Block Markov Superposition Transmission: Construction of Big Convolutional Codes from Short Codes

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Sun Yat-sen University,

Hong Kong, October, 2013

Outline

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Construction of Long Codes from Short Codes

Short Convolutional Codes

Convolutional codes with short constraint lengths: e.g.,

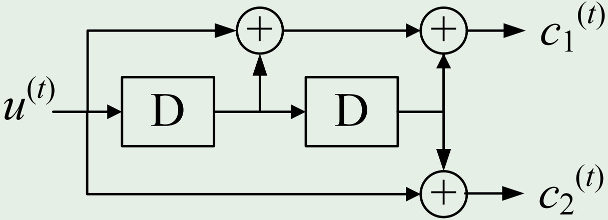

Figure: A $(2, 1, 2)$ convolutional code encoder.

Short Block Codes

Block codes with short length: repetition codes, single parity-check codes, Hamming codes, etc. We are actually interested in Cartesian product of short block codes. For example $[2, 1, 2]^{5000}$, $[6, 5, 2]^{2000}$, $[7, 4, 3]^{2500}$;

Construction of Long Codes from Short Codes

Long Codes from Short Codes

- Product codes;
- Concatenated codes;
- Turbo codes: parallel concatenated convolutional codes (PCCC) and serial concatenated convolutional codes (SCCC);
- (Irregular) Repeat accumulate (RA) codes;
  Accumulate-repeat-accumulate (ARA) codes;
- Concatenated zigzag codes; Precoded concatenated zigzag codes;

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- Convolutional LDPC codes;
- Polar codes: concatenation of a series of simple transformation;
- $\cdots$

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Superposition Block Markov Encoding (SBME) in the Relay Channel

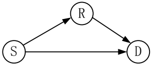

Figure: The Relay Channel.

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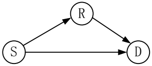

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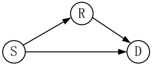

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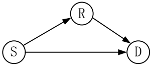

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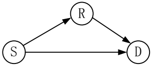

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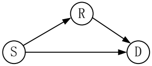

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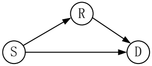

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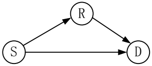

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- We apply a similar strategy (SBME) to the single-user communication system, resulting in the block Markov superposition transmission (BMST) scheme.

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Block Markov Superposition Transmission

Encoding

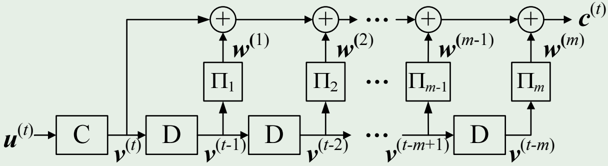

Figure: Encoding structure of BMST with memory m.

Recursive Encoding of BMST

1. *Initialization*: For $t < 0$, set $\mathbf{v}^{(t)} = \mathbf{0} \in \mathbb{F}_2^n$.
2. *Recursion*: For $t = 0, 1, \cdots, L-1$,
    - Encode $\mathbf{u}^{(t)}$ into $\mathbf{v}^{(t)} \in \mathbb{F}_2^n$ by the encoding algorithm of the basic code $\mathscr{C}$;
    - For $1 \le i \le m$, interleave $\mathbf{v}^{(t-i)}$ by the i-th interleaver Π_i into $\mathbf{w}^{(i)}$;
    - Compute $\mathbf{c}^{(t)} = \mathbf{v}^{(t)} + \sum_{1\le i\le m} \mathbf{w}^{(i)}$, which is taken as the t-th block of transmission.
3. *Termination*: For $t = L, L+1, \cdots, L+m-1$, set $\mathbf{u}^{(t)} = \mathbf{0} \in \mathbb{F}_2^k$ and compute $\mathbf{c}^{(t)}$ recursively.

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- For a rate $R = k/n$ general convolutional code, information sequence $\boldsymbol{u} = \left(\boldsymbol{u}^{(0)}, \boldsymbol{u}^{(1)}, \cdots\right)$is encoded into code sequence $\boldsymbol{c} = \left(\boldsymbol{c}^{(0)}, \boldsymbol{c}^{(1)}, \cdots\right)$ by
$$\boldsymbol{c}^{(t)} = \boldsymbol{u}^{(t)}\boldsymbol{G}_0 + \boldsymbol{u}^{(t-1)}\boldsymbol{G}_1 + \cdots + \boldsymbol{u}^{(t-m)}\boldsymbol{G}_m, t \geq 0,$$
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- Let $\boldsymbol{G}$ be the generator matrix of the basic code. Let $\Pi_1, \cdots, \Pi_m$ be m permutation matrices. The generator matrix of the BMST has a simple form
$$G_{\text{BMST}} = \begin{bmatrix} \boldsymbol{G} & \boldsymbol{G\Pi}_1 & \cdots & \boldsymbol{G\Pi}_m & & & & & \\ & \boldsymbol{G} & \ddots & \vdots & \ddots & & & & \\ & & \ddots & \boldsymbol{G\Pi}_1 & \ddots & \boldsymbol{G\Pi}_m & & & \\ & & & \boldsymbol{G} & \ddots & \vdots & \ddots & & \\ & & & & \ddots & \boldsymbol{G\Pi}_1 & \ddots & \boldsymbol{G\Pi}_m & \\ & & & & & \boldsymbol{G} & \cdots & \boldsymbol{G\Pi}_{m-1} & \boldsymbol{G\Pi}_m \end{bmatrix}.$$

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- The BMST defines a *special* class of convolutional codes. Specialities: *possibly extremely* large constraint length; $\boldsymbol{G\Pi}_i$ instead of $\boldsymbol{G}_i$.

Block Markov Superposition Transmission

Normal Graph

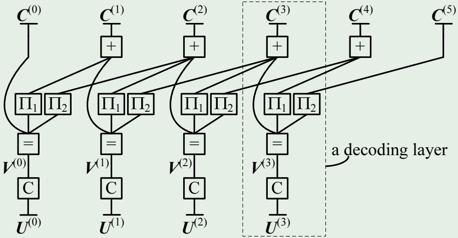

Figure: The normal graph of a BMST system with $L = 4$ and $m = 2$.

Decoding

- An iterative forward-backward decoding schedule is used for basic codes with small L;
- An iterative sliding-window decoding schedule is used for basic codes with large L;
- Four types of nodes: C, $=$, $+$, and $\prod$;
- Messages are processed and passed through different decoding layers forward and backward over the normal graph.

Outline

Coding Gain Analysis of the BMST

Genie-Aided Lower Bound on BER

- Imagine that $\mathbf{u}' = \{\mathbf{u}^{(i)}, t - m \le i \le t + m, i \ne t\}$ are known at the receiver.
- This is equivalent to transmitting $\boldsymbol{u}^{(t)}$ for $m + 1$ times.
- The coding gain of the BMST can not be larger than

$$10 \log_{10}(m + 1) - 10 \log_{10}(1 + m/L) \text{ dB}.$$

- Noticing that $\Pr\{\boldsymbol{u}'|\boldsymbol{y}\} \approx 1$ in the low error rate region, we can expect that the maximal coding gain $10 \log_{10}(m + 1) - 10 \log_{10}(1 + m/L)$ dB.

Upper Bound on BER

- The input-output weight enumerating function (IOWEF) of the BMST system can be computed from that of the basic code.
- The BER can be upper-bounded by an improved union bound.
- Notice that an incomplete (truncated) IOWEF is sufficient for upper bounds. (See Xiao Ma, Jia Liu and Baoming T-COMM 2013).

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Simulation Results

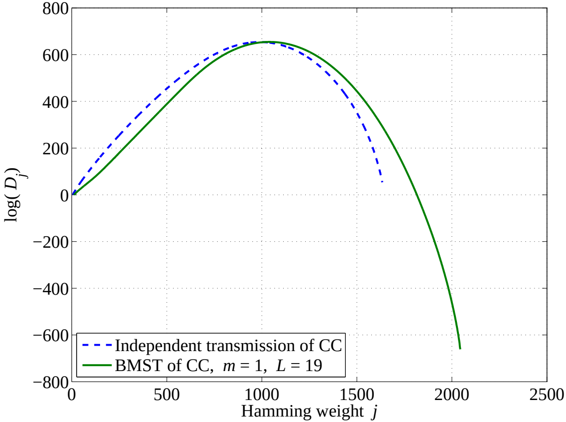

Figure: Comparison of the weight spectrum between the independent transmission system and the ensemble of the BMST system. The basic code is a terminated systematic encoded 4-state $(2, 1, 2)$ convolutional code defined by the polynomial generator matrix $G(D) = [1, (1 + D + D^2)/(1 + D^2)]$. The BMST system encodes $L = 19$ sub-blocks of data with memory $m = 1$.

