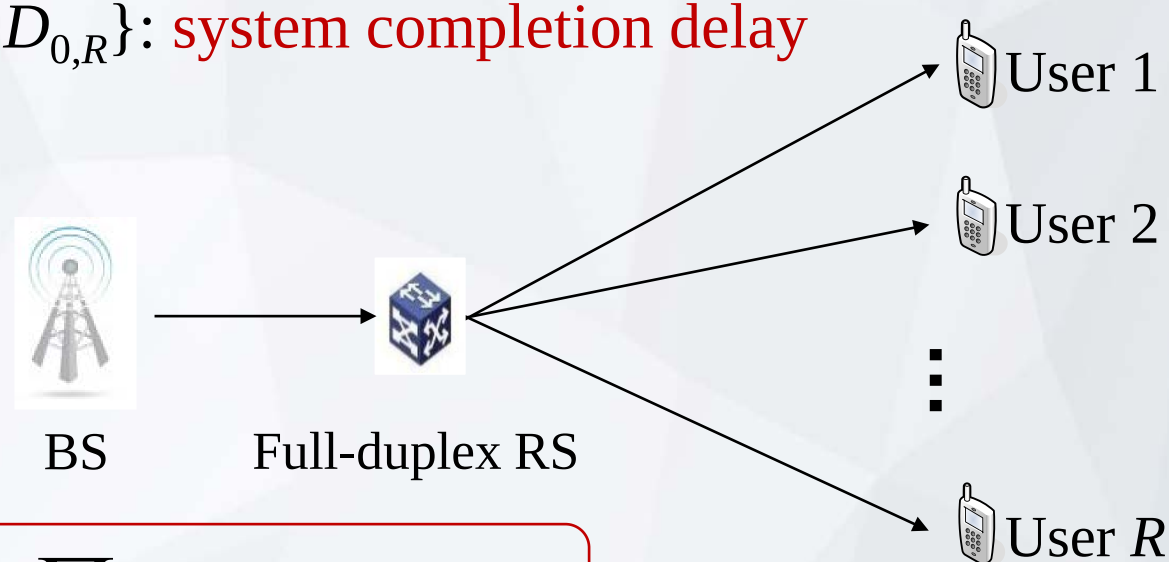
➢ Perfect RLNC without buffer

- No buffer and thus no coding at RS
- RS directly forwards what it just receives.
- A fundamental performance guarantee for perfect RLNC

$D_{0,r}$: completion delay of a single receiver r

$D_0 = \max\{D_{0,1}, D_{0,2}, \ldots, D_{0,R}\}$: system completion delay



Propositions.

