A Computationally Informed Hierarchical Theory of Network Coding Rate Regions

John MacLaren Walsh

Department of Electrical and Computer Engineering

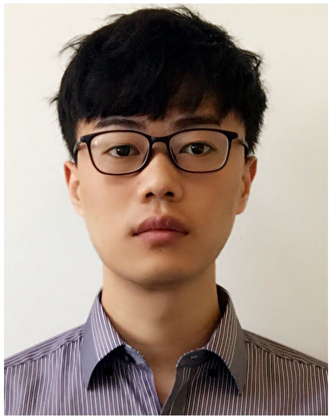
Drexel University

Philadelphia, PA

jwalsh@ece.drexel.edu

Thanks to NSF CCF-1421828 & NSF CCF-1053702.

Collaborators & Co-Authors



Yirui Liu
Ph.D. Candidate,
ASPITRG
Drexel University

Common information based network combination operators

Congduan Li, Ph.D.
Postdoctoral Fellow,
City Univ. of Hong Kong

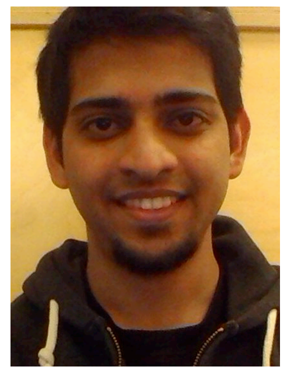
rate region database
network operators
forbidden net. minors

Steven Weber, Ph.D.
Professor, Dept. of ECE
Drexel University

co-advisor for Congduan Li

Alejandro Erick Trofimoff
Ph.D. Candidate,
ASPITRG
Drexel University

ordering probabilistic supports for mapping nonlinear EVs



Jayant Apte, Ph.D.
Data Scientist,
HVH Precision Analytics

network symmetry
software: ITCP & ITAP

Yunshu Liu, Ph.D.
Senior Applied Researcher
eBay

nonlinear entropic vectors & codes

What is a network coding problem?

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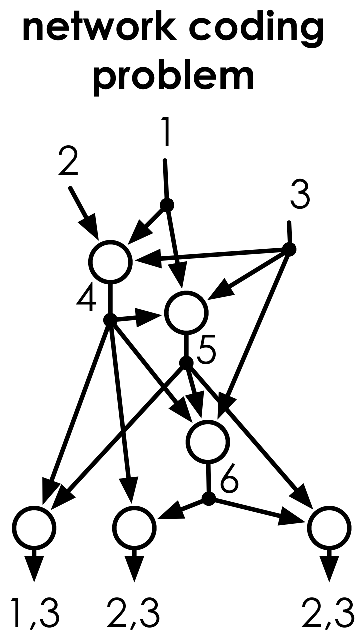

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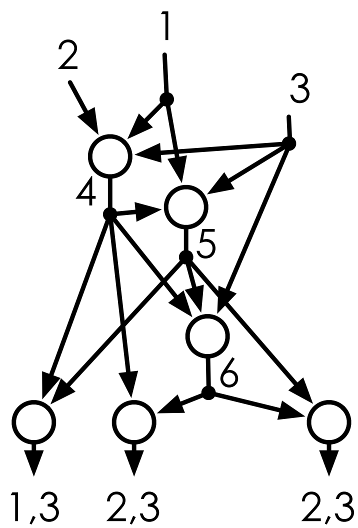
What is a Network Coding Problem?

A labelled directed acyclic hypergraph, including:

network coding problem



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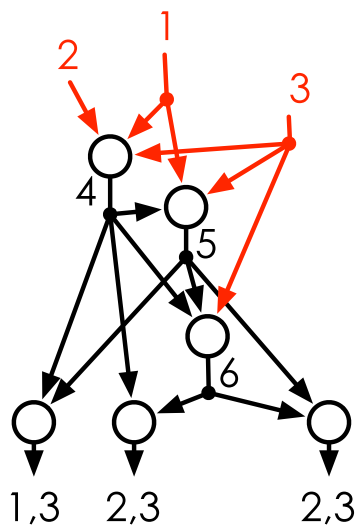
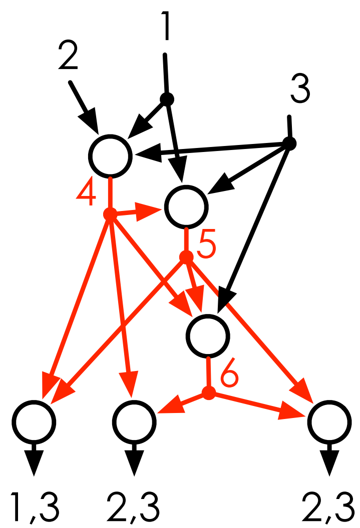


1. independent sources

What is a network coding problem?



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1. independent sources

2. messages: outgoing encoded from incoming

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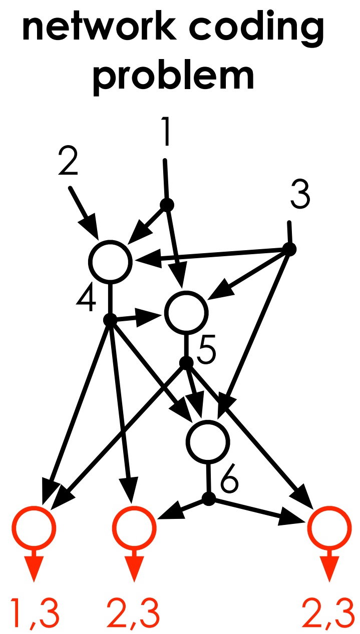

1. independent sources

2. messages: outgoing encoded from incoming

3. sinks: subsets of sources decoded from messages

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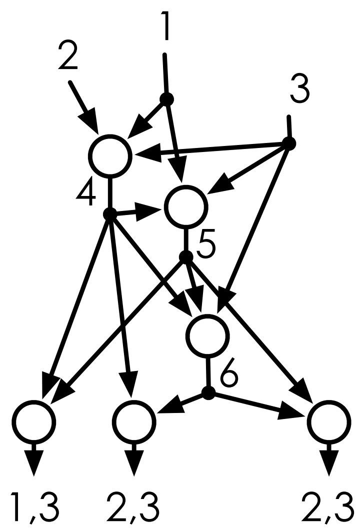

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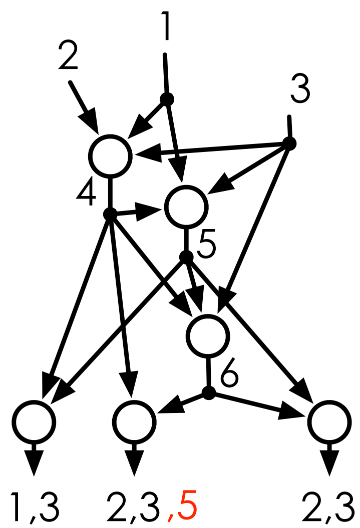

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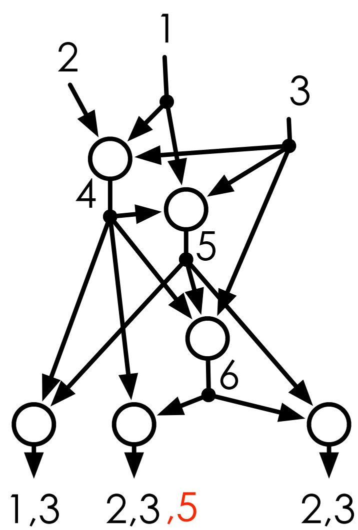

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By drawing the right graph, this includes:

1. index coding
2. Distributed storage (exact & functional repair)
3. Coded Caching

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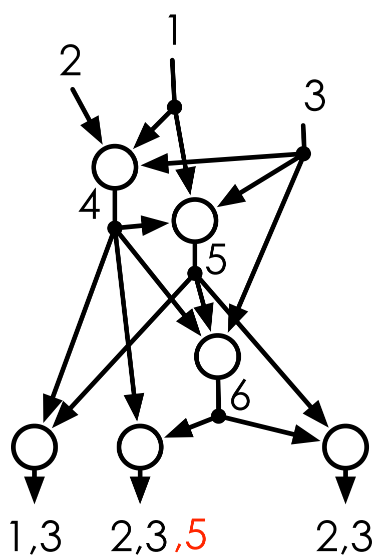

A labelled directed acyclic hypergraph, including:

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Also, core class of multiterminal information theory problems: embedded special cases

1. no noise. messages overheard perfectly
2. sources independent
3. sources reproduced perfectly

What is a network coding capacity region?

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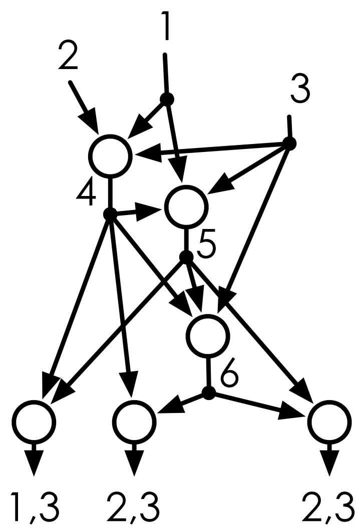

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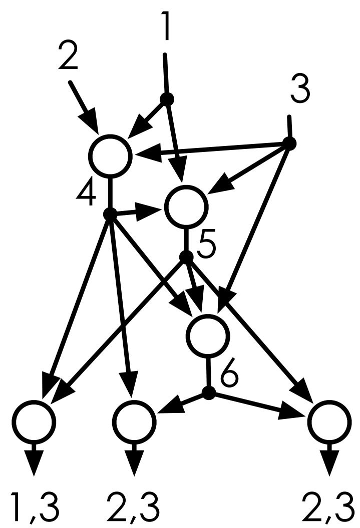

Source rates $H(Y_k)$, $k \in \{1, \ldots, K\}$ and edge rates $R_e, e \in \{K+1, \ldots, N\}$ are *achievable* if:

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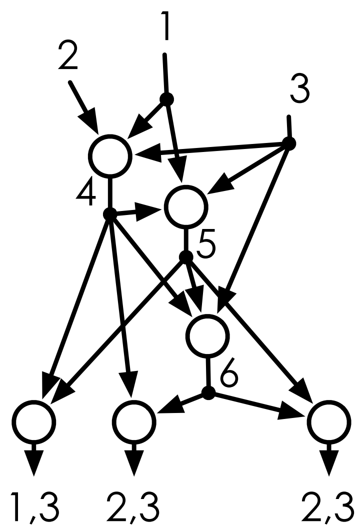

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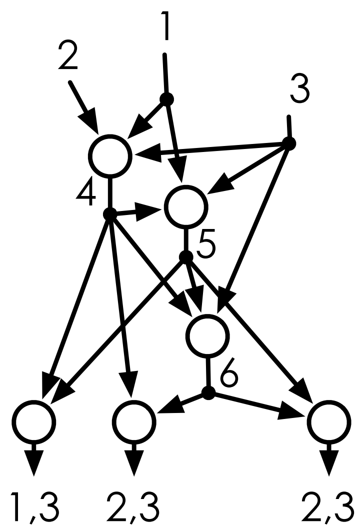

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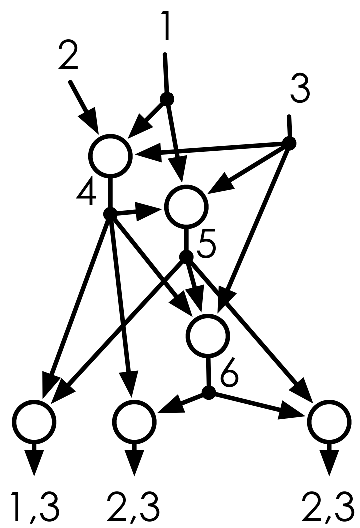

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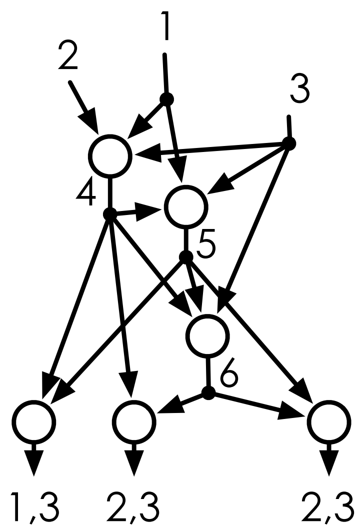

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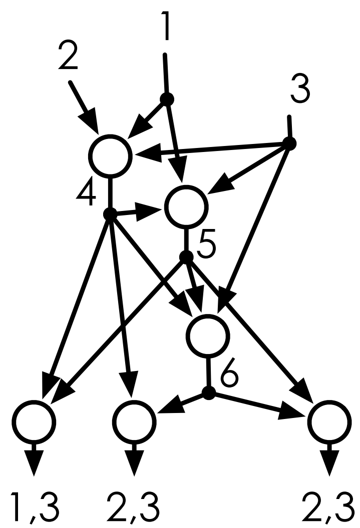

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Closure of set of all such achievable vectors $\mathbf{r} = [H(Y_k), R_e | k \in \{1, \dots, K\}, e \in \{K+1, \dots, N\}]$ is *capacity region*, $\mathcal{R}^*$, a convex cone.