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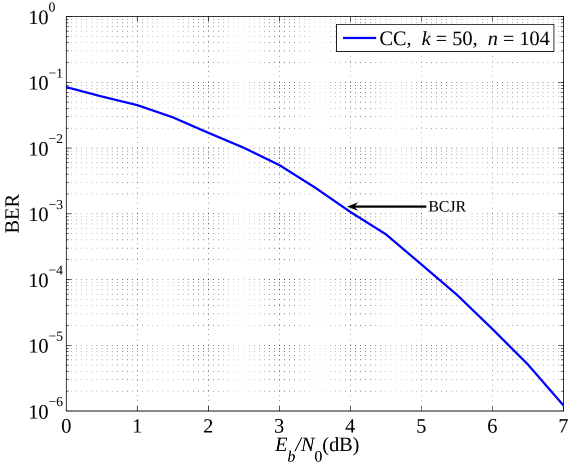

Figure: Coding gain analysis of the BMST system. The basic code is a terminated convolutional code (CC) with the polynomial generator matrix $[1, \frac{1+D+D^2}{1+D^2}]$. The coding parameters of the BMST system are $m = 1$, $L = 19$, $d = 19$, and $I_{\max} = 18$. The decoding algorithm is performed after all 20 transmitted sub-blocks are received (forward-backward schedule).

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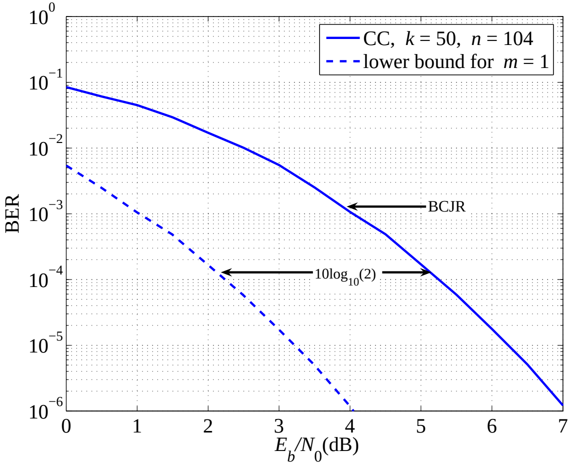

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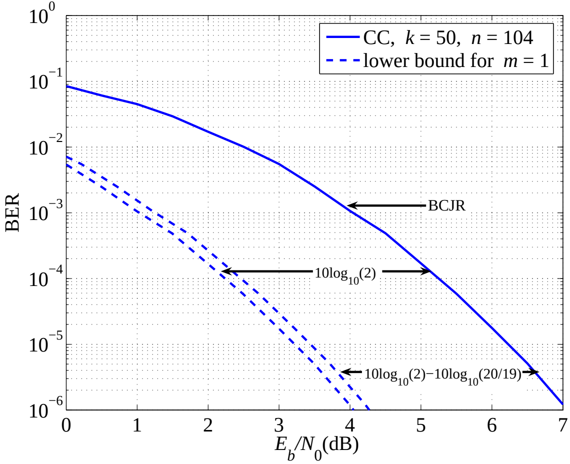

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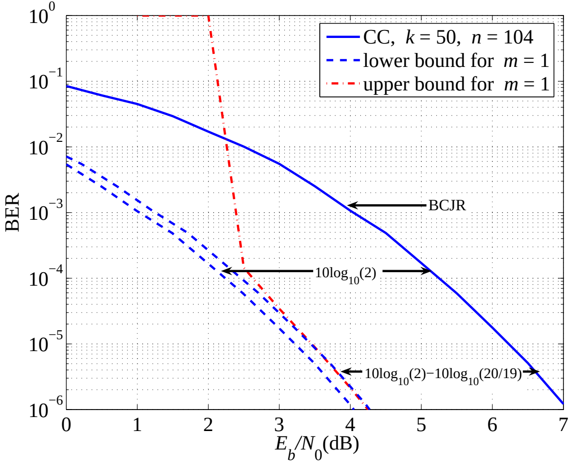

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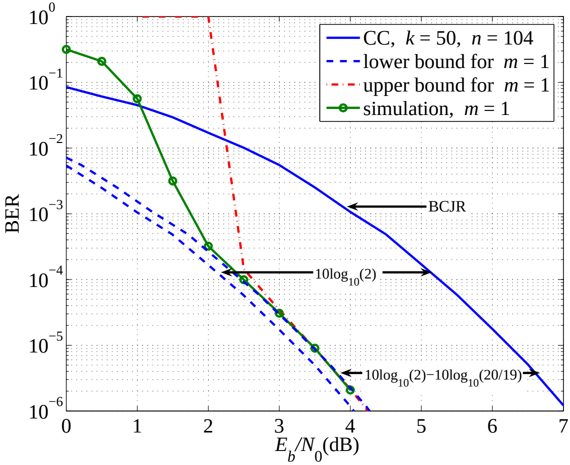

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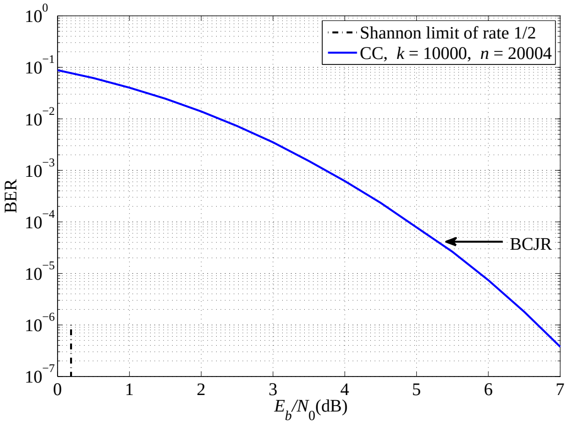

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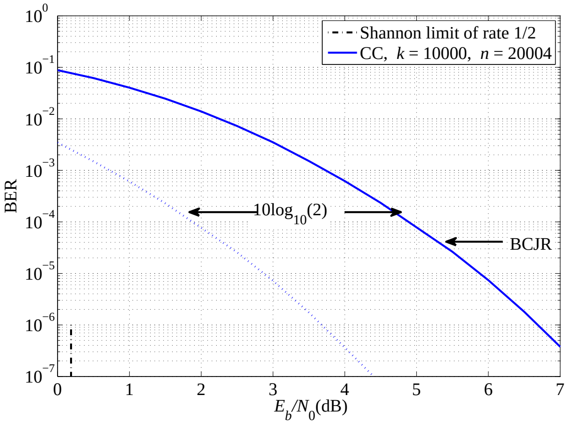

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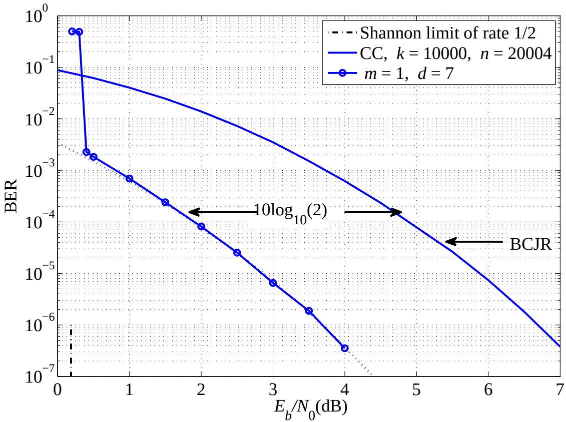

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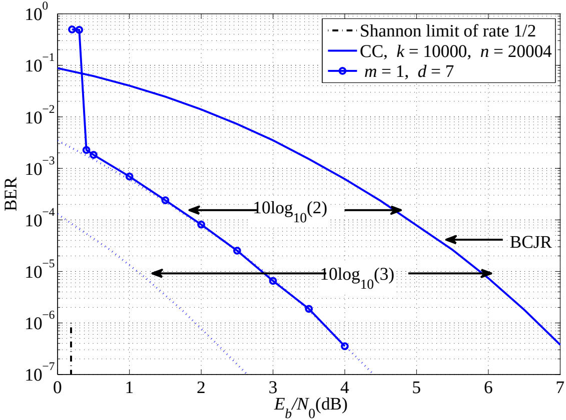