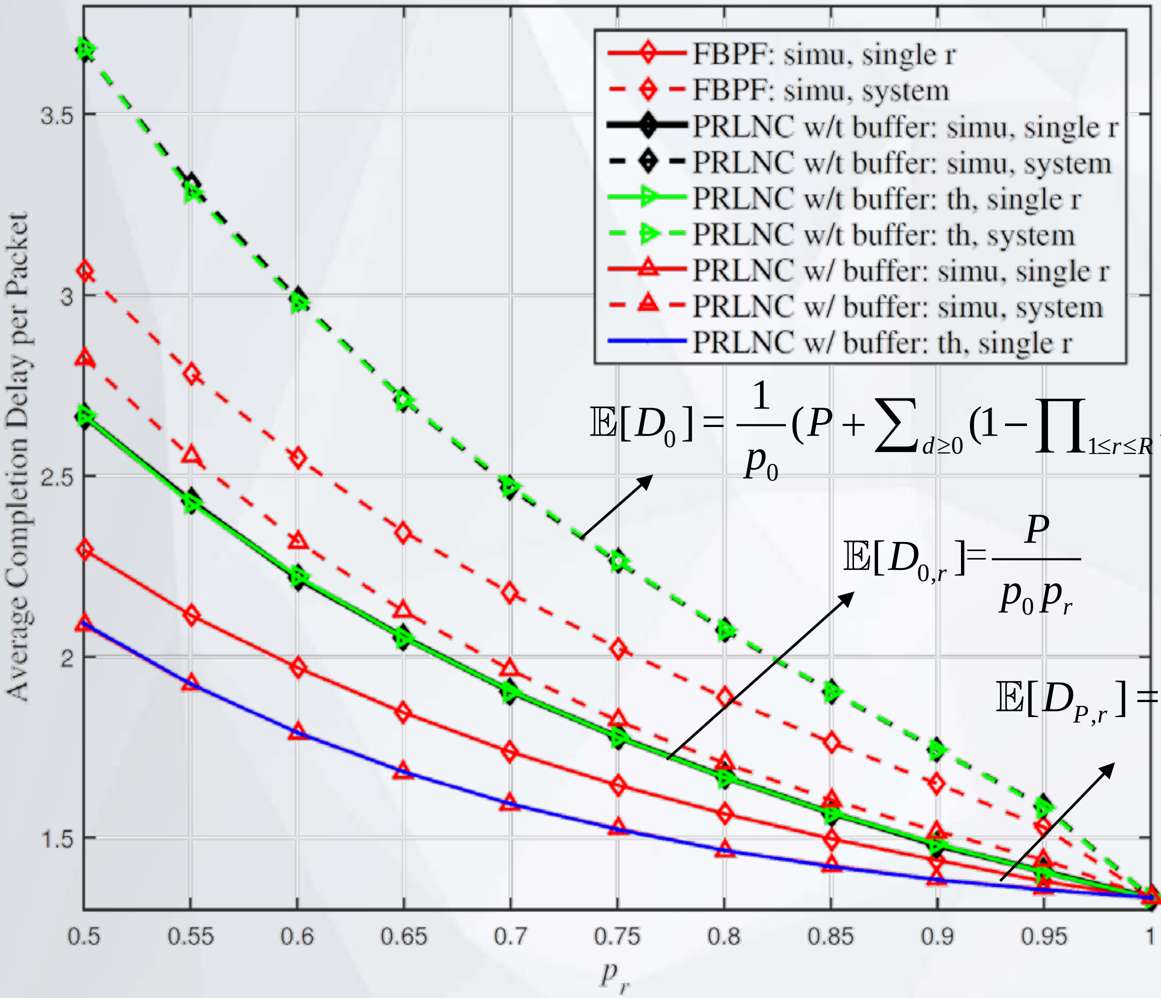
- $\mathbb{E}[D_{0,r}] = \dfrac{P}{p_0 p_r}$

- $\mathbb{E}[D_0] = \dfrac{1}{p_0}\left(P + \sum_{d \geq 0}\left(1 - \prod_{1 \leq r \leq R} I_{p_r}(P, d+1)\right)\right).$

// perfect RLNC in *classical wireless broadcast*

Numerical validation and analysis

- Setting: $P = 10$, $R = 10$, $p_0 = 0.75$



$$\mathbb{E}[D_0] = \frac{1}{p_0}\left(P + \sum_{d \geq 0}\left(1 - \prod_{1 \leq r \leq R} I_{p_r}(P, d+1)\right)\right)$$