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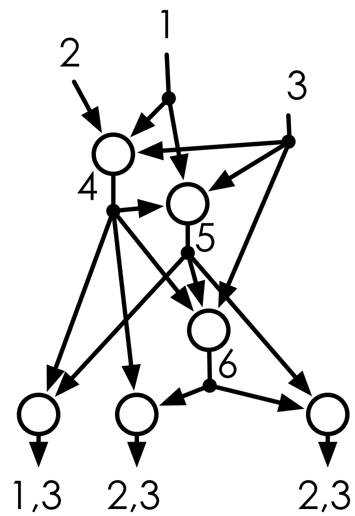

inequality description

$$R_4 \geq H(Y_2)$$
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rate region

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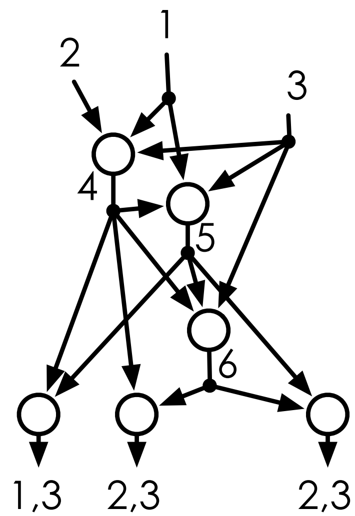

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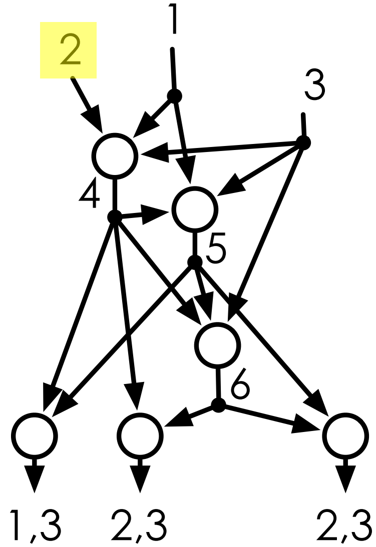

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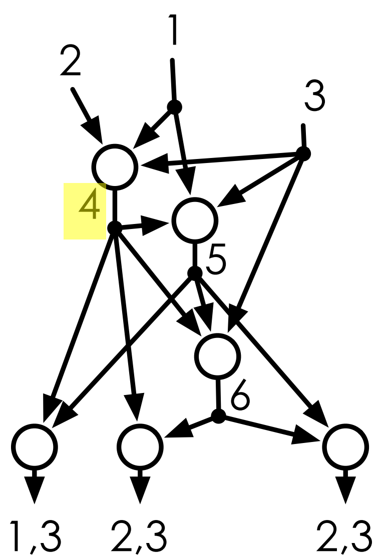

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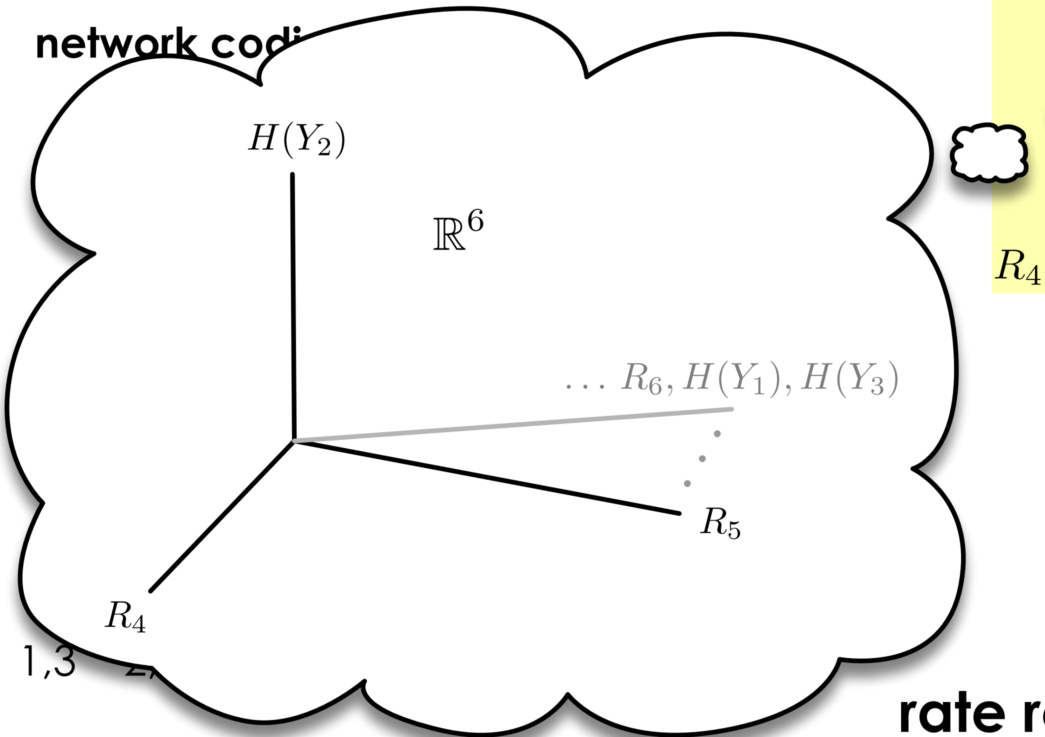

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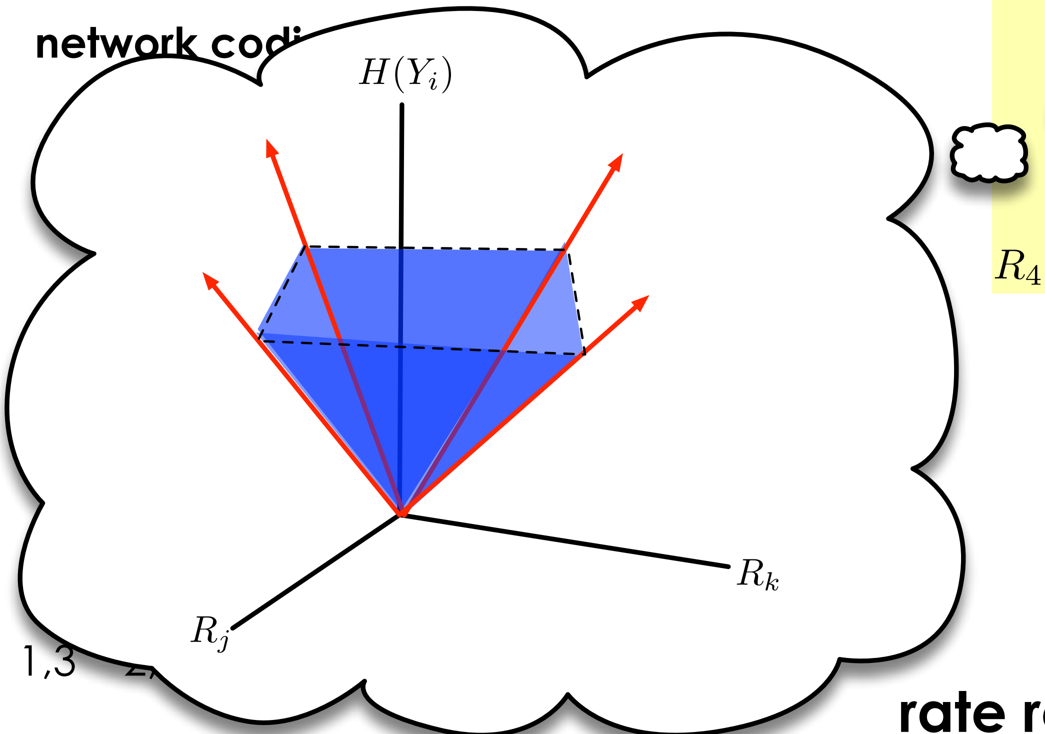

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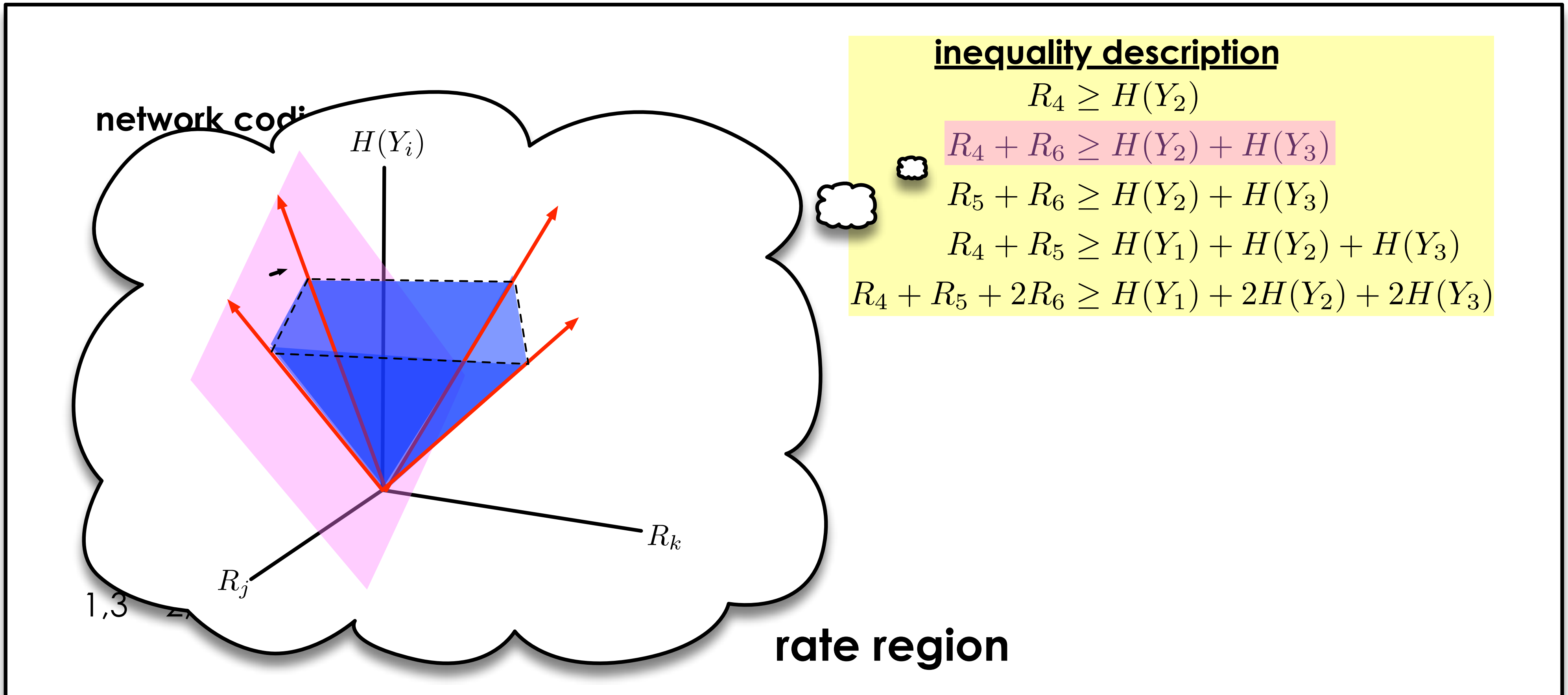

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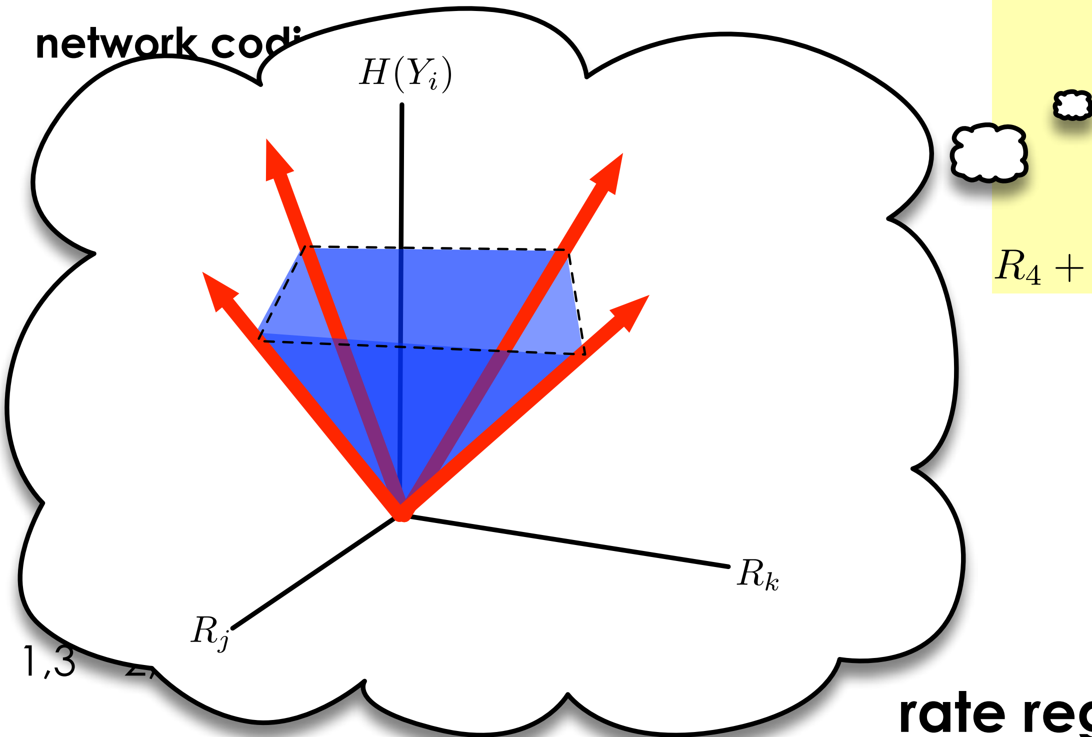

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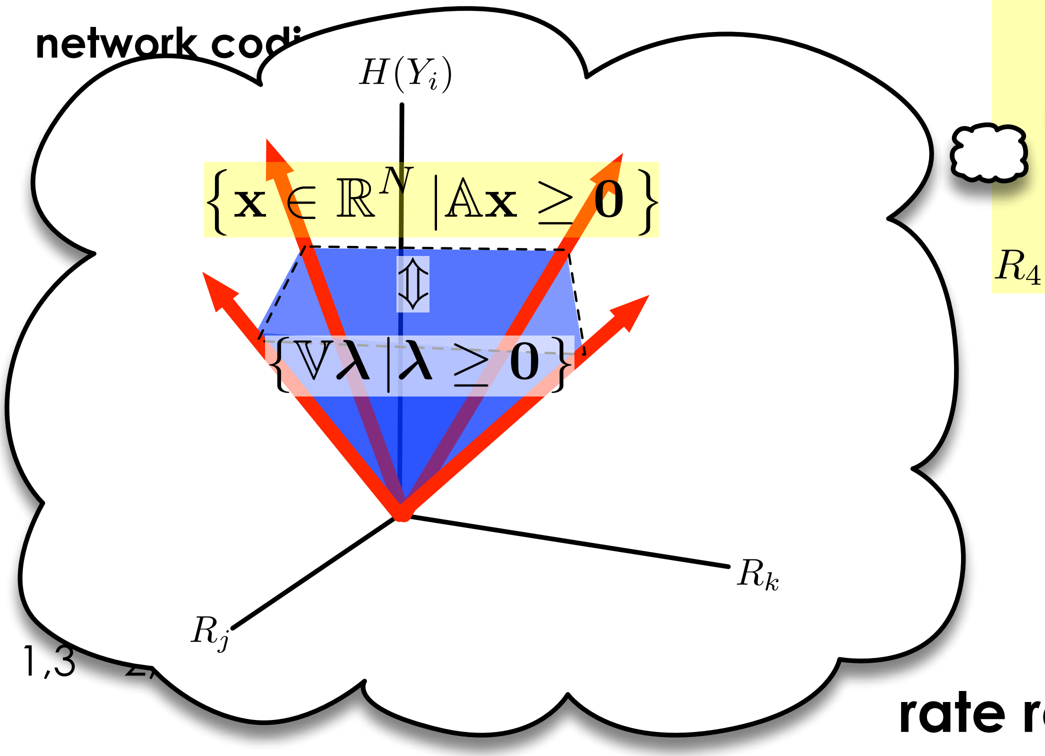