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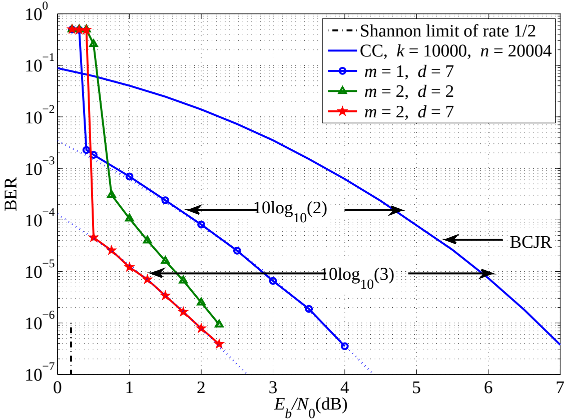

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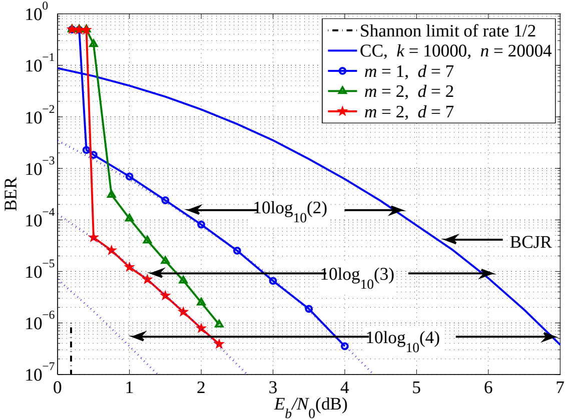

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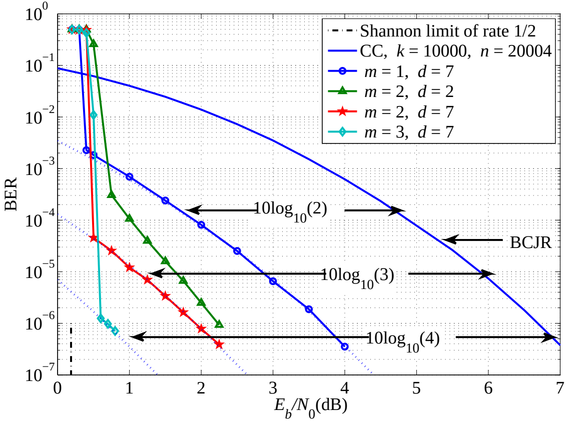

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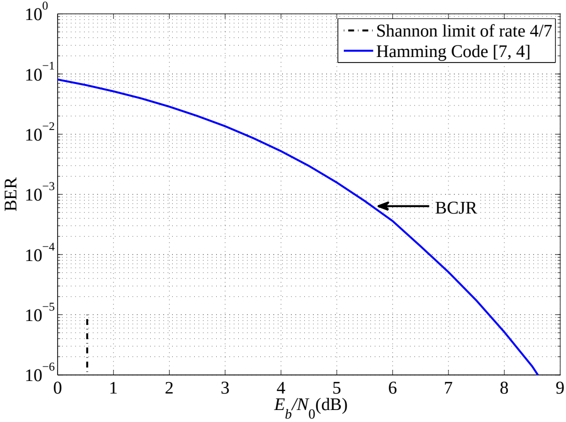

Figure: The basic code is the Cartesian product of Hamming code $[7, 4]^{2500}$. The system encodes $L = 1000$ sub-blocks of data and the iterative sliding-window decoding algorithm is performed, where the encoding memories and the decoding delays are specified in the legends.

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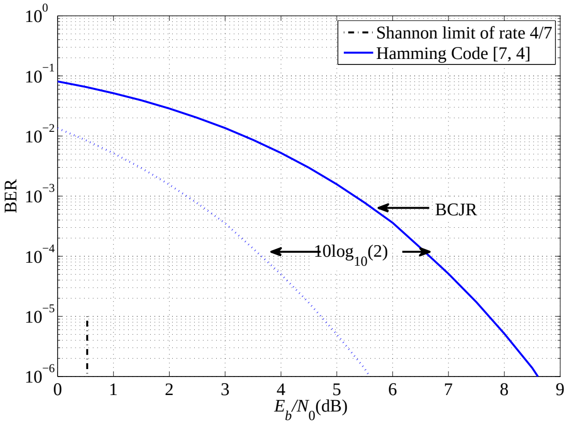

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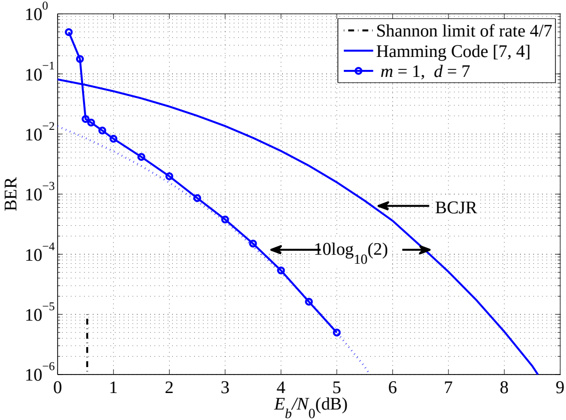

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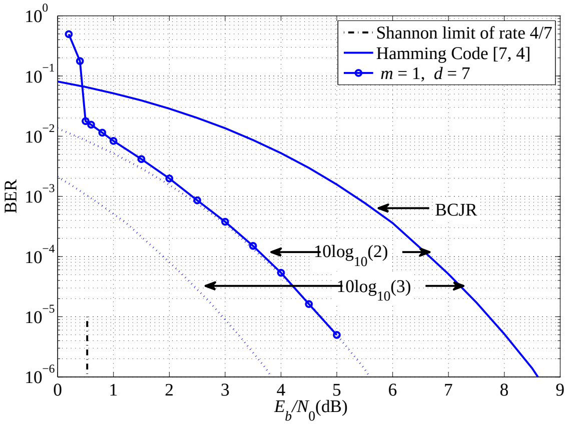

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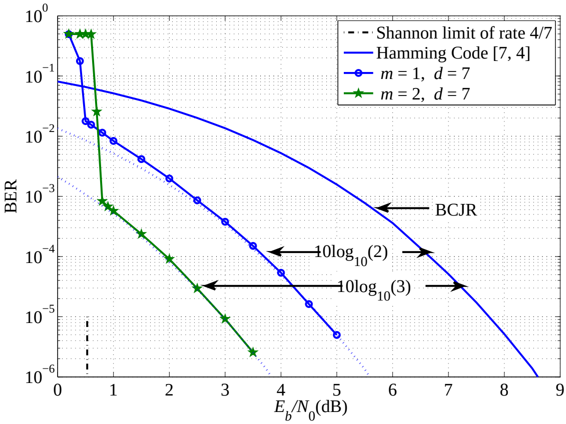

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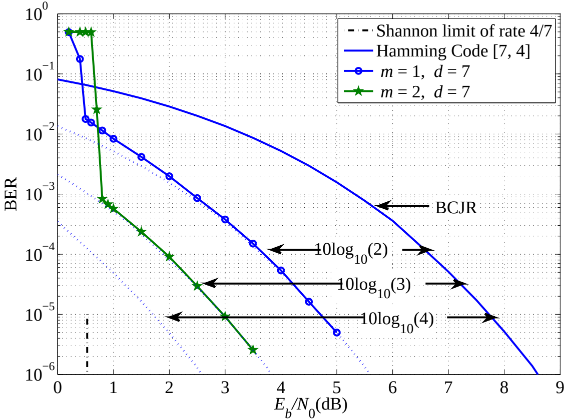

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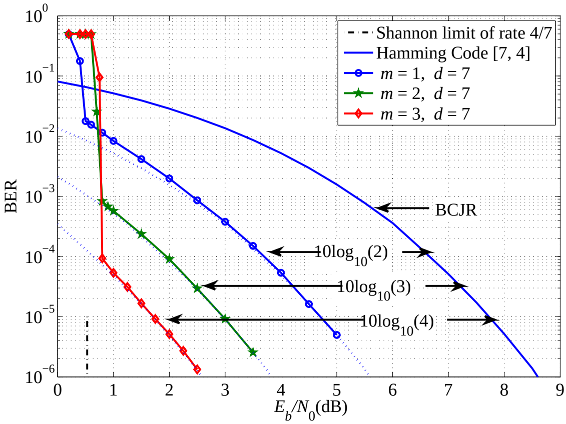

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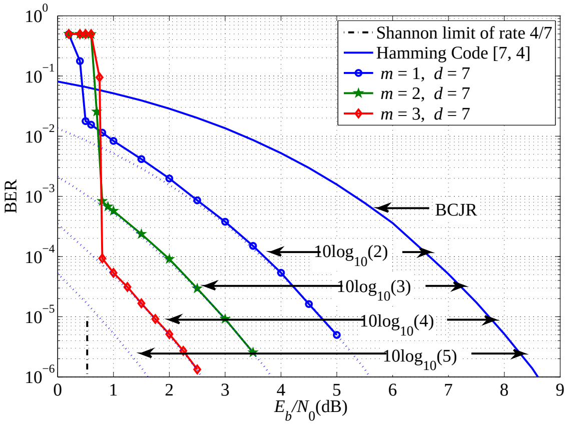