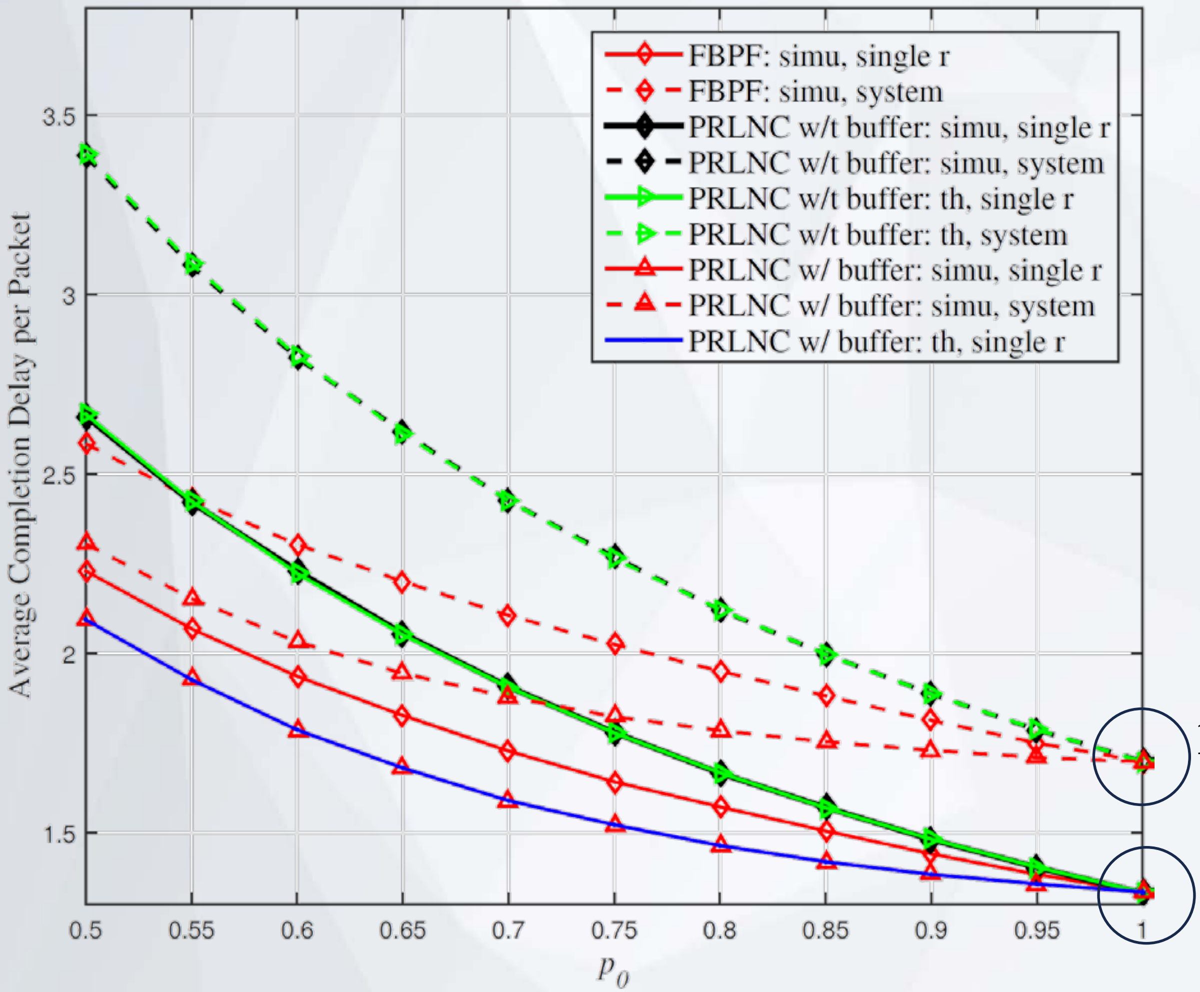
$$\mathbb{E}[D_{0,r}] = \frac{P}{p_0 p_r}$$

$$\mathbb{E}[D_{P,r}] = \frac{P}{p_0} + \frac{P}{p_r} - 1 + \sum_{i=0}^{P-2}\sum_{j=0}^{i}\frac{(P-i-1)T_{i,j}(p_0 p_r)^i}{(p_0 p_r - p_0 - p_r)^{i+j+1}}$$

- When $p_r \uparrow$, $D\downarrow$.
- Theoretical results are numerically verified.

Numerical validation and analysis

- Setting: $P = 10$, $R = 10$, $p_r = 0.75$



$$1+\frac{1}{P}\sum_{d\geq 0}(1-\prod_{1\leq r\leq R} I_{p_r}(P,d+1))$$

$$1/p_r$$

- When $p_0 \uparrow$, $D\downarrow$.
- Theoretical results are numerically verified.

Completion delay characterizations

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Theoretical contributions:

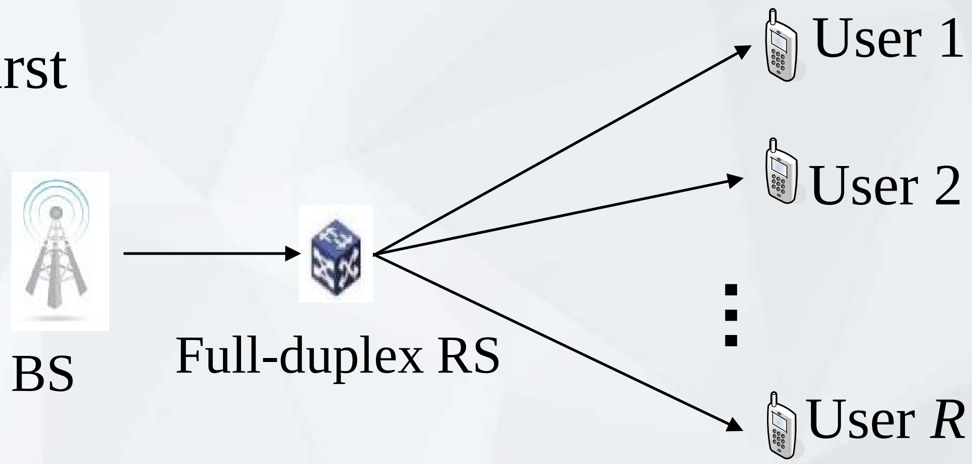
For different RLNC schemes, explicit formulae of the expected completion delay are derived.