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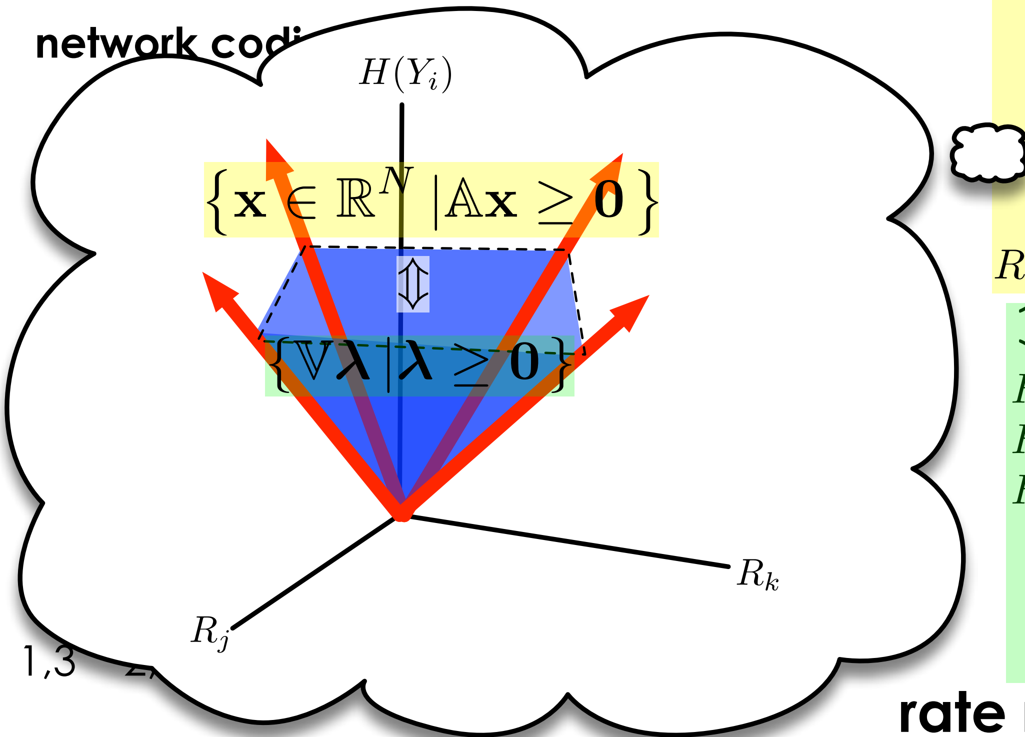

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$\Updownarrow$ **extreme ray description** $\Updownarrow$

$H(Y_1)$	1	0	0	0	0	1	0	0	0	1	0	0	0
$H(Y_2)$	0	1	0	0	1	2	1	0	0	0	0	0	0
$H(Y_3)$	0	0	0	1	1	0	0	2	1	0	0	0	1
R_4	0	1	1	1	1	2	1	1	1	1	0	0	0
R_5	1	1	0	1	1	1	0	1	0	0	0	1	1
R_6	0	0	0	0	1	1	1	1	1	0	1	0	1

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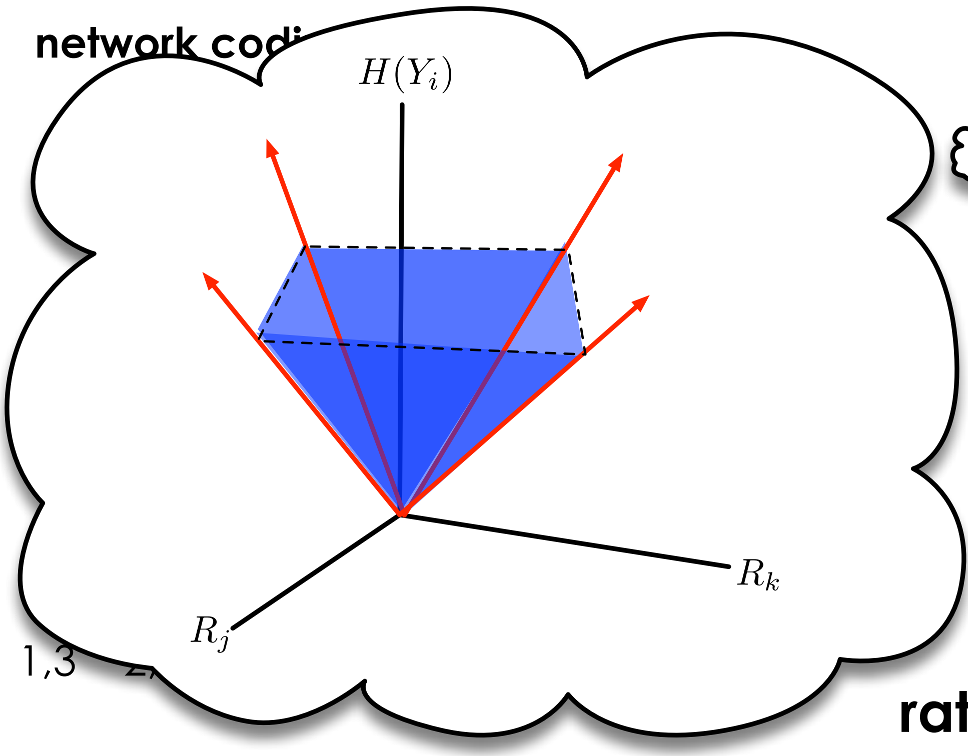

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R_4	0	1	1	1	1	2	1	1	1	1	0	0	0
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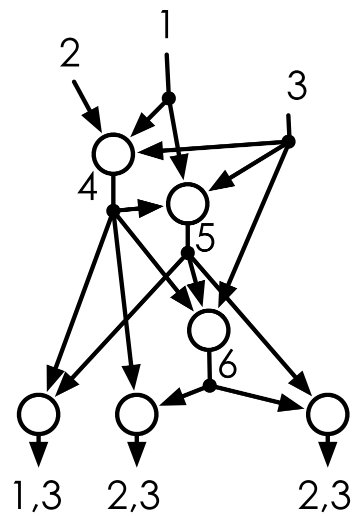

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R_4	0	1	1	1	1	2	1	1	1	1	0	0	0
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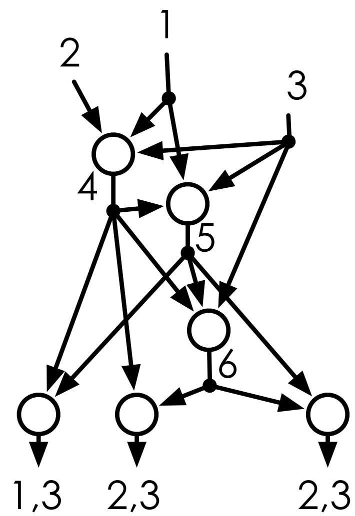

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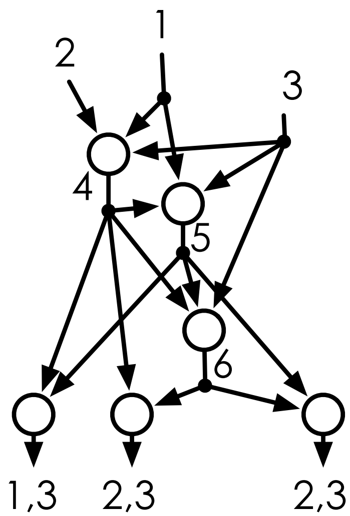

converse proofs

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Proof:

$R_4 \geq H(U_4)$

⇕ **extreme ray description** ⇕

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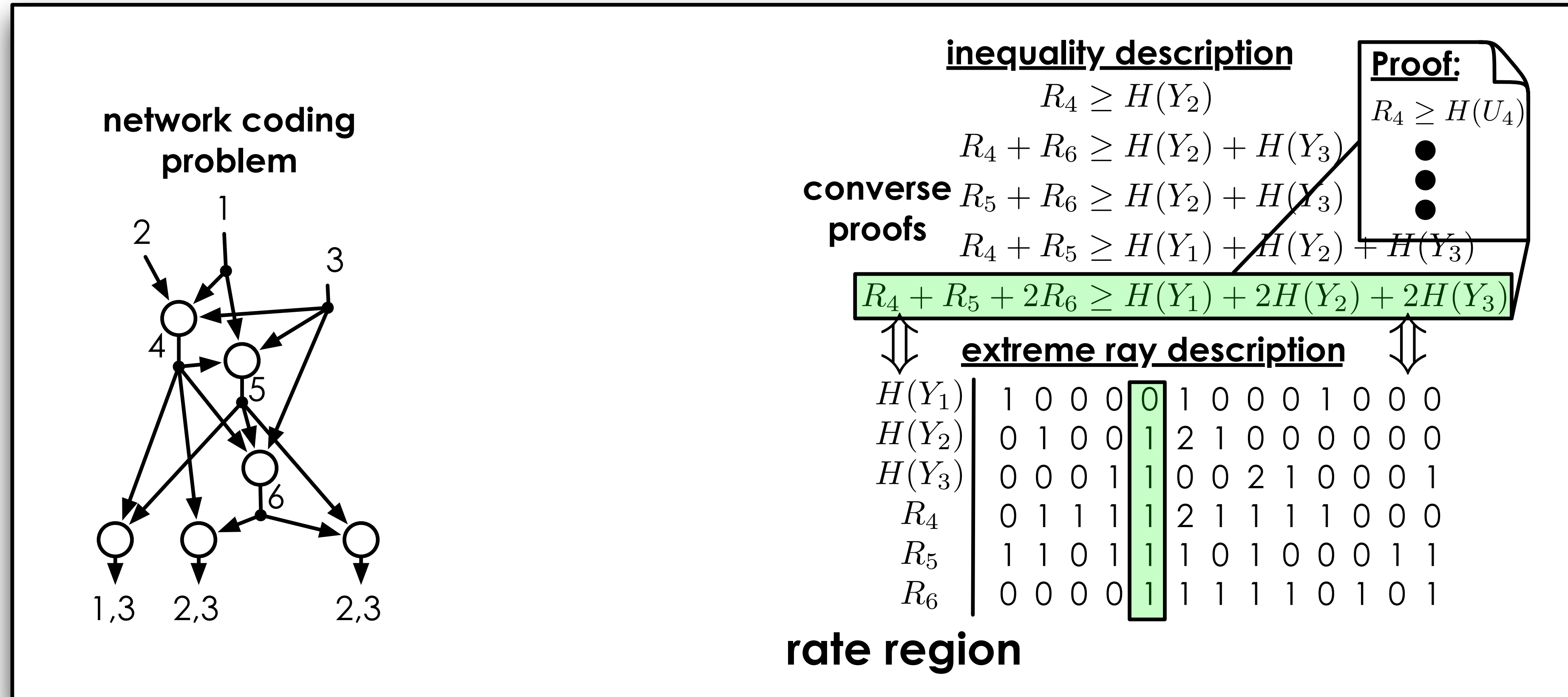
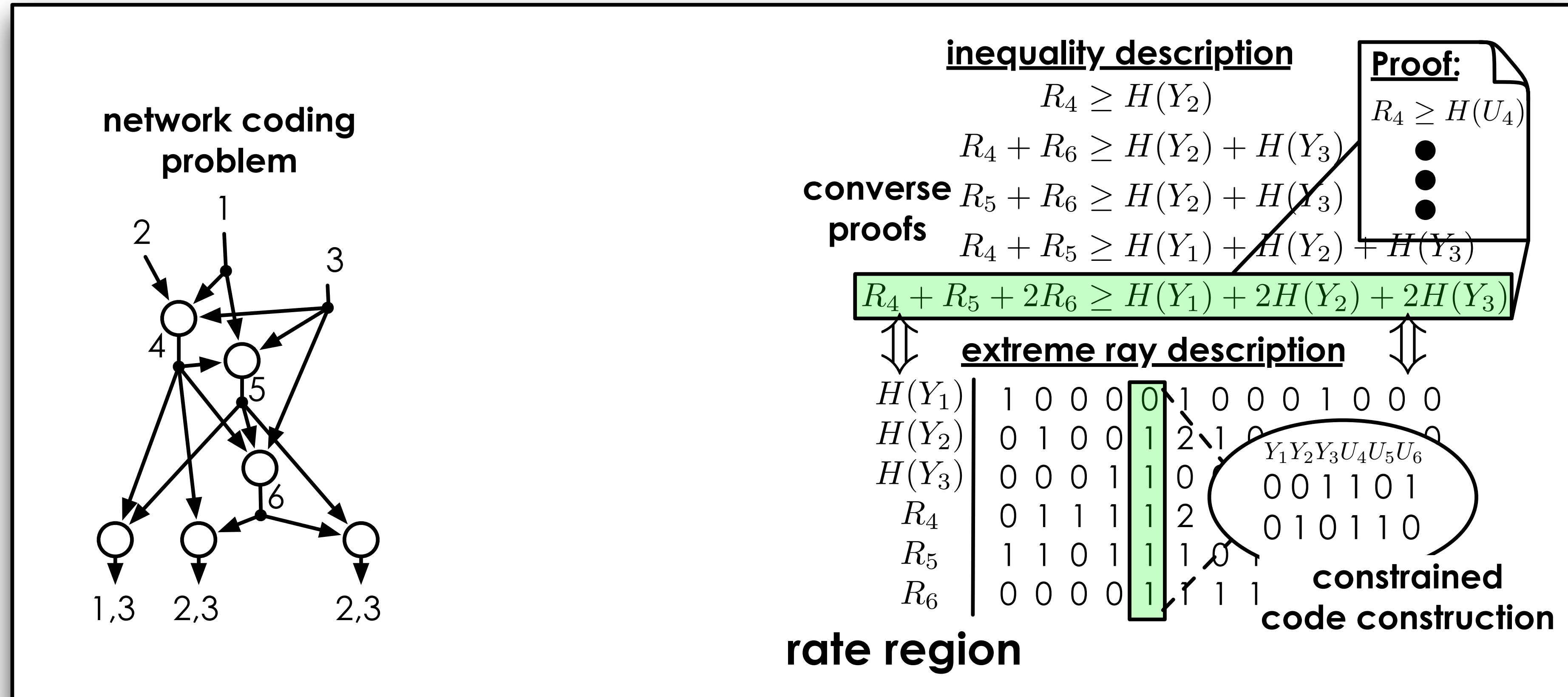
$H(Y_3)$	0	0	0	1	1	0	0	2	1	0	0	0	1
R_4	0	1	1	1	1	2	1	1	1	1	0	0	0
R_5	1	1	0	1	1	1	0	1	0	0	0	1	1
R_6	0	0	0	0	1	1	1	1	1	0	1	0	1