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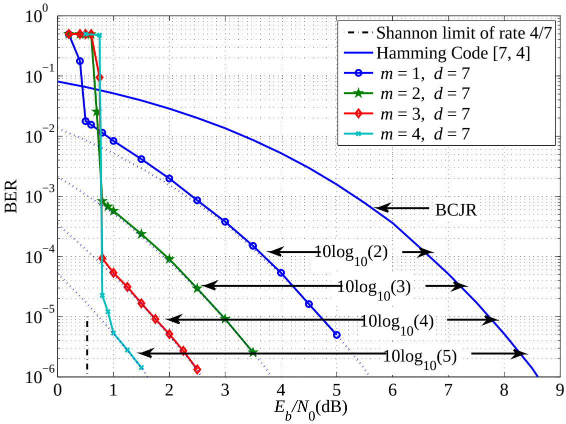

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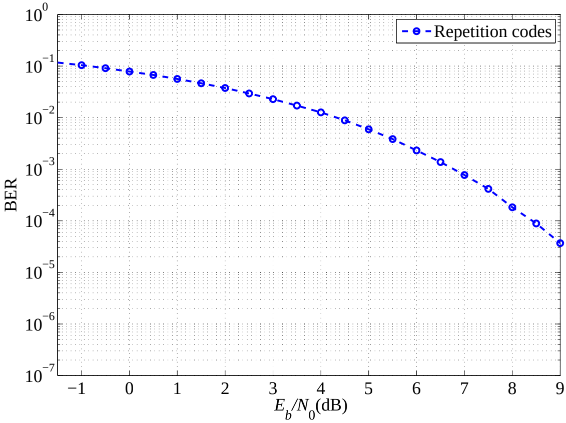

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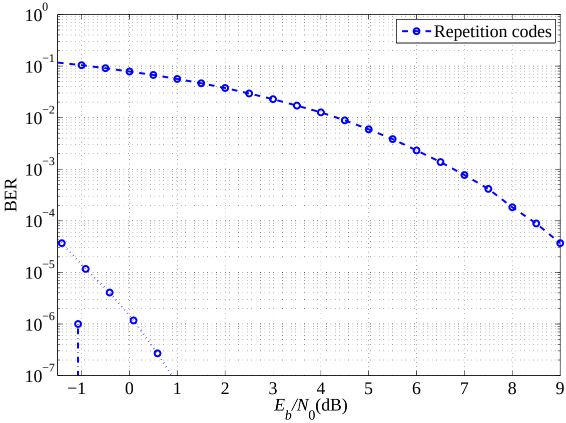

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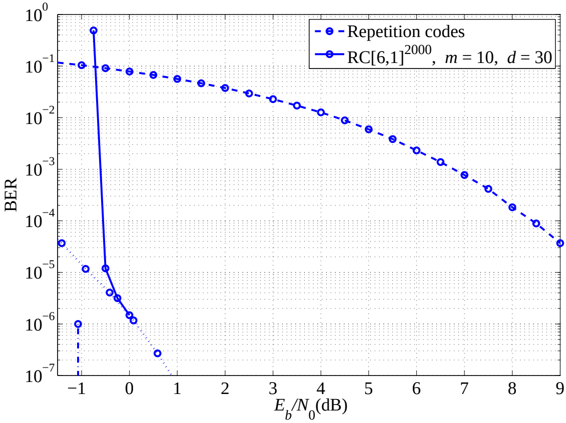

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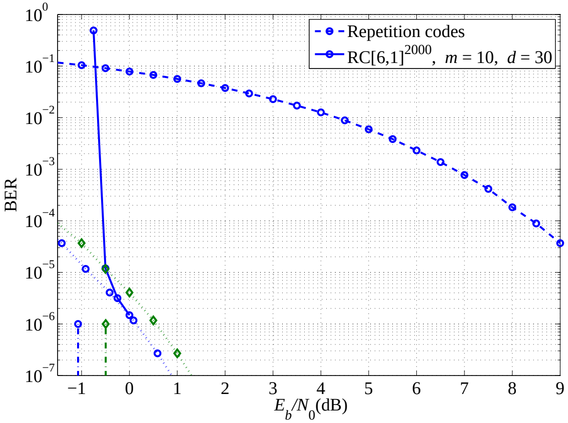

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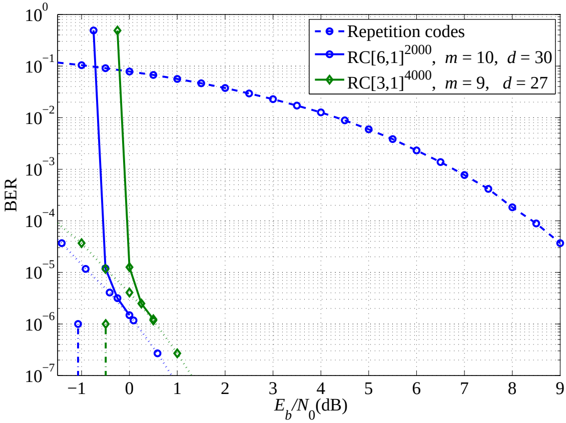

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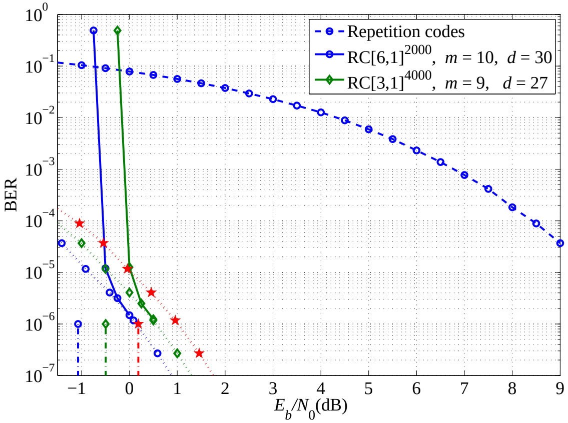

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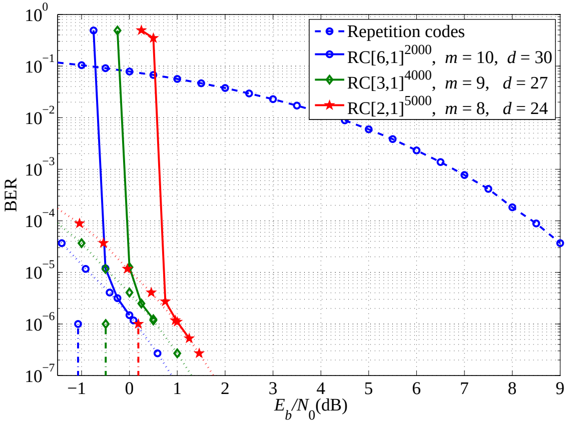

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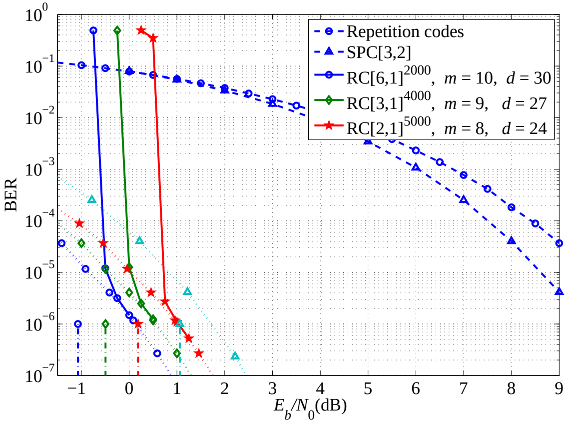

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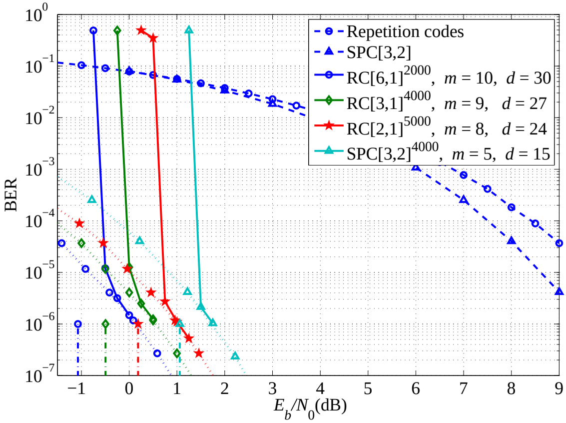