Transmission Scenarios	Schemes	$\mathbb{E}[D_r]$ Characterization	$\mathbb{E}[D]$ Characterization
Classical Wireless Broadcasts	Conventional Scalar RLNC	Exact	Exact
Full-duplex Relay Broadcasts	Perfect RLNC with Buffer	Exact	Approximate
	Perfect RLNC without Buffer	Exact	Exact

General FBPF RLNC Scheme

➢ **Original FBPF**

- **F**ewest **B**roadcast **P**ackets **F**irst
- A **perfect** RLNC scheme
- Unlimited buffer
- No coding



The search of a proper packet at the RS takes high complexity.

➢ **General FBPF**

- Limited and arbitrary buffer size *B*
- Consider the buffer size *B* as a new parameter

General FBPF RLNC Scheme

$D_{\infty,r}$: completion delay at a single receiver r

Theorem: For the original FBPF scheme with unlimited buffer,

$$\lim_{P\to\infty} \mathbb{E}[D_{\infty,r}]/P = \frac{1}{p_r(1+p_0p_r-p_r)-A}$$

$$\text{where } A=\begin{cases} 0 & \text{if } p_0 \geq 0.5 \\ p_r - p_0 - (1-p_0)p_r^2 + (1-p_r)^{\left\lfloor \frac{1}{p_0} \right\rfloor \left(p_0 - p_r + p_0 p_r \left\lfloor \frac{1}{p_0} \right\rfloor\right)} & \text{if } p_0 < 0.5 \end{cases}$$

Theorem: For the general FBPF scheme with buffer B, we provide an upper bound:

$$\lim_{P\to\infty} \mathbb{E}[D_{B,r}]/P \leq \begin{cases} \dfrac{1}{p_r(1+p_0p_r-p_r)-A_1} & \text{if } p_0 = 0.5 \\ \dfrac{1}{p_r(1+p_0p_r-p_r)-A_2} & \text{if } p_0 \neq 0.5 \end{cases}$$

$$\text{where } A_1 = \frac{(1-p_0)(1-p_r)p_r}{B+1}\left(2-\frac{1}{p_0+p_r-p_0p_r}\right), A_2 = \frac{p_r^2(1-p_r)(1-2p_0)}{\left(1-\left(\dfrac{p_0}{1-p_0}\right)^{B+1}\right)(p_0+p_r-p_0p_r)}.$$

Criterion of Selecting Buffer Size

- Obtain a lower bound of B to satisfy the performance constraint:

$$\frac{\mathbb{E}[D_{B,r}]}{\mathbb{E}[D_{\infty,r}]} \le 1+\varepsilon$$

→ The acceptable performance loss

Denote $C_\varepsilon = \dfrac{(1+\varepsilon)(1-p_r)}{\left(\varepsilon\left(1+p_0p_r-p_r\right)+A_\infty/p_r\right)\left(p_0+p_r-p_0p_r\right)}$,

Proposition: For the general FBPF scheme with buffer size B, for sufficiently large P, as long as B satisfies

$$B \ge \begin{cases} \left\lceil \frac{1}{2} p_r C_\varepsilon - 1 \right\rceil & \text{if } p_0 = 0.5 \\ \left\lceil \log_{\frac{p_0}{1-p_0}} \left(1+p_r\left(2p_0-1\right)C_\varepsilon\right) - 1 \right\rceil & \text{if } p_0 \ne 0.5 \end{cases}$$

we have $\dfrac{\mathbb{E}[D_{B,r}]}{\mathbb{E}[D_{\infty,r}]} \le 1+\varepsilon.$

Criterion of Selecting Buffer Size

Table I: Minimum B based on the criterion with $\varepsilon = 0.02$ and different p_0 , p_r

p_r \ p_0	0.5	0.6	0.7	0.8	0.9
0.5	11	3	2	1	1
0.6	10	3	2	1	1
0.7	9	3	2	1	1

- **Insight of the Proposition**

It presents a **criterion** on the optimal selection of the buffer size B **under a quality of service constraint**, so that the buffer size will be significantly reduced while its completion delay performance is comparable to that of the original FBPF scheme.