rate region

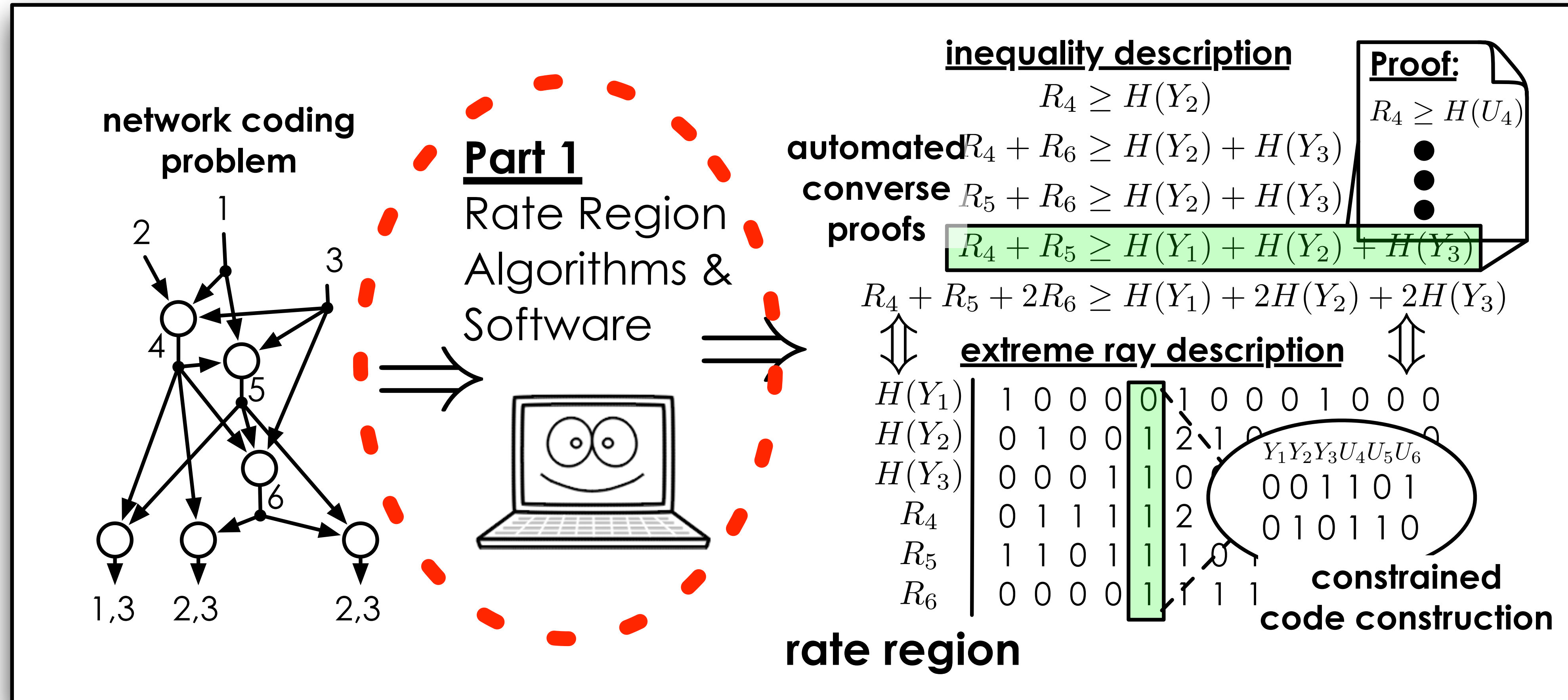
What is a network coding capacity region?



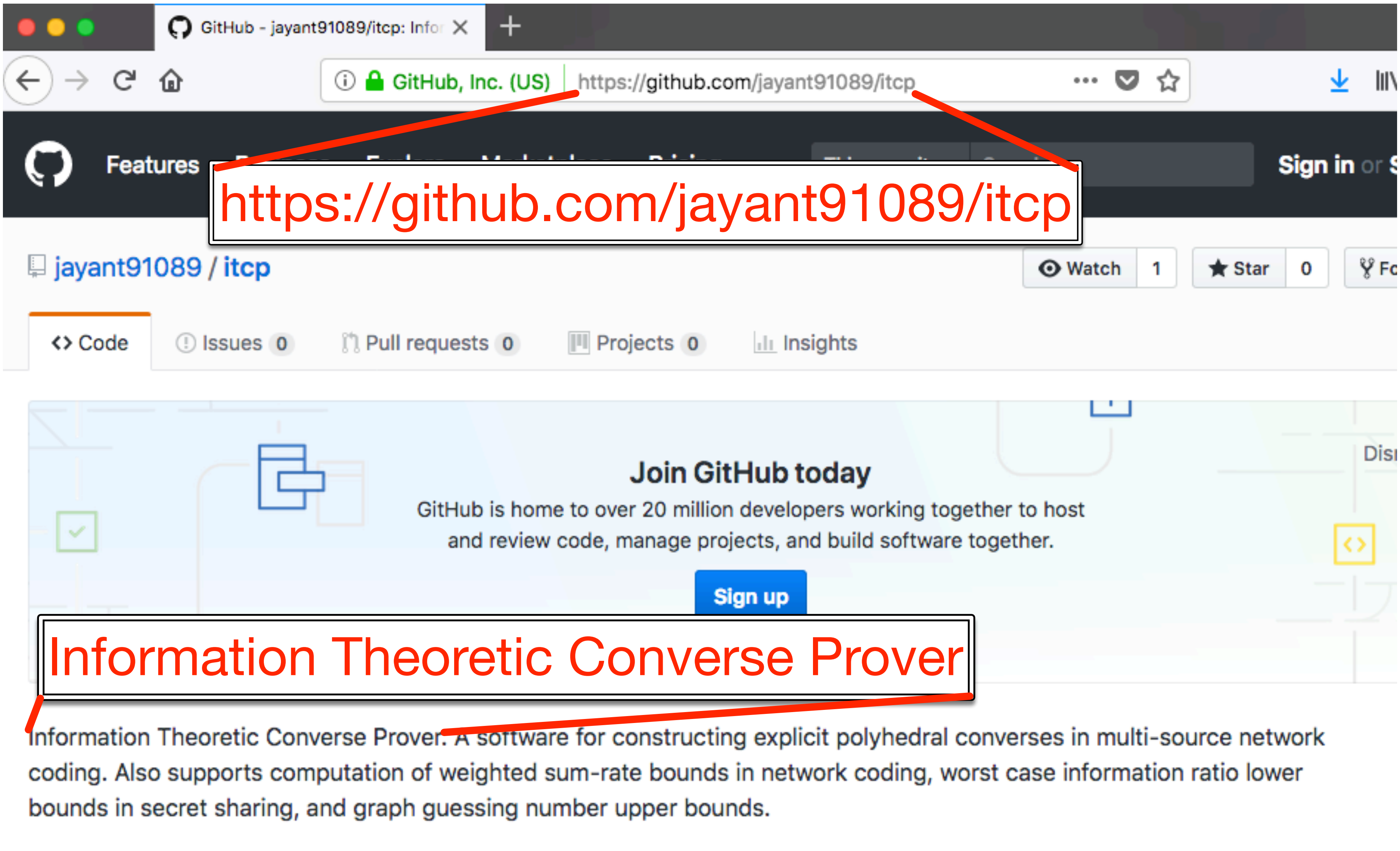
What is a network coding capacity region?



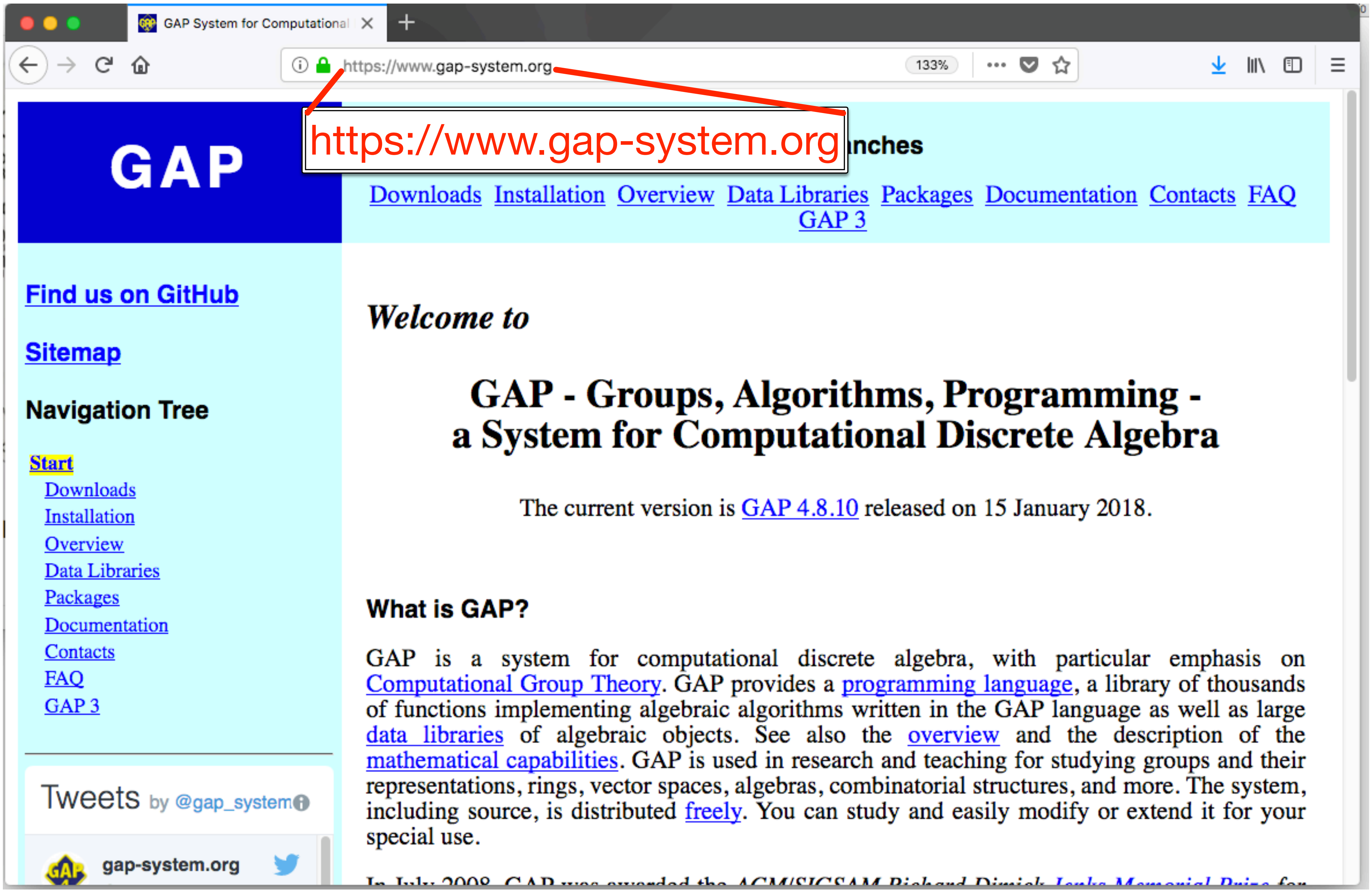
Computationally Enabled Research Agenda: #1 = Prove Regions



The Information Theoretic Converse Prover – ITCP (github)

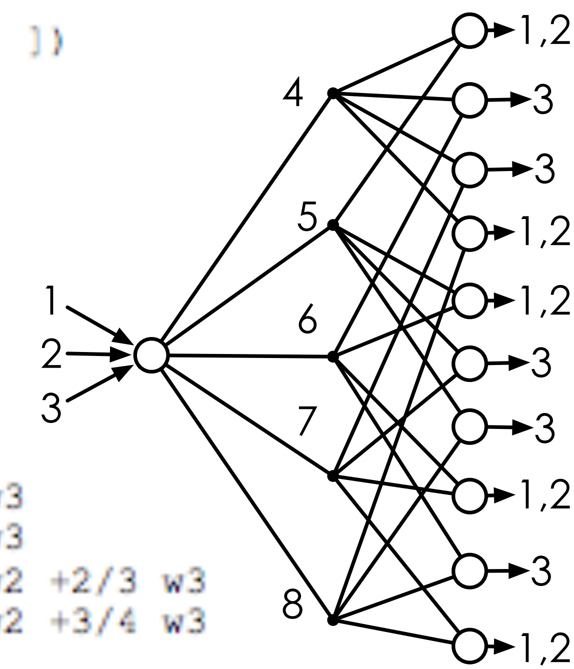


The Information Theoretic Converse Prover – ITCP is a GAP package!