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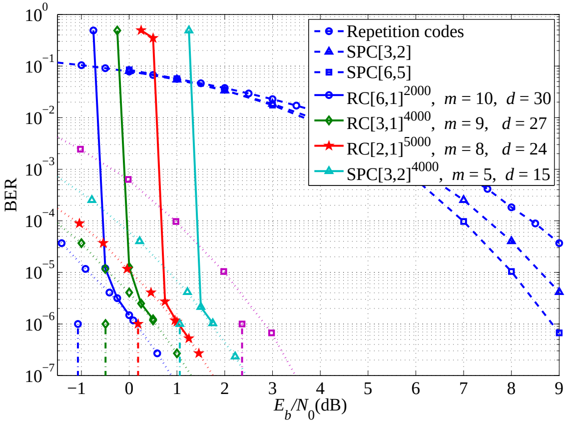

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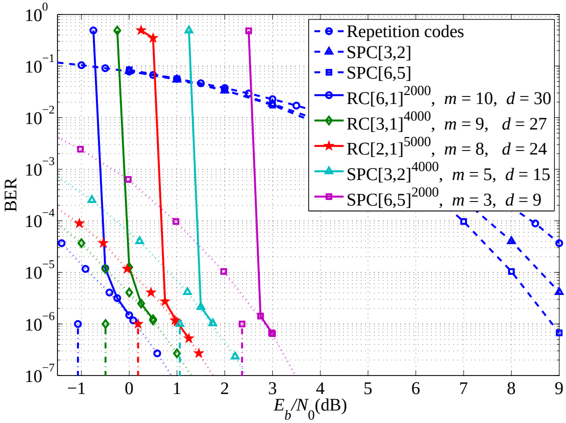

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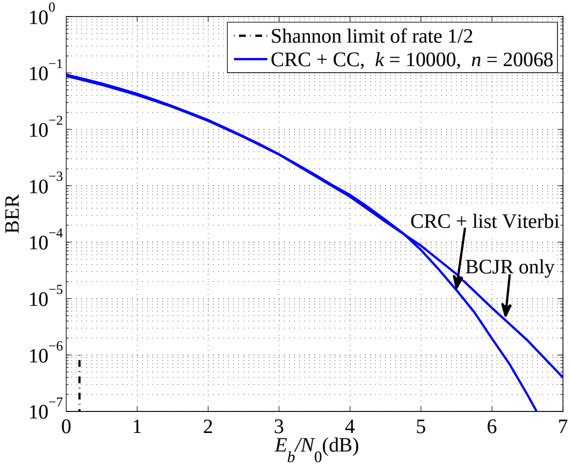

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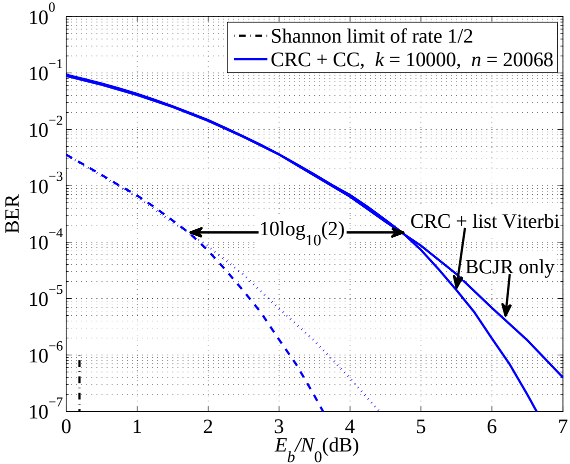

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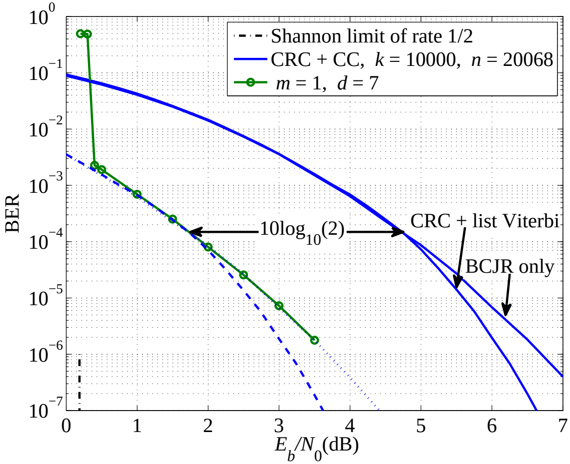

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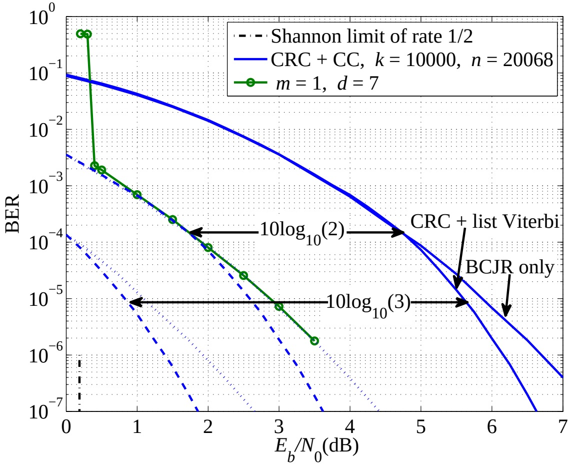

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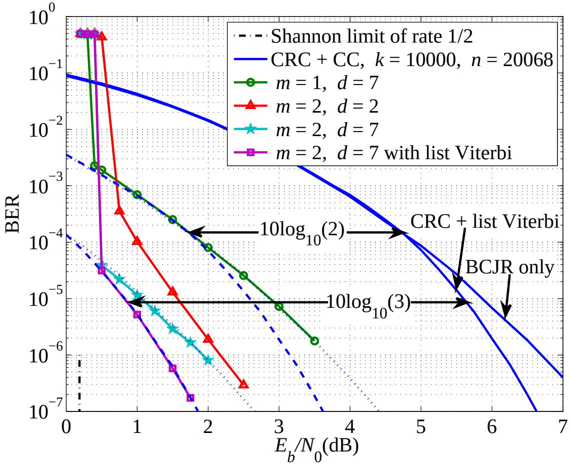

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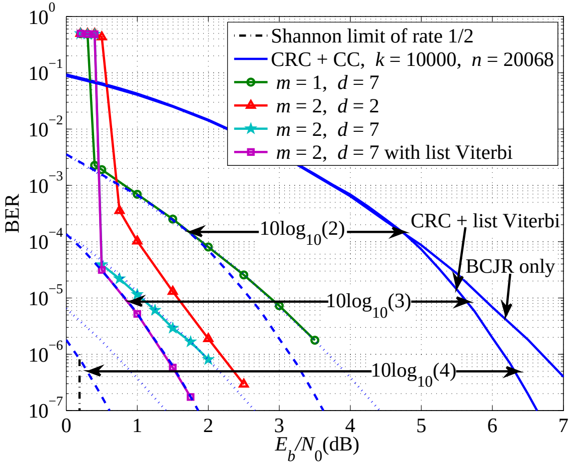

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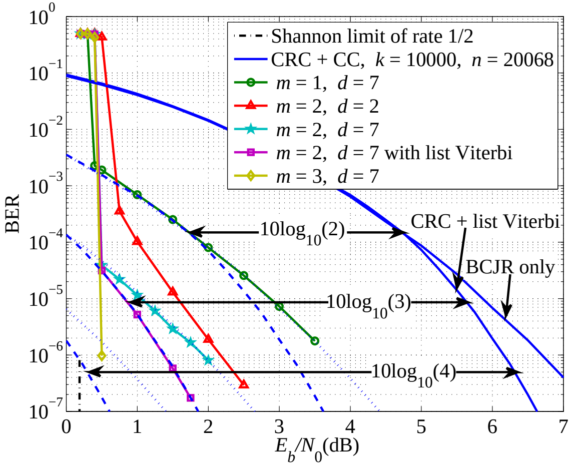

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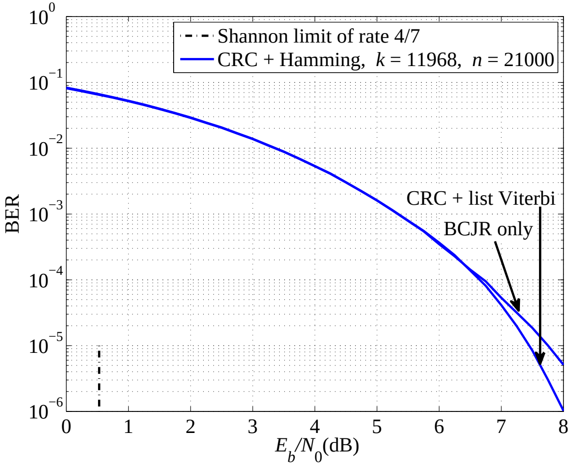

Figure: The basic code $\mathscr{C}$ is a concatenated code of dimension $k = 11968$ and length $n = 21000$, where the outer code is a 32-bit CRC code and the inner code is the Cartesian product of Hamming code $[7, 4]^{3000}$. Other coding parameters of the BMST system are $L = 1000$ and $I_{\text{max}} = 18$.