- ✓ **When $p_0 \geq 0.8$, $p_r \geq 0.5$, setting $B = 1$ is sufficient to guarantee that the performance loss is within 2%.**
- ✓ **The criterion improves the practicability of the FBPF scheme.**

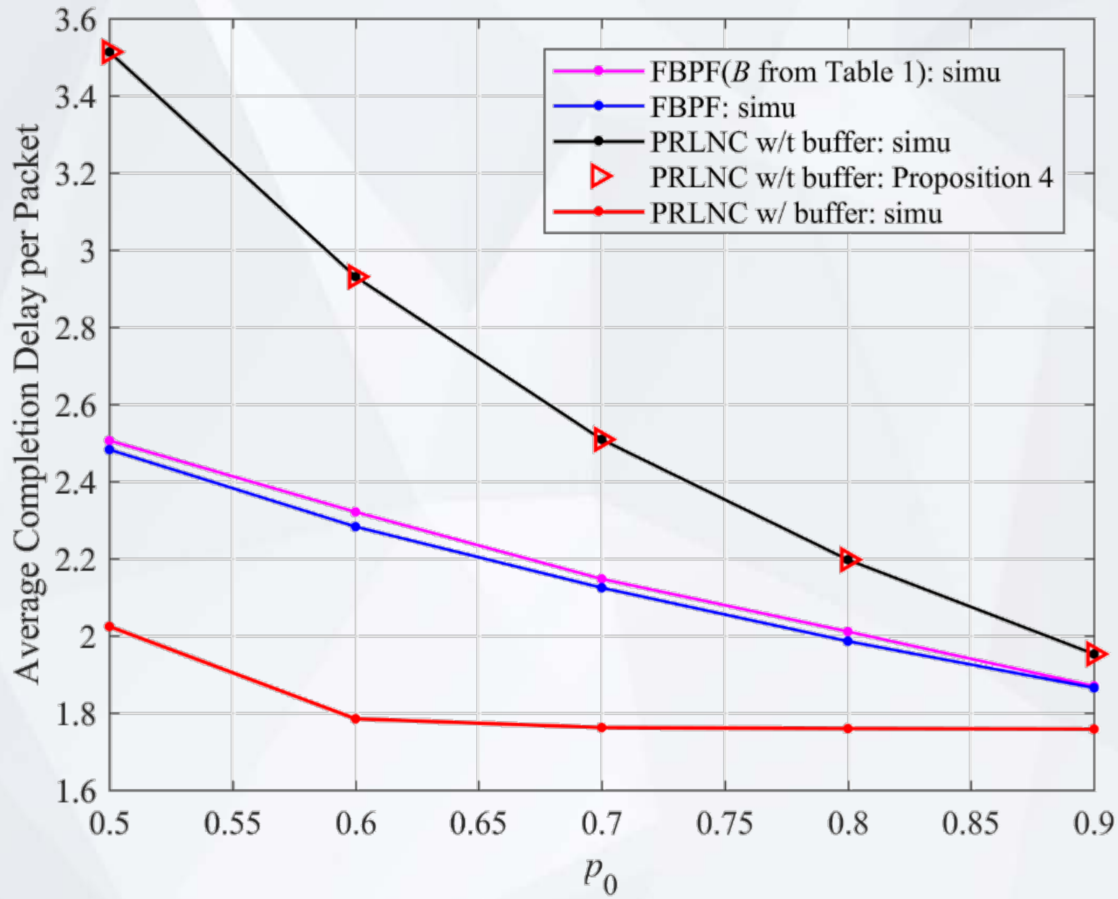
Numerical validation and analysis

RLNC schemes in full-duplex relay broadcast networks:

- ✓ Original FBPF RLNC ($B = \infty$)
- ✓ General FBPF RLNC ($B > 0$)
- ✓ Perfect RLNC without buffer ($B = 0$)
- ✓ Perfect RLNC with buffer ($B = P$)