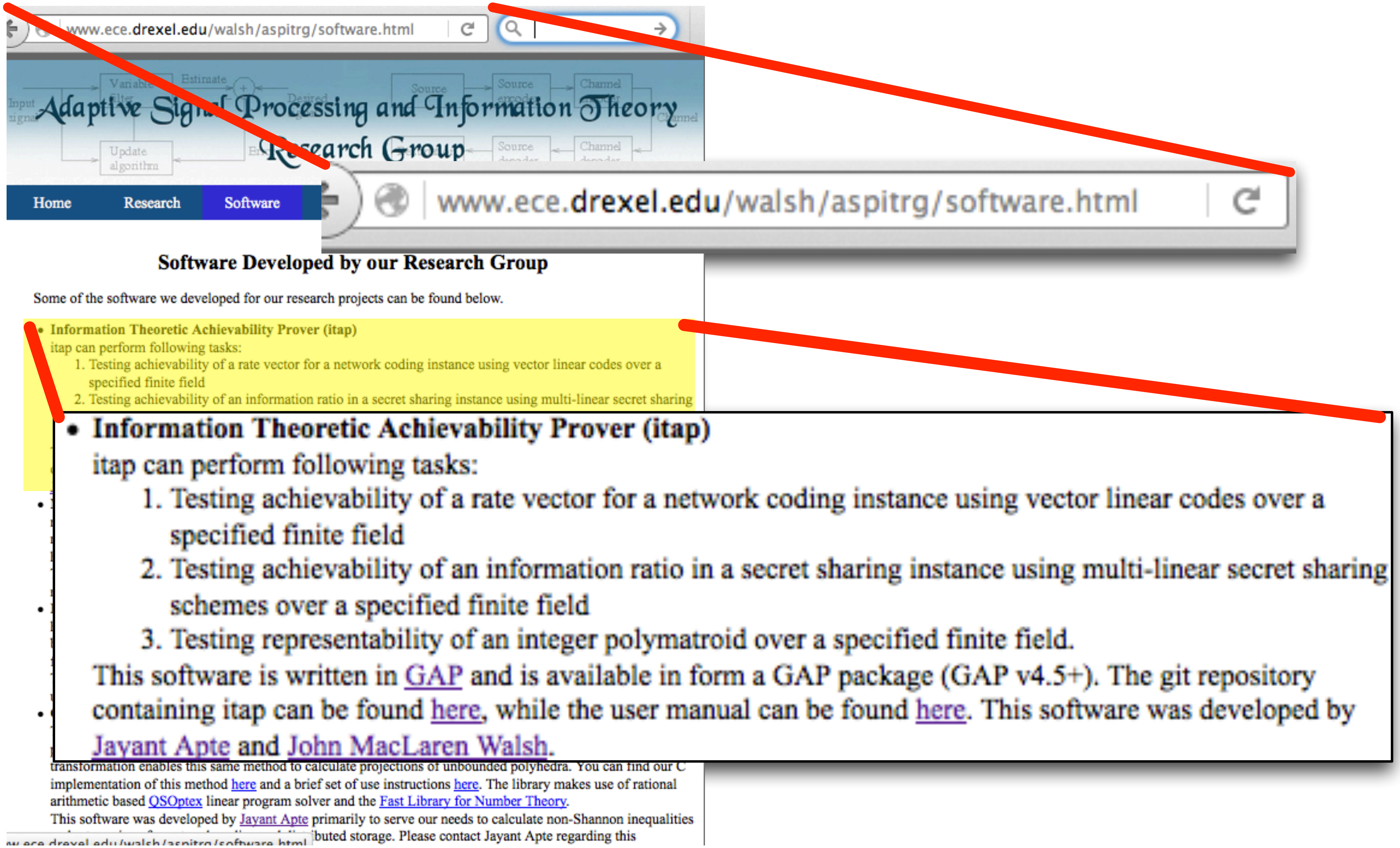


The Information Theoretic Converse Prover – ITCP (github)

```
gap> # Define a size 8 IDSC instance
> idsc:=[ [ [ [ 1, 2, 3 ], [ 1, 2, 3, 4, 5, 6, 7, 8 ] ],\
>         [ [ 4, 5 ], [ 1, 2, 4, 5 ] ], [ [ 5, 6 ], [ 1, 2, 5, 6 ] ],\
>         [ [ 6, 7 ], [ 1, 2, 6, 7 ] ], [ [ 7, 8 ], [ 1, 2, 7, 8 ] ],\
>         [ [ 4, 8 ], [ 1, 2, 4, 8 ] ], [ [ 4, 6 ], [ 3, 4, 6 ] ],\
>         [ [ 5, 8 ], [ 3, 5, 8 ] ], [ [ 4, 7 ], [ 3, 4, 7 ] ],\
>         [ [ 5, 7 ], [ 3, 5, 7 ] ], [ [ 6, 8 ], [ 3, 6, 8 ] ] ], 3, 8 ];
gap> G:=NetSymGroup(idsc);
Group([ (5,8)(6,7), (4,5)(6,8), (4,6)(7,8), (1,2) ])
gap> Size(G);
20
gap> rlist1:=NCRateRegionOB2(idsc,true,[]);;
gap> Display(rlist1[2]);
0 >= -w2
0 >= -w3
+R4 >= 0
+R4 +R6 >= +w3
+R4 +R5 >= +w1 +w2
+R4 +1/2 R5 +1/2 R8 >= +w1 +w2 +1/2 w3
+1/2 R4 +1/2 R5 +1/2 R6 +1/2 R7 >= +w1 +w2 +1/2 w3
+2/3 R4 +2/3 R5 +1/3 R6 +1/3 R8 >= +w1 +w2 +2/3 w3
+2/3 R4 +1/3 R5 +1/3 R6 +1/3 R7 +1/3 R8 >= +w1 +w2 +2/3 w3
+1/2 R4 +1/2 R5 +1/2 R6 +1/4 R7 +1/4 R8 >= +w1 +w2 +3/4 w3
+R4 +1/2 R5 +1/2 R6 +1/2 R7 >= +w1 +w2 +w3
+R4 +1/2 R5 +1/2 R6 +1/2 R8 >= +w1 +w2 +w3
+R4 +1/3 R5 +1/3 R6 +1/3 R7 +1/3 R8 >= +w1 +w2 +w3
+2/3 R4 +2/3 R5 +1/3 R6 +2/3 R7 +1/3 R8 >= +w1 +w2 +4/3 w3
+R4 +1/2 R5 +1/2 R6 +R7 >= +w1 +w2 +3/2 w3
+R4 +1/2 R5 +1/2 R6 +1/2 R7 +1/2 R8 >= +w1 +w2 +3/2 w3
+2 R4 +R6 +R7 >= +w1 +w2 +2 w3
+R4 +R5 +R6 +R7 >= +w1 +w2 +2 w3
```



The Information Theoretic Achievability Prover (ITAP)



www.ece.drexel.edu/walsh/aspitrg/software.html

Adaptive Signal Processing and Information Theory Research Group

Home | Research | Software

Software Developed by our Research Group

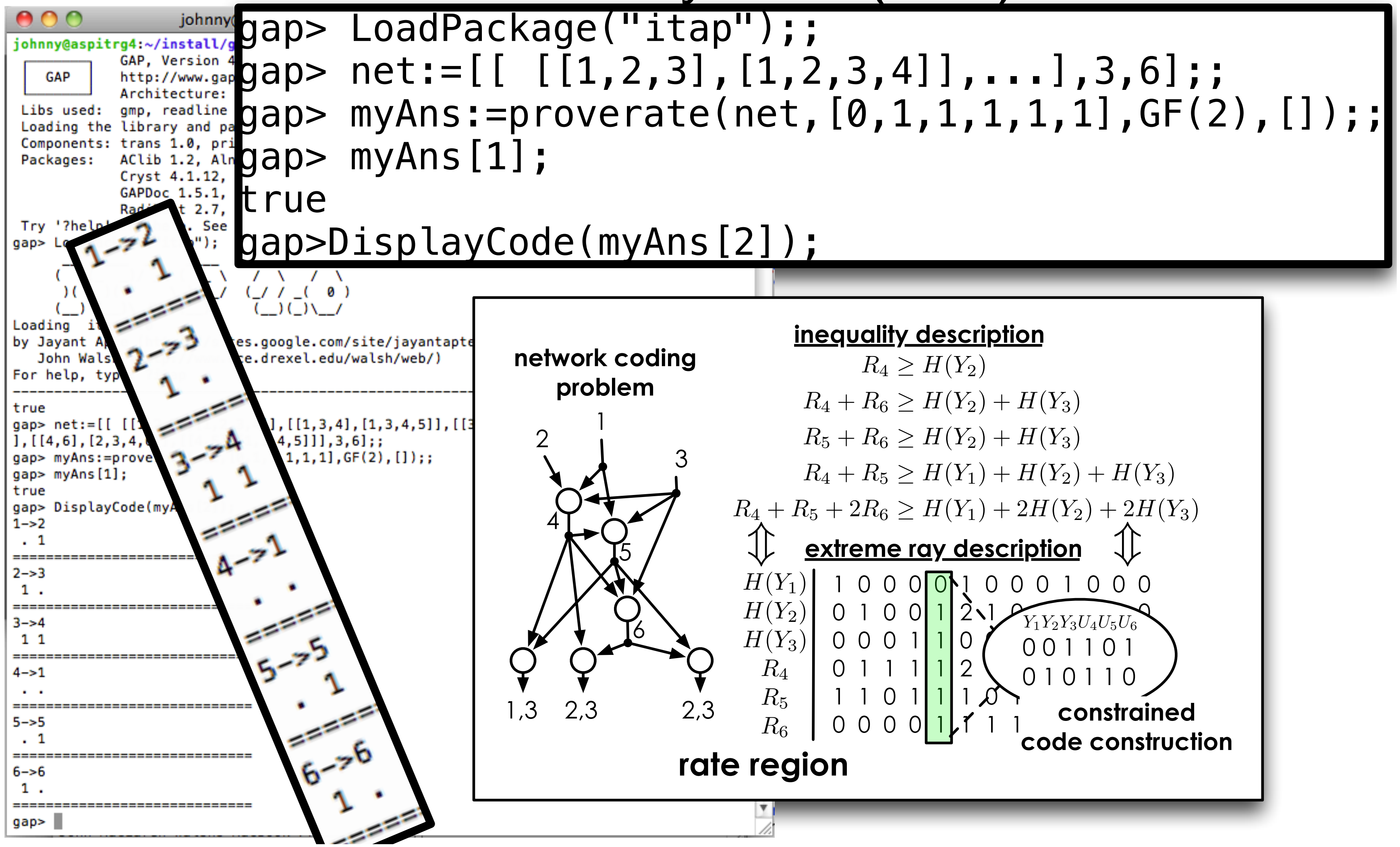
Some of the software we developed for our research projects can be found below.

- **Information Theoretic Achievability Prover (itap)**
  itap can perform following tasks:
  1. Testing achievability of a rate vector for a network coding instance using vector linear codes over a specified finite field
  2. Testing achievability of an information ratio in a secret sharing instance using multi-linear secret sharing schemes over a specified finite field
  3. Testing representability of an integer polymatroid over a specified finite field.

  This software is written in GAP and is available in form a GAP package (GAP v4.5+). The git repository containing itap can be found here, while the user manual can be found here. This software was developed by Jayant Apte and John MacLaren Walsh.

transformation enables this same method to calculate projections of unbounded polyhedra. You can find our C implementation of this method here and a brief set of use instructions here. The library makes use of rational arithmetic based QSOptex linear program solver and the Fast Library for Number Theory.
This software was developed by Jayant Apte primarily to serve our needs to calculate non-Shannon inequalities ... buted storage. Please contact Jayant Apte regarding this

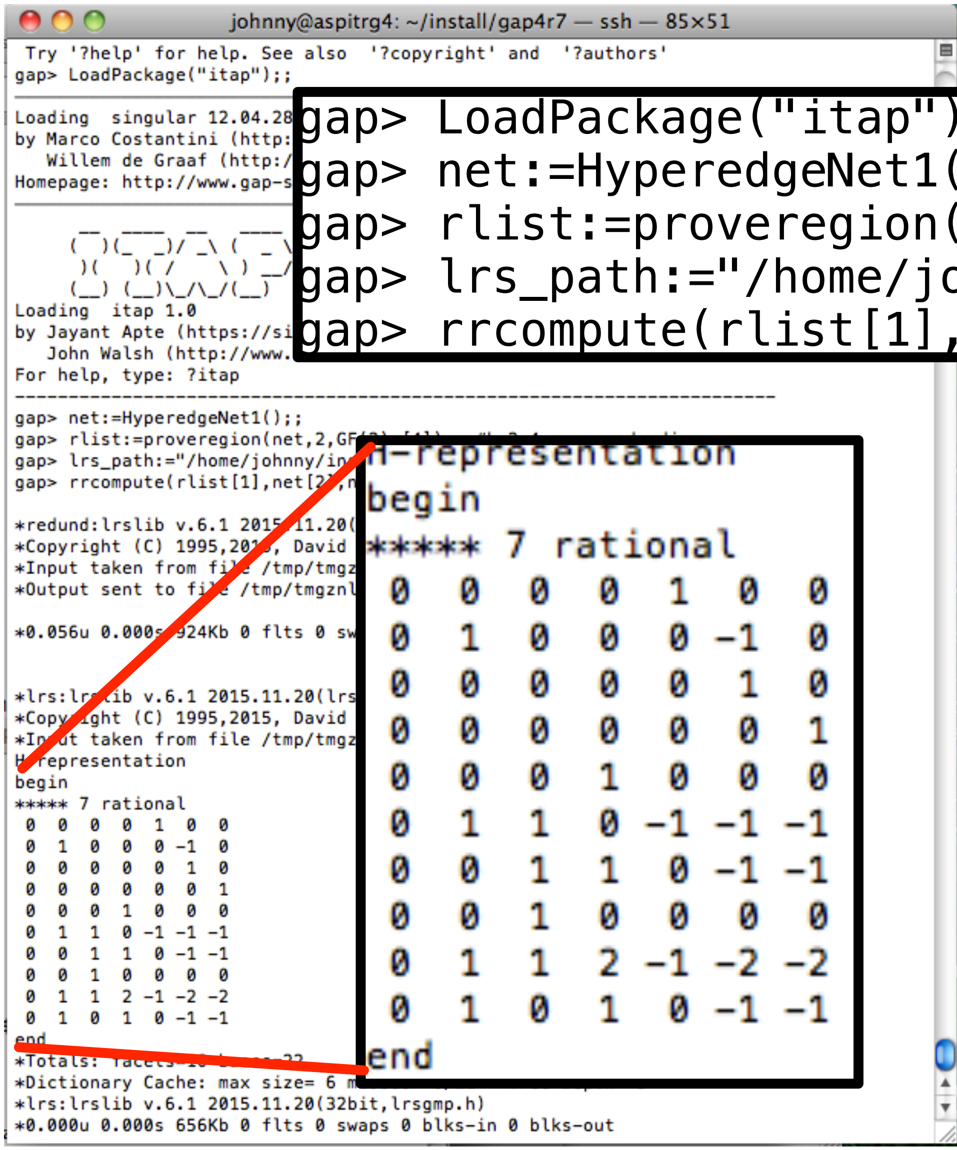
The Information Theoretic Achievability Prover (ITAP) – Rate Vector Verification



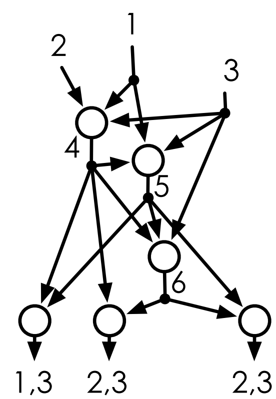
Verification user interface shown. Listing interface available as well.

Although enumeration oriented, when used as a verification algorithm (w/ specified rate vector) it can still be faster than the Groebner basis (w/ Singular) based path-gain verification of Subramanian & Thangaraj! (also included)

The Information Theoretic Achievability Prover (ITAP) – Rate Vector Verification



network coding problem



inequality description

$R_4 \geq H(Y_2)$

$R_4 + R_6 \geq H(Y_2) + H(Y_3)$

$R_5 + R_6 \geq H(Y_2) + H(Y_3)$

$R_4 + R_5 \geq H(Y_1) + H(Y_2) + H(Y_3)$

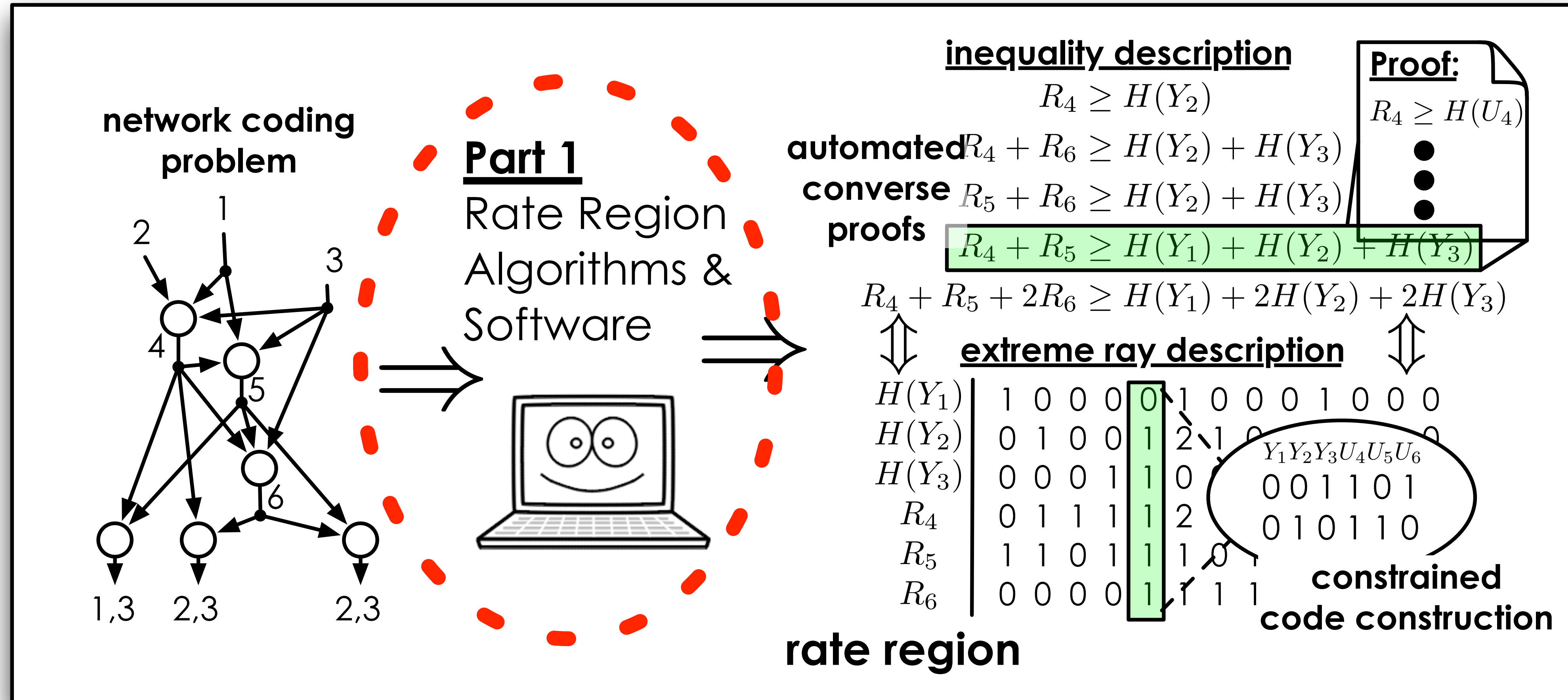
$R_4 + R_5 + 2R_6 \geq H(Y_1) + 2H(Y_2) + 2H(Y_3)$