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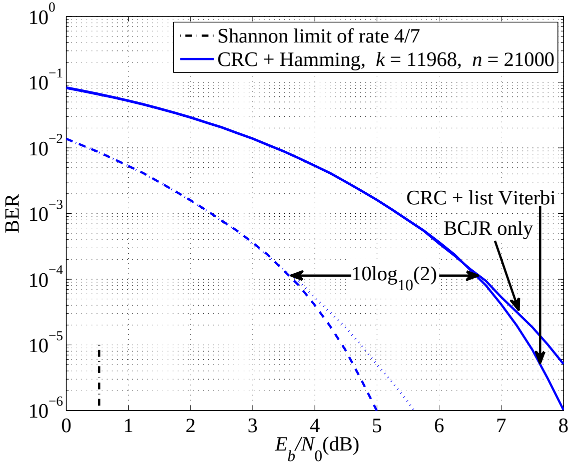

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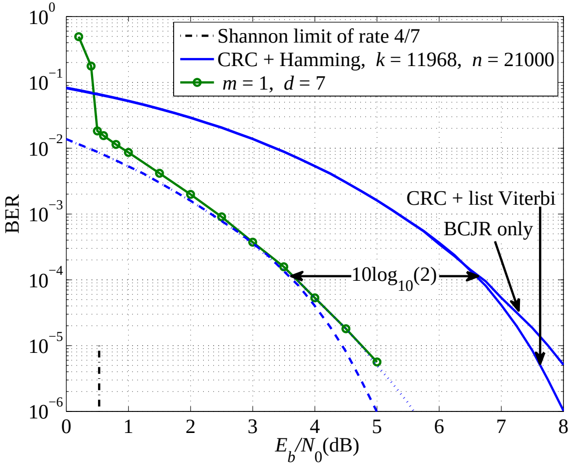

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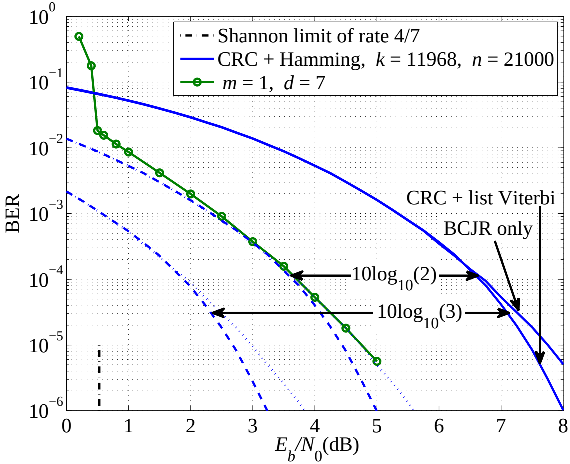

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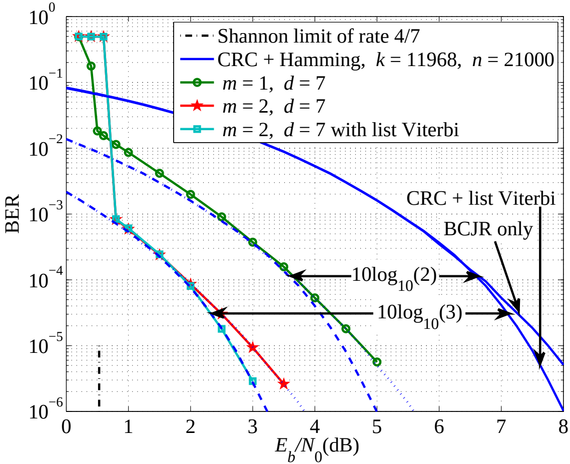

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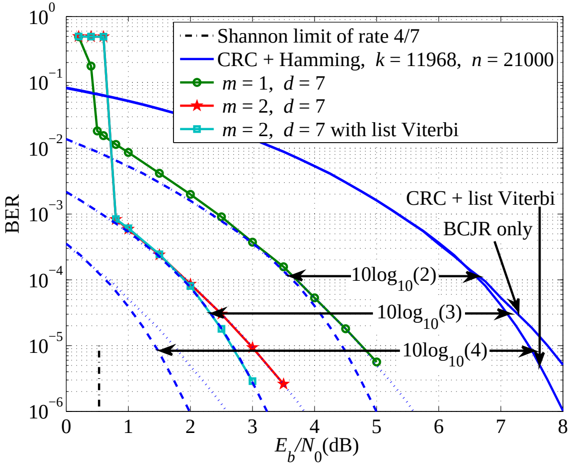

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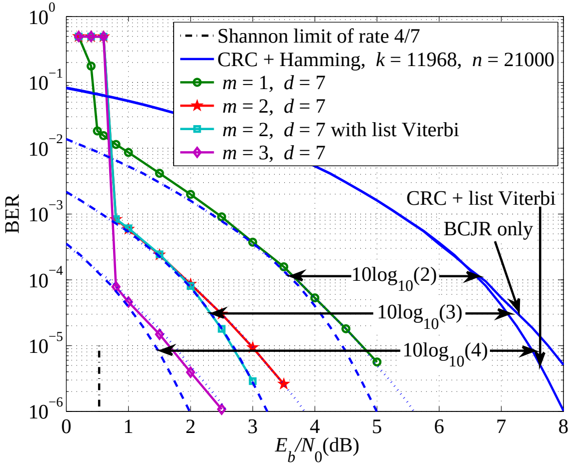

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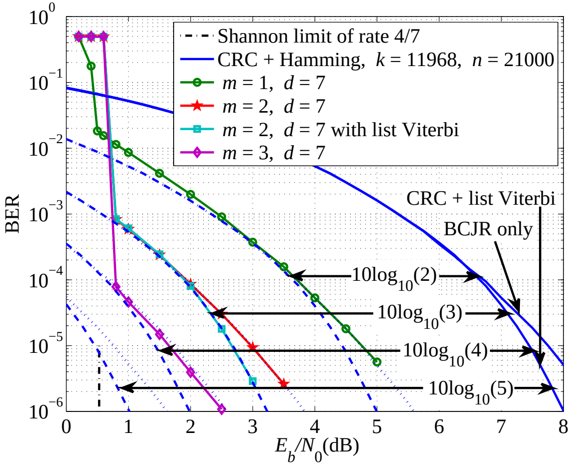

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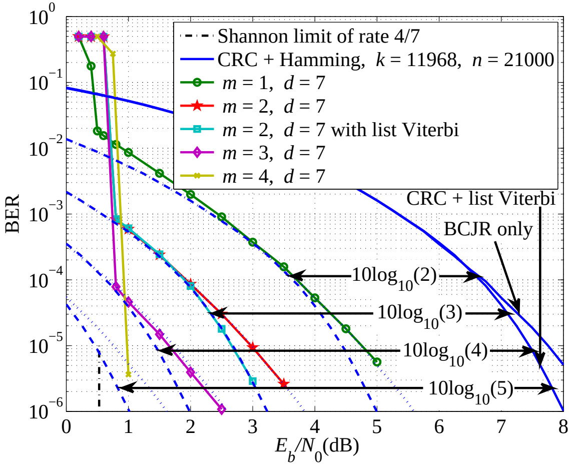

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Outline

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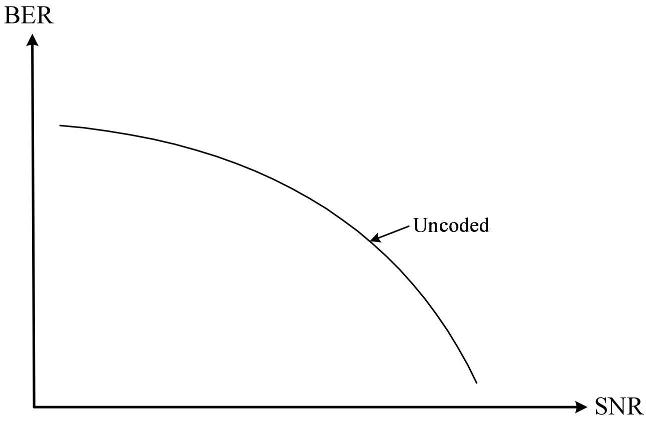