Numerical validation and analysis

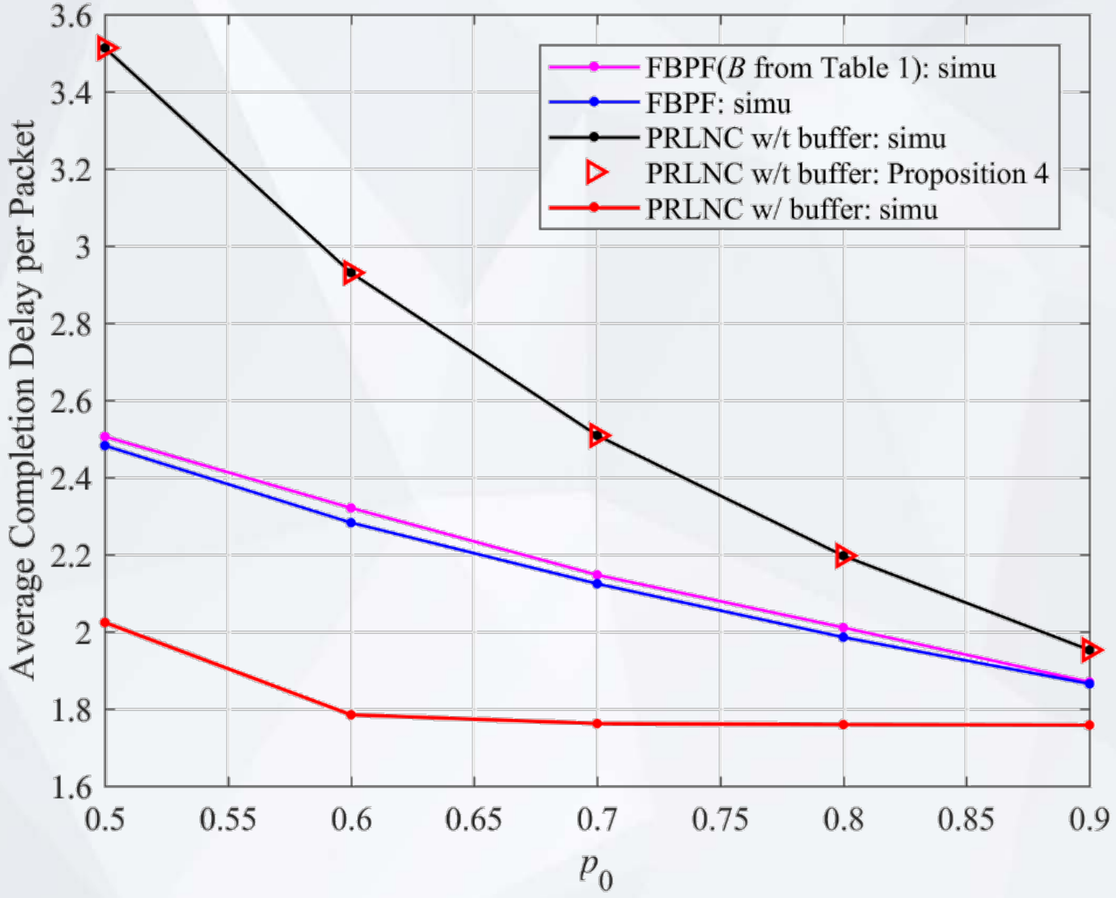
- Setting: : $P = 500$, $R = 20$, $p_r = 0.6$



The average completion delay of FBPF is upper bounded by that of **perfect RLNC without buffer** and lower bounded by that of **perfect RLNC with buffer**.

Numerical validation and analysis

- Setting: : $P = 500$, $R = 20$, $p_r = 0.6$



The difference between the system completion delay of FBPF for the case that B is prescribed by Table 1 and that for the case $B = \infty$ is within **2%**.

Summary

We proposed circular-shift (vector) RLNC in classical wireless broadcasts

- A much better trade-off between completion delay and encoding/ decoding complexity.

We investigate the performance limit of RLNC in full-duplex relay (broadcast) networks

- Explicit formulae of completion delay are derived.
- The average completion delay of FBPF is lower bounded by that of perfect RLNC with buffer.

We generalize the FBPF RLNC in full-duplex relay (broadcast) networks

- Explicit formulae of completion delay are derived.
- Improve the practicability of FBPF RLNC.

Thanks for attention.