$\Updownarrow$ **extreme ray description** $\Updownarrow$

$H(Y_1)$	1	0	0	0	0	1	0	0	0	1	0	0	0
$H(Y_2)$	0	1	0	0	1	2	1	0	0	0	0	0	0
$H(Y_3)$	0	0	0	1	1	0	0	2	1	0	0	0	1
R_4	0	1	1	1	1	2	1	1	1	1	0	0	0
R_5	1	1	0	1	1	1	0	1	0	0	0	1	1
R_6	0	0	0	0	1	1	1	1	1	0	1	0	1

rate region

Computationally Enabled Research Agenda: #1 = Prove Regions



Computationally Enabled Research Agenda:

1. Train a computer to calculate network coding capacity regions and their proofs.

2. Build a database of all network coding capacity regions up to a certain size.

3. Organize this database to learn from it, and then to use it to create solutions to networks too large for the computer calculate directly.

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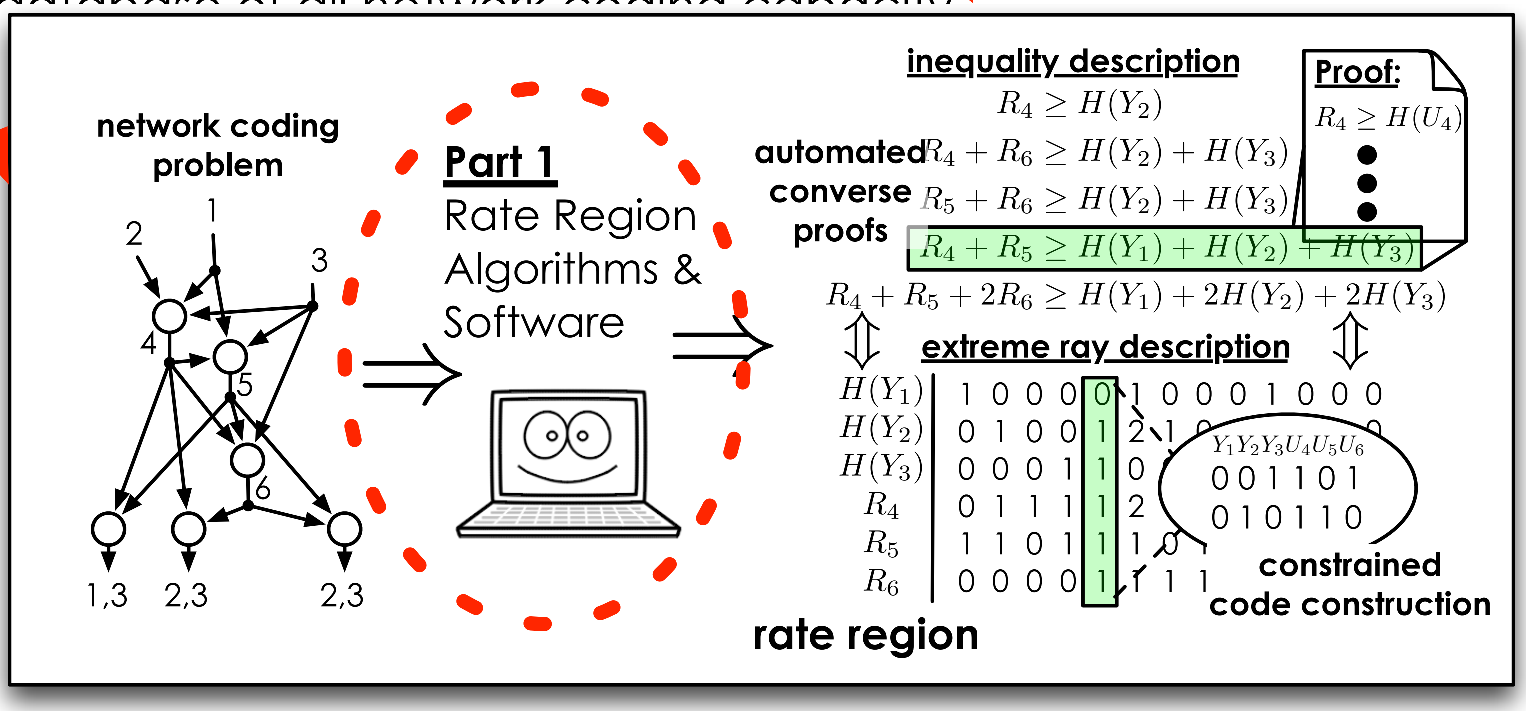

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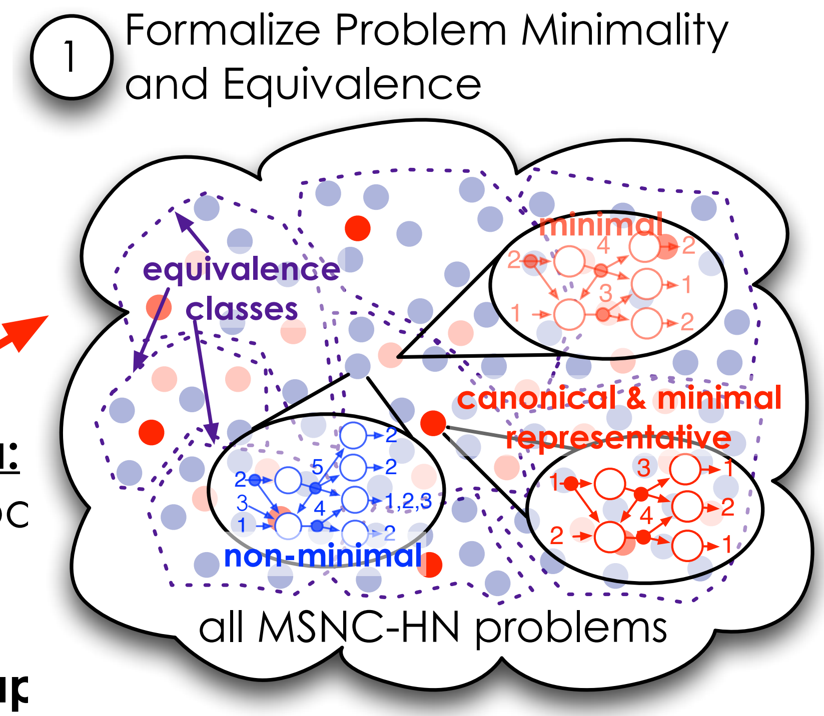
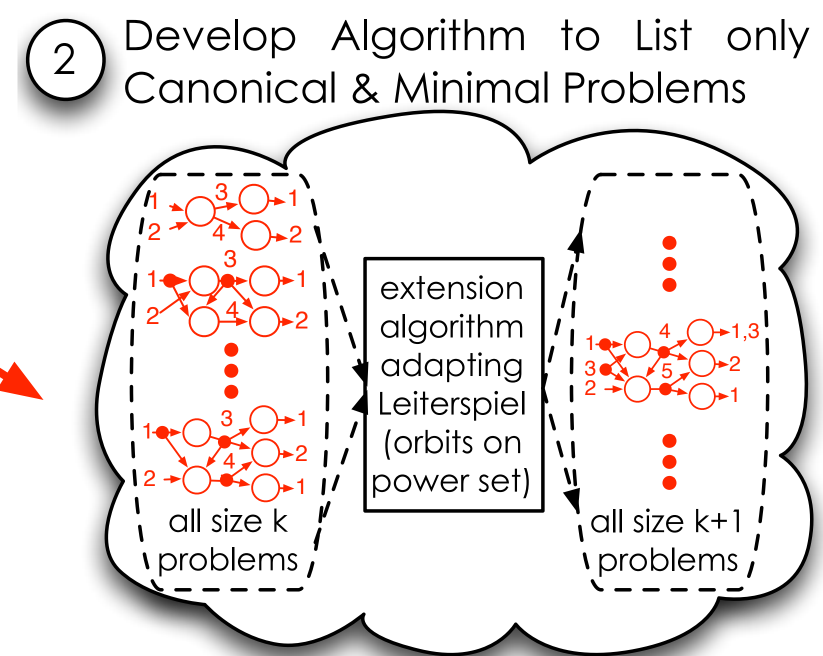
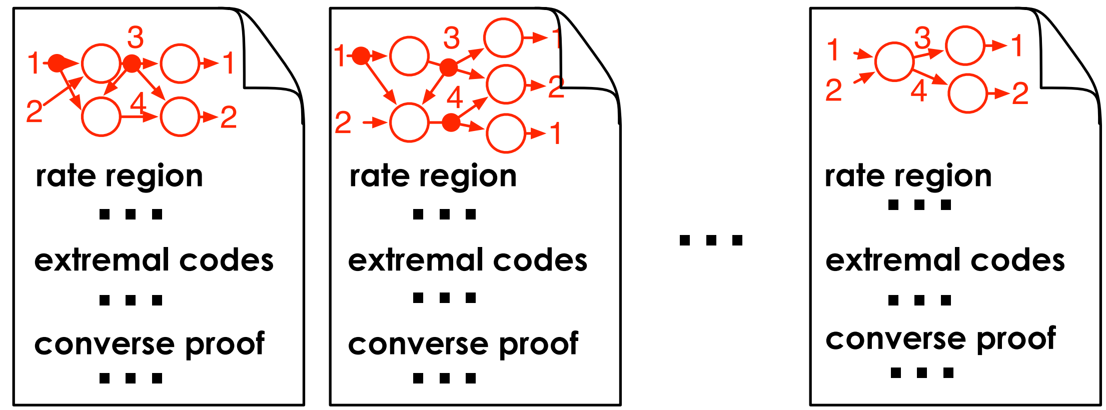




Network Coding Rate Region Database



Database of ~**7000** Rate Regions of >**100k** Networks (Trans IT Jan. '17)
Database of ~**744k** Rate Regions of > 7M or ~**2.3T** Minimal Networks (Trans IT Nov. '17)

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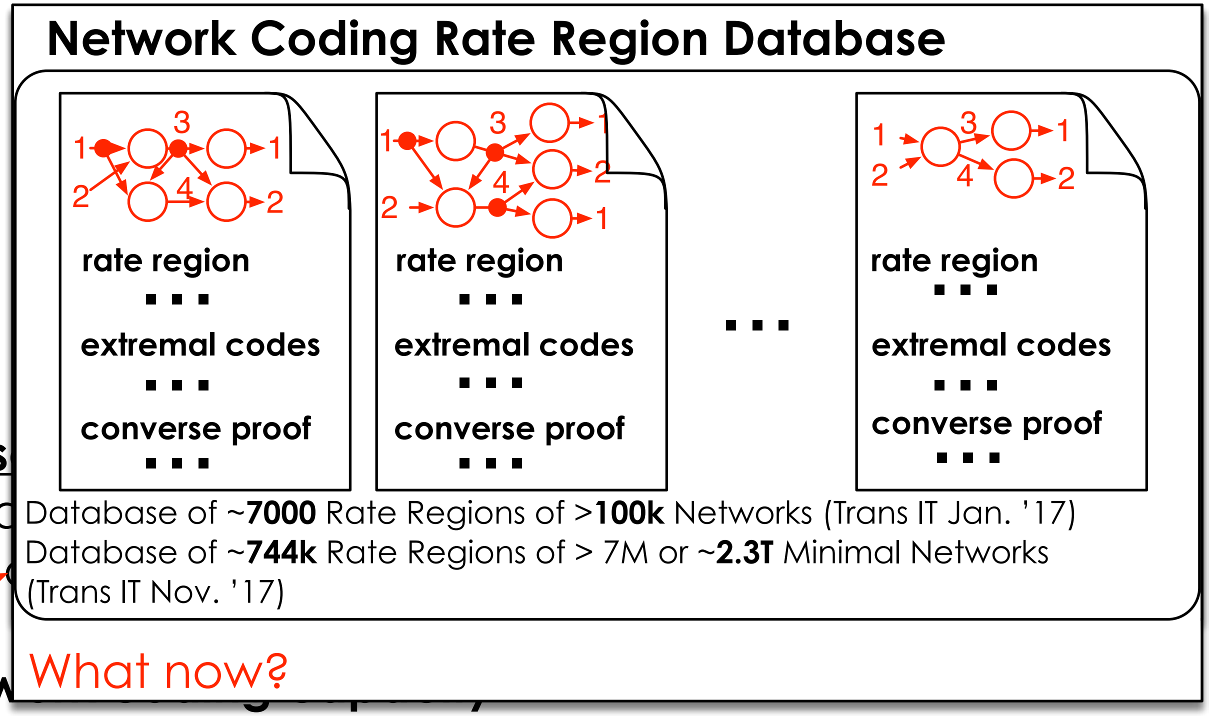

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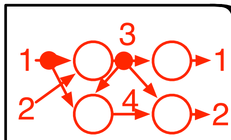
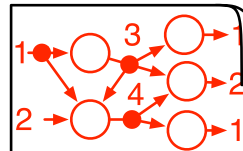


rate region
...
extremal codes
...
converse proof
...



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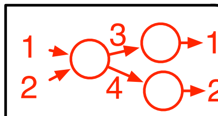

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What now? Submit 744,000 transactions papers?

<u>Computationally Enabled Res</u>

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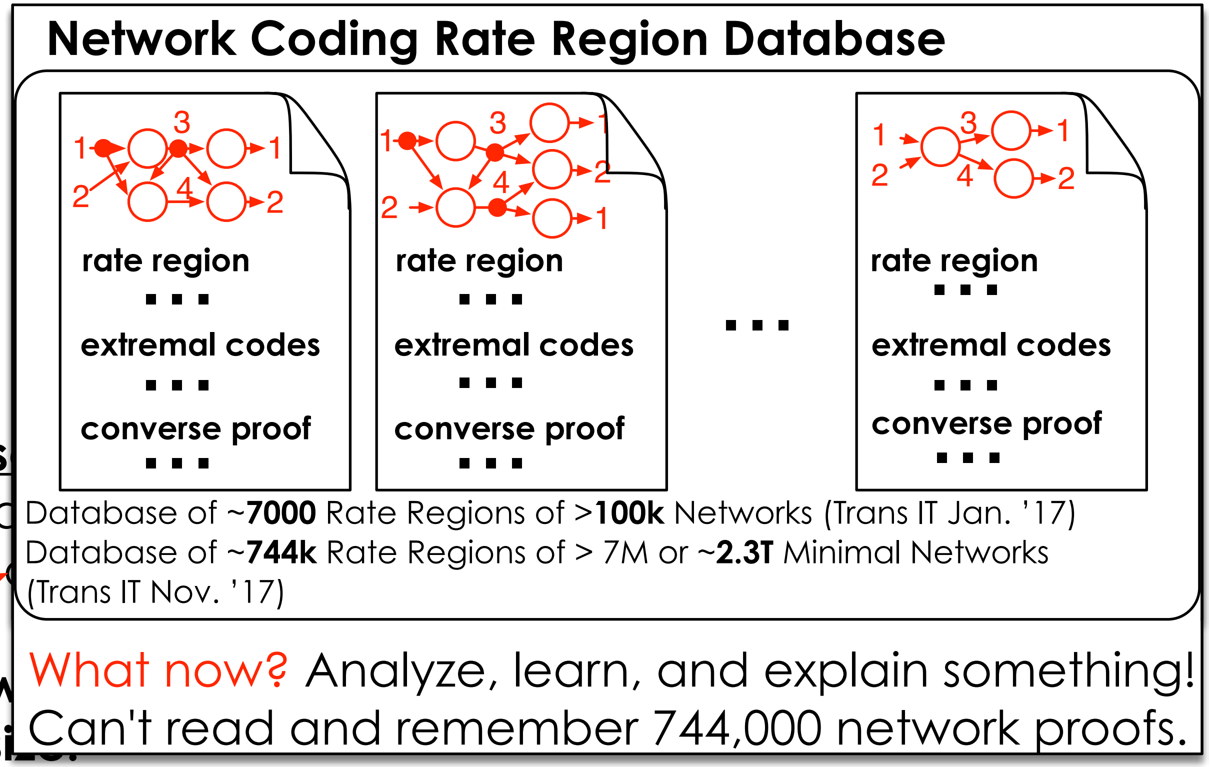

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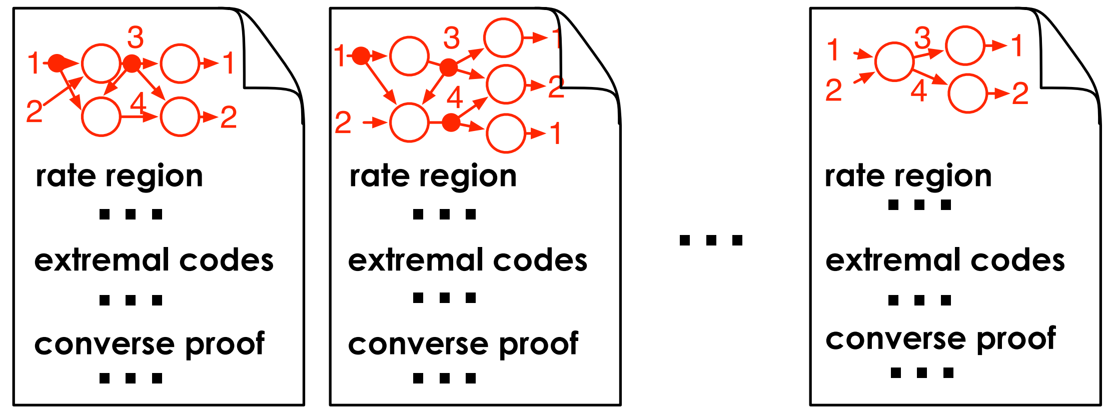

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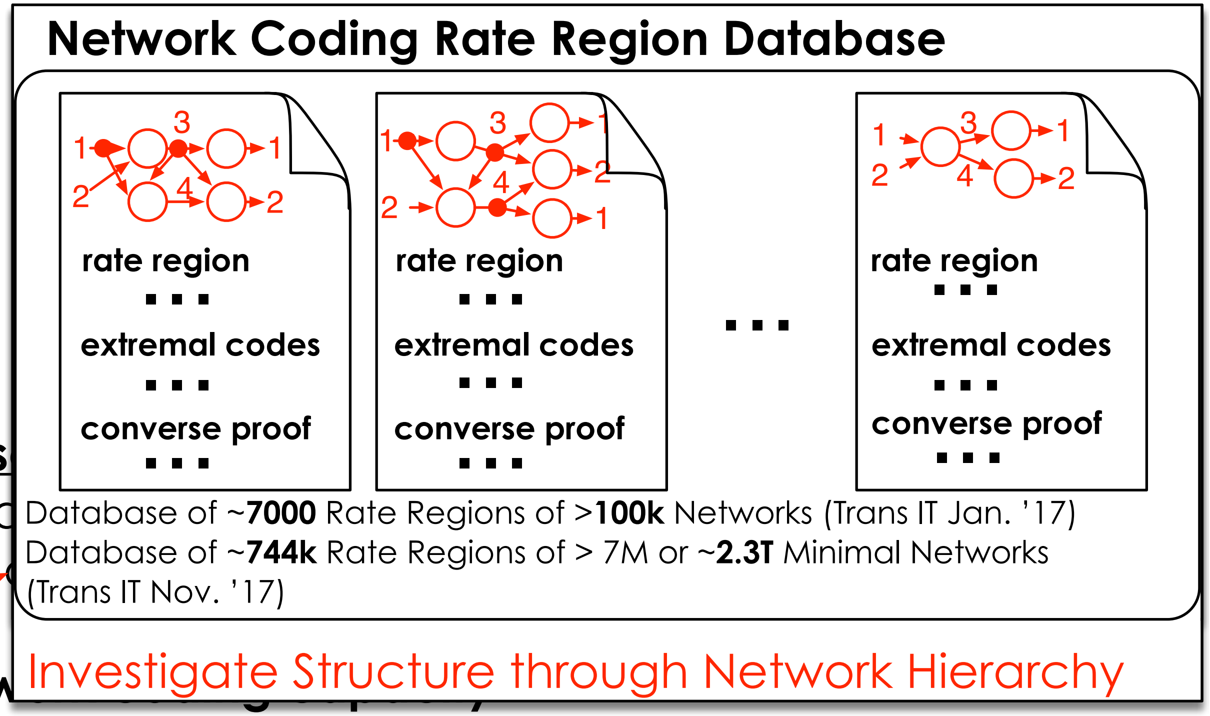
What now? Analyze, learn, and explain something! Can't read and remember 744,000 network proofs.

Software handles only small nets (max 8-10 edges).

Community taste (caching, storage) is for large graphs and low dim. projections of rate regions



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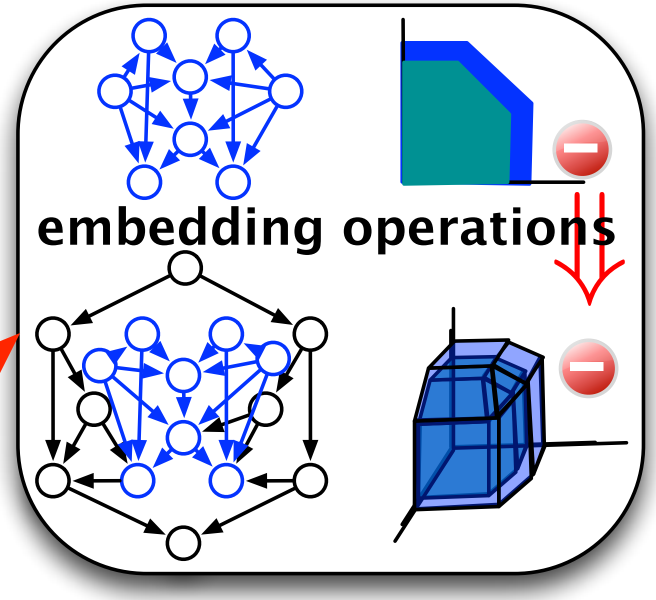

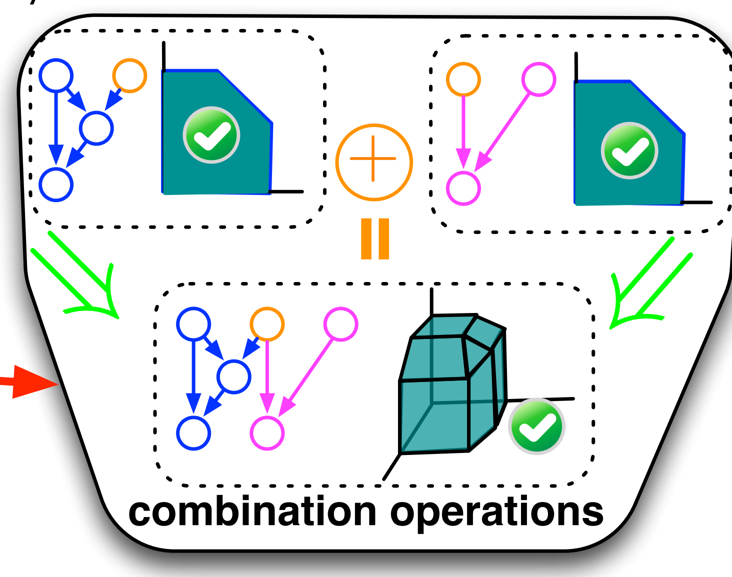

Background: Inspiration for Hierarchy – Well quasi-ordering of Graphs

Definition 1 (Graph Minor)**:** A graph $\mathcal{G}_1(\mathcal{V}_1, \mathcal{E}_1)$ is a *minor* of another graph $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{E}_2)$ if $\mathcal{G}_1$ can be obtained through a sequence of node deletions, edge deletions, and contractions.

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Theorem 1 (Kuratowski/ Wagner)**:** A graph $(\mathcal{V}, \mathcal{E})$ is planar if and only if it has no $K_{3,3}$ or K_5 minor.

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Observe: the set of planar graphs is closed under the operation of taking minors.

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Theorem 2 (Robertson-Seymour Theorem)**:** Any family of graphs that is closed under the operation of taking minors has at most a finite series of forbidden minors.

Equivalent to stating that there are no infinite anti-chains (any infinite sequence of graphs must have a pair with a minor relationship) and no infinite descending chains (WQO).

Background: Inspiration for Hierarchy – Forbidden Minors Continued

- network coding problems are part graph based (already enough)

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- network coding problems are part graph based (already enough)
- Network coding rate regions are built not only from labelled graphs, but also from entropy functions, which are polymatroids, $\supseteq$ matroids $\supseteq$ independence in graphs

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- roughly, set sources = independent uniform RVs, call what is set on an edge e a RV U_e. Collect all RV.s into $\mathbf{X} = (Y_s, U_e | s \in \mathcal{S}, e \in \mathcal{E})$.

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$$h(\mathcal{A}) = H(\mathbf{X}_{\mathcal{A}}),\ \mathcal{A} \subseteq \mathcal{N} = \mathcal{S} \cup \mathcal{E} \tag{4}$$

is a *polymatroid*.

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Definition 2 (polymatroid)**:** A set function $\rho : 2^{\mathcal{N}} \to \mathbb{R}_{\geq 0}$ is a polymatroid if $\forall \mathcal{A}, \mathcal{B}$, $\rho(\mathcal{A}) + \rho(\mathcal{B}) \geq \rho(\mathcal{A} \cup \mathcal{B}) + \rho(\mathcal{A} \cap \mathcal{B})$ – submodular, and for any $\mathcal{C} \subseteq \mathcal{D}$, $\rho(\mathcal{C}) \leq \rho(\mathcal{D})$ – non-decreasing.

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Definition 3 (matroid)**:** A matroid is a polymatroid ρ taking values in $\mathbb{Z}_{\geq 0}$ for whom $\rho(\mathcal{A}) \leq |\mathcal{A}|$.

Background: Inspiration for Hierarchy – Rota's Conjecture

Definition 4 (Matroid Deletion)**:** $\rho' : 2^{\mathcal{N}'} \to \mathbb{Z}_{\geq 0}$ is obtained by deleting $e \in \mathcal{N}$ from ρ if $\mathcal{N} = \mathcal{N} \setminus \{e\}$ and $\rho'(\mathcal{A}) = \rho(\mathcal{A}), \forall \mathcal{A} \subseteq \mathcal{N}'$.

Definition 5 (Matroid contraction)**:** $\rho' : 2^{\mathcal{N}'} \to \mathbb{Z}_{\geq 0}$ is obtained by contracting $e \in \mathcal{N}$ from ρ if $\mathcal{N} = \mathcal{N} \setminus \{e\}$ and $\rho'(\mathcal{A}) = \rho(\mathcal{A} \cup \{e\}) - \rho(\{e\}), \forall \mathcal{A} \subseteq \mathcal{N}'$. (condition entropy on X_e).

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Theorem 4 (Rota's Conjecture (1970) Proved 2013 by Geelen, Gerards, Whittle)**:** The set of matroids representable over $\mathbb{F}_q$ (translation: set of $h(\mathcal{A})$ arising from scalar codes over $\mathbb{F}_q$) has a most a finite number of forbidden minors.

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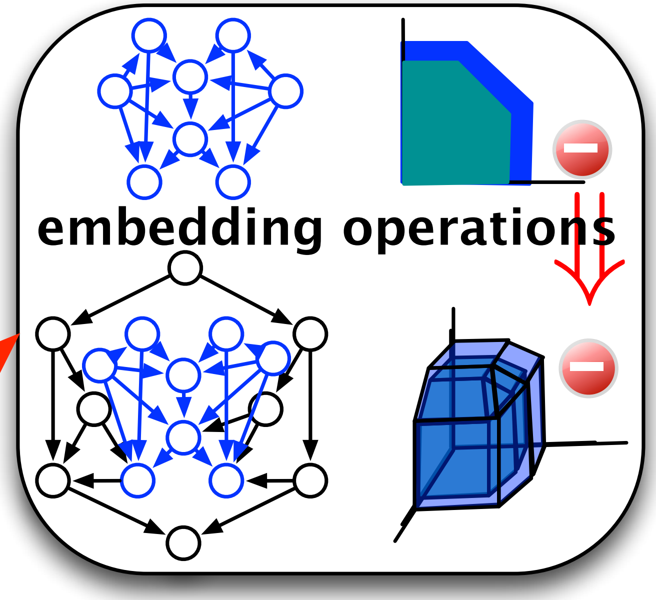
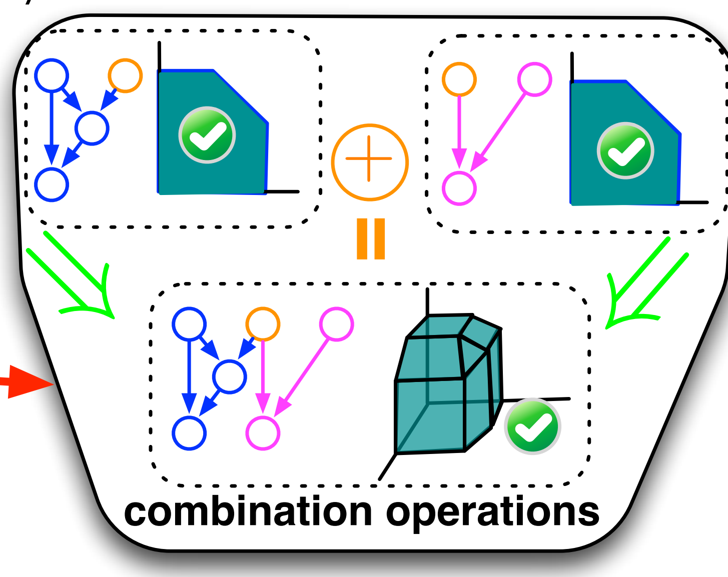
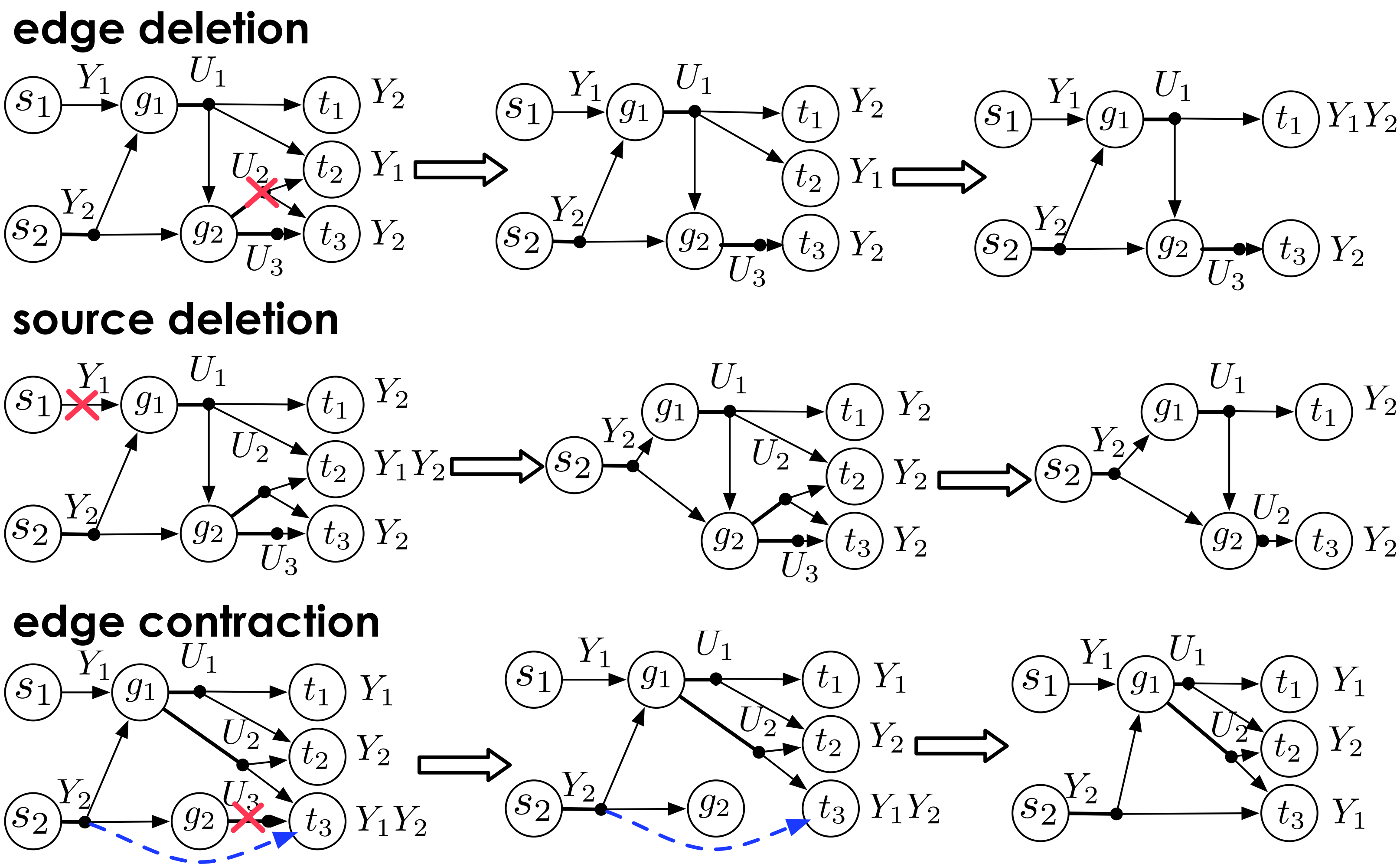


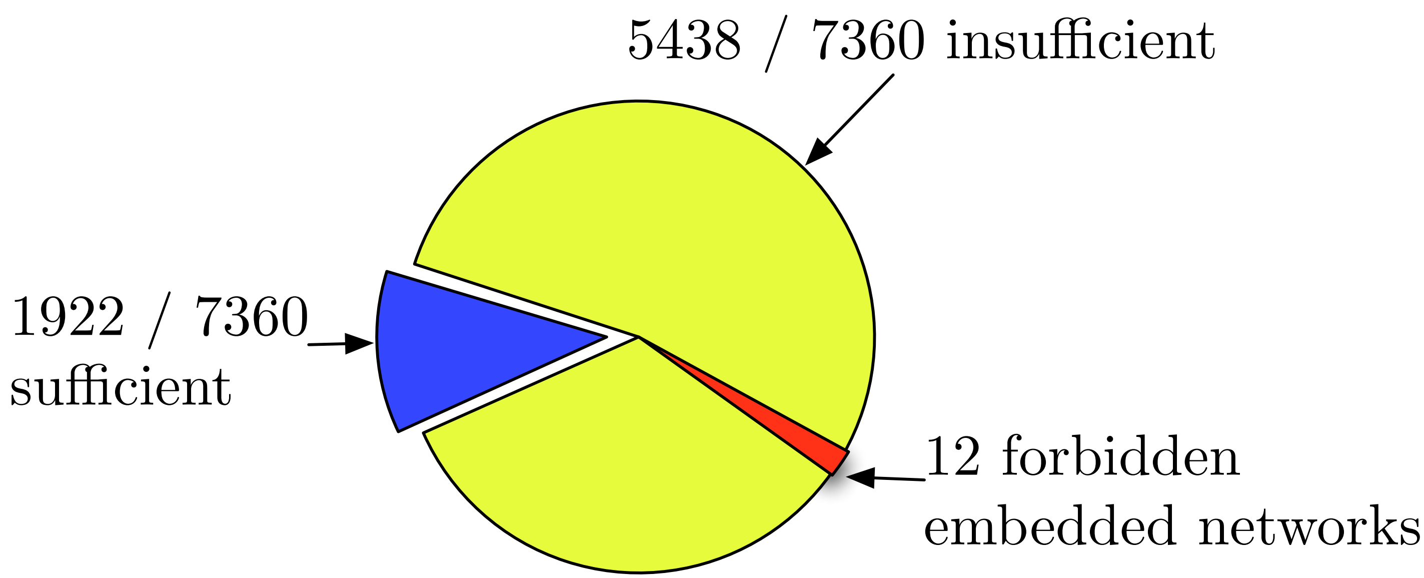


Computationally Enabled Research Agenda – Hierarchy: Embedding Operators



- Rate region (bound) of embedded network can be directly obtained from rate region (bound) of parent network.
- Insufficiency of class of codes of small $\Longrightarrow$ insufficiency of class of codes of big. (*forbidden network minor*)

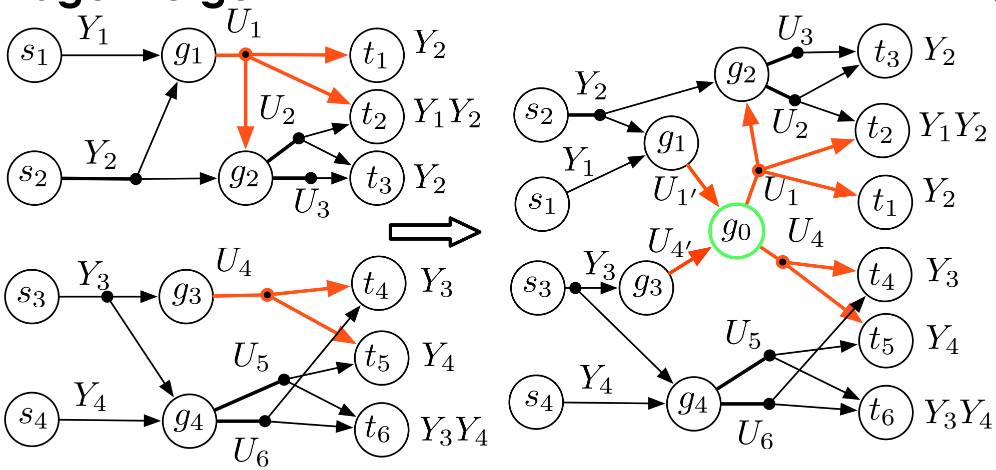
Computationally Enabled Research Agenda – Hierarchy: Embedding Operators



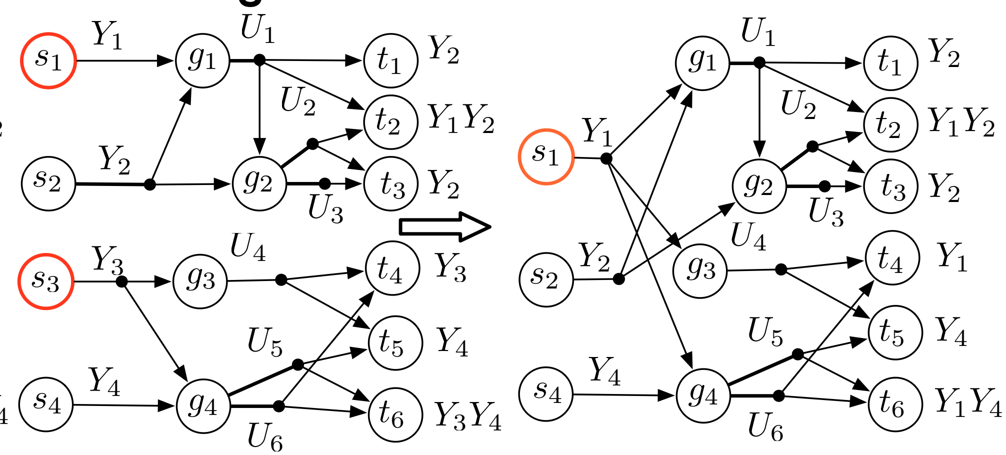
- First database: 5438 canonical MDCS for which scalar binary codes are insufficient can be boiled down to 12 forbidden minor networks.

Computationally Enabled Research Agenda – Hierarchy: Combination Operators

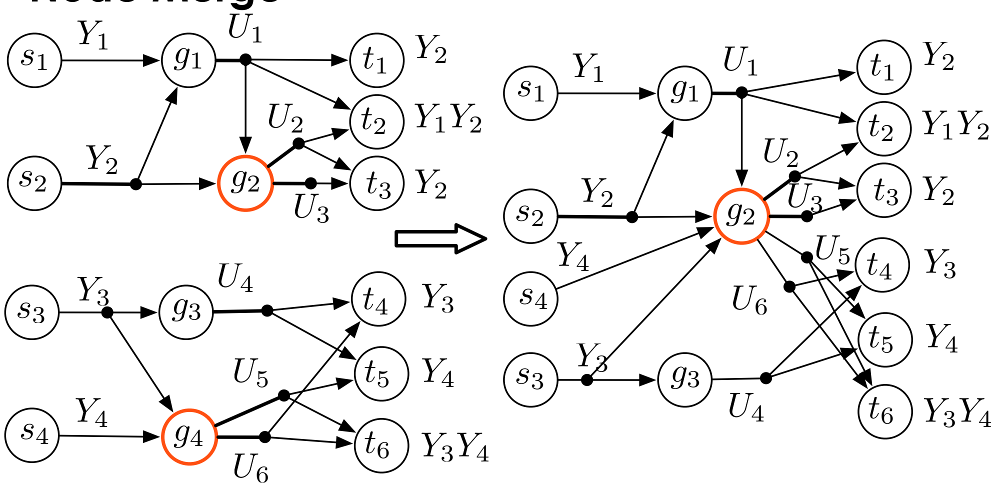
Edge Merge



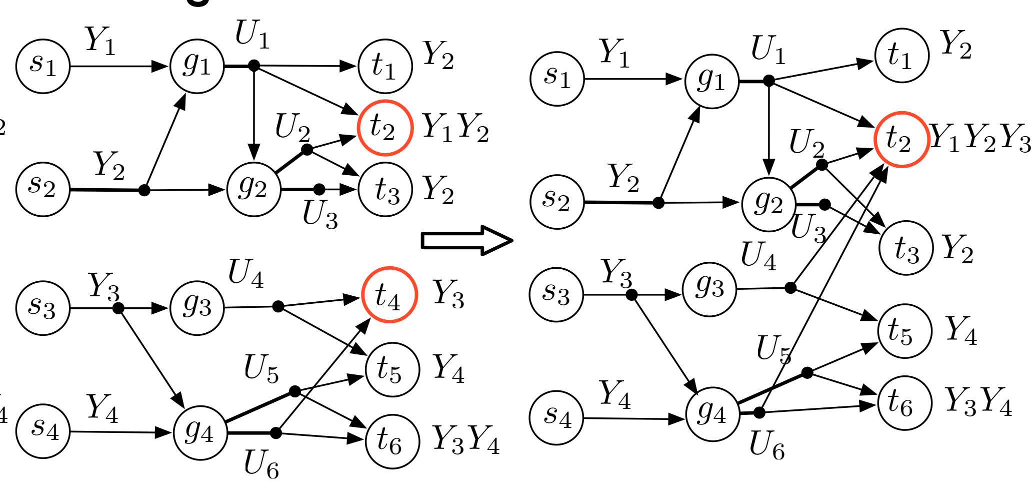
Source Merge



Node Merge