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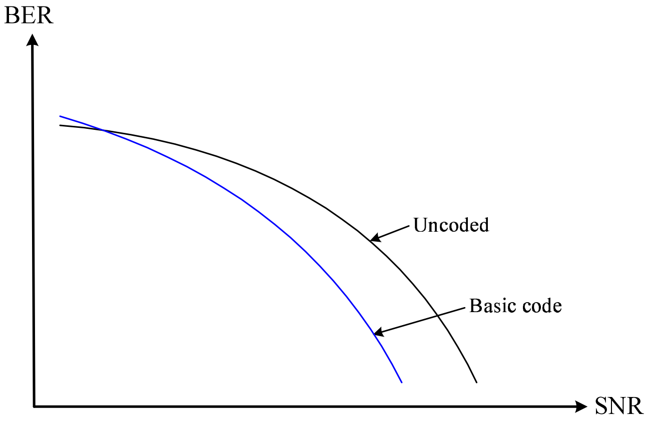

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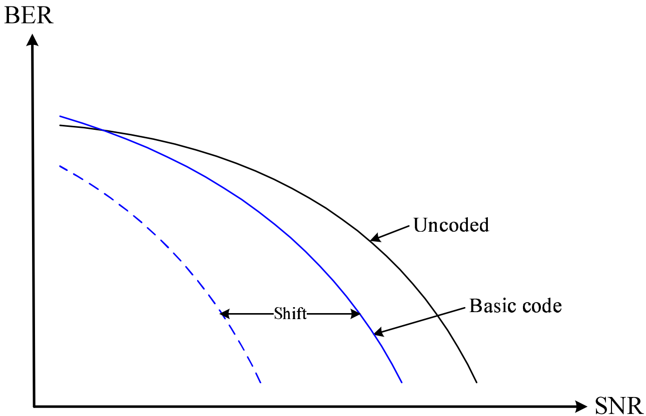

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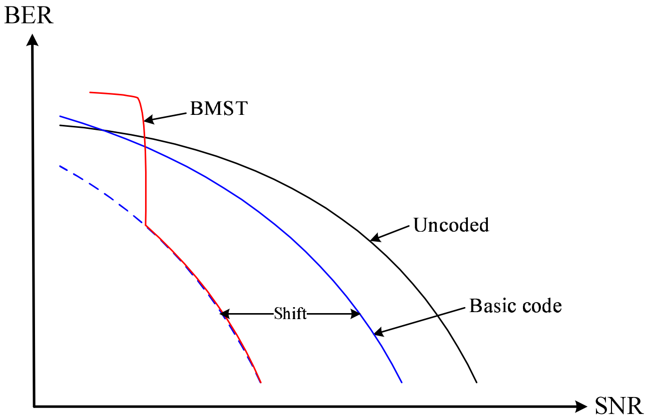

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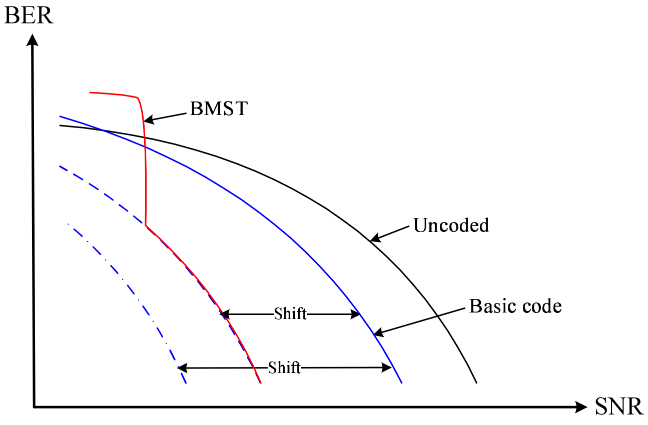

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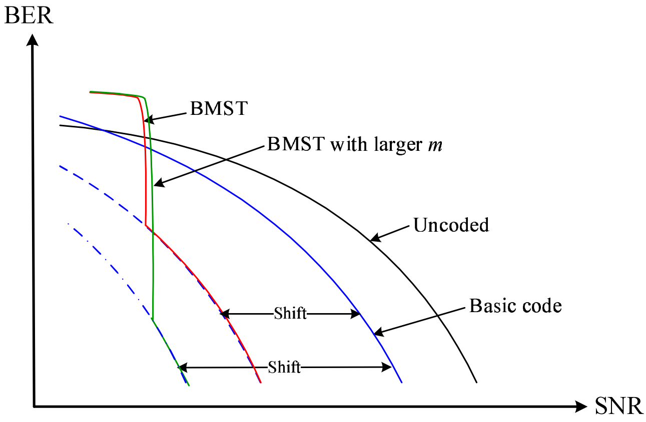

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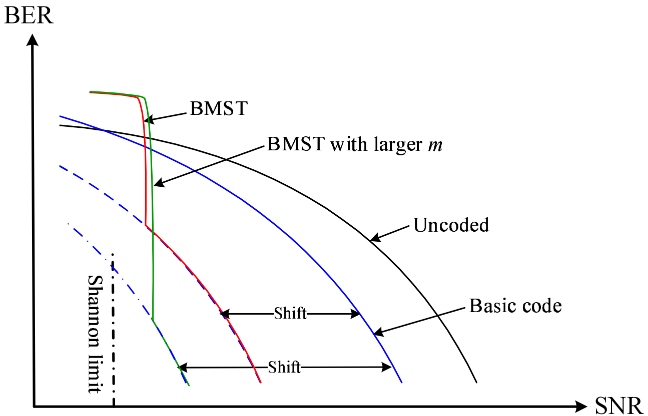

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- Define a sequence of matrices $\boldsymbol{P}_0 = \begin{cases} \mathbf{0}, & t \leq -1, \\ \boldsymbol{I} & t = 0, \\ \sum_{1 \leq \ell \leq m} \boldsymbol{P}_{t-\ell} \boldsymbol{\Pi}_\ell & t \geq 1, \end{cases}$ where $\boldsymbol{I}$ is the identity matrix of order n and $\mathbf{0}$ is the zero matrix of order n.

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- The parity-check matrix of the BMST system is given by

$$\boldsymbol{H}_{\text{BMST}} = \text{diag}\{\underbrace{\boldsymbol{H}, \cdots, \boldsymbol{H}}_{L}, \underbrace{\boldsymbol{I}, \cdots, \boldsymbol{I}}_{m}\} \boldsymbol{P}^{\mathrm{T}},$$

where, the superscript T denotes “transpose” and

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{P}_1 & \boldsymbol{P}_2 & \cdots & \boldsymbol{P}_{L+m-1} \\ & \boldsymbol{I} & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{L+m-2} \\ & & \ddots & \ddots & \vdots \\ & & & \boldsymbol{I} & \boldsymbol{P}_1 \\ & & & & \boldsymbol{I} \end{bmatrix}.$$

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- What do we really care about?

What we really care about is whether or not the basic code has efficient encoding/decoding algorithms.

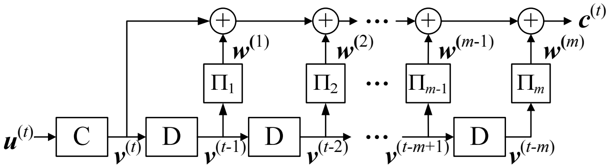


Figure: Encoding of BMST with memory m.

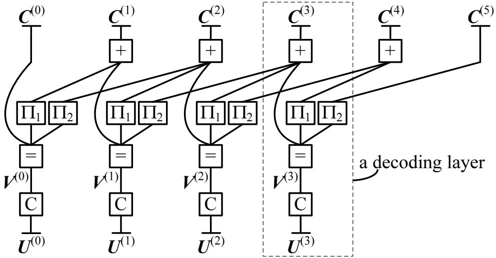


Figure: Sliding-window decoding over the normal graph.

Simulation Results

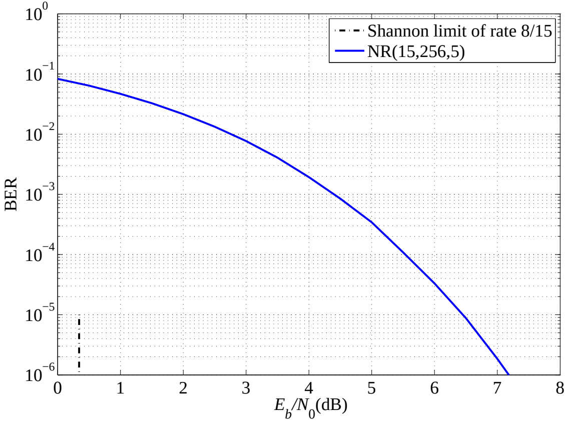
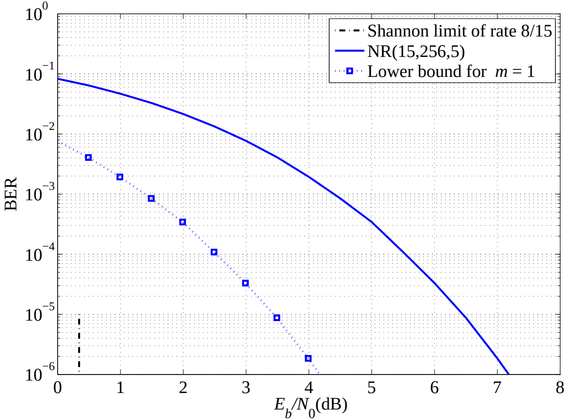


Figure: The basic code is the Cartesian product of the optimum Nordstrom-Robinson nonlinear code $(15, 256, 5)^{800}$. The system encodes $L = 1000$ sub-blocks of data and the iterative sliding-window decoding algorithm is performed, where the encoding memories and the decoding delays are specified in the legends.



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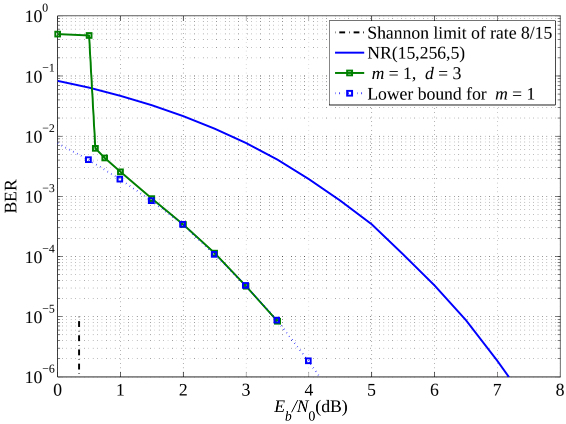

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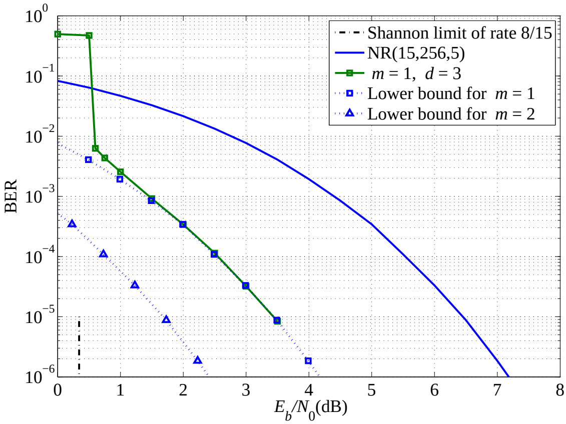

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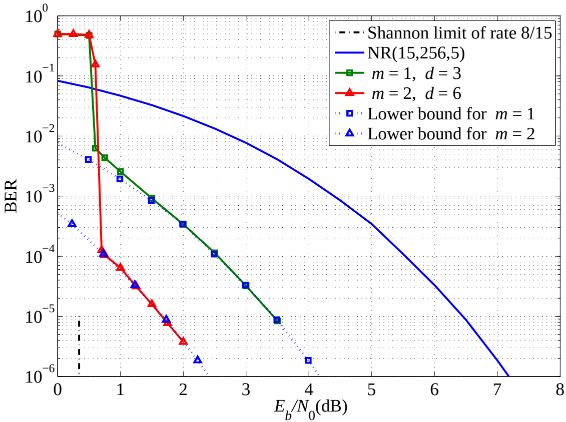

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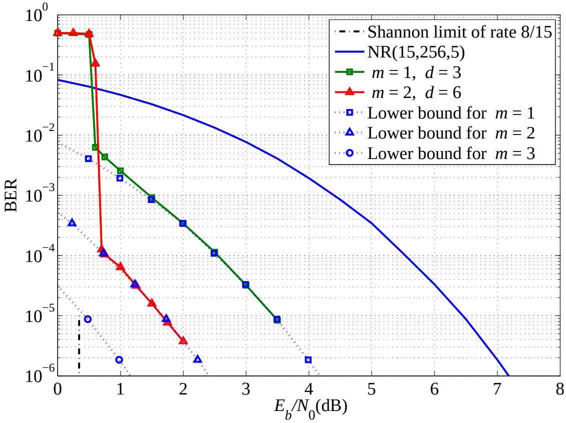

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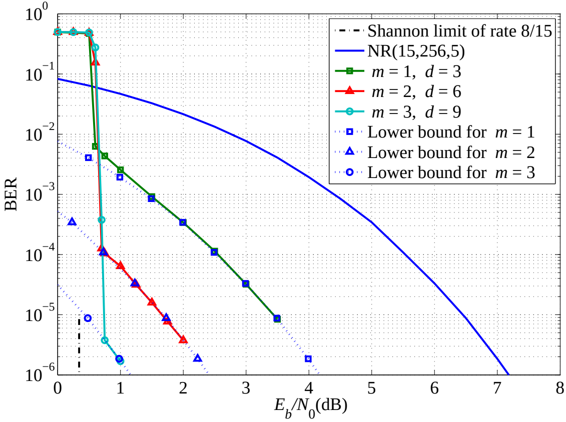

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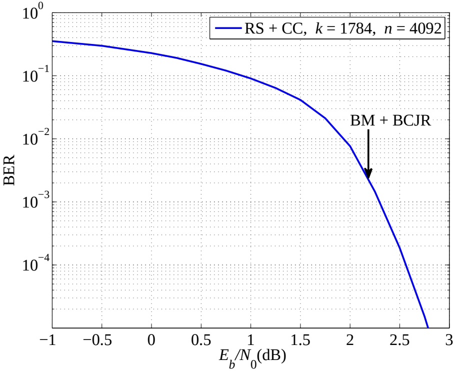
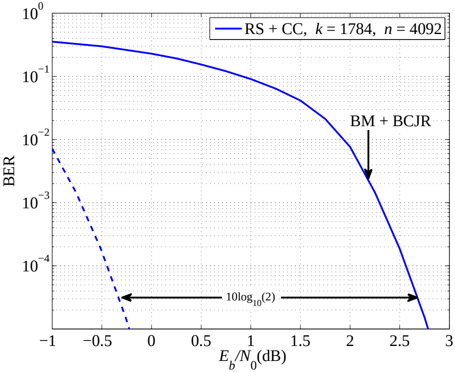


Figure: The basic code $\mathscr{C}$ is the Consultative Committee on Space Data System (CCSDS) standard code of dimension $k = 1784$ and length $n = 4092$, where the outer code is a [255, 223] Reed-Solomon (RS) code over $\mathbb{F}_{256}$ and the inner code is a terminated convolutional code with the polynomial generator matrix $G(D) = [1 + D + D^2 + D^3 + D^6,\ 1 + D^2 + D^3 + D^5 + D^6]$. Other coding parameters of the BMST system are $L = 100$ and $I_{\max} = 18$.



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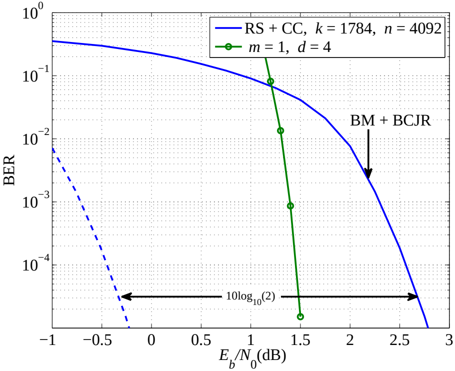

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Outline

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- We presented a new method for constructing long codes from short codes;
- The encoding process can be as fast as the short code, while the decoding has a fixed but tunable delay.
- With an iterative sliding-window decoding algorithm, the performance of BMST can approach the derived lower bound in low error rate region;
- This scheme can be generalized, for example, to non-binary codes, lattice codes, and so on.
- In principle, any code can be the basic code as long as it has efficient encoding algorithm and (exact or approximated) soft-in soft-out (SISO) decoding algorithm.

Acknowledgements

- This work is supported by the 973 Program (No.2012CB316100) and the NSF (No.61172082) of China.

Thank You for Your Attention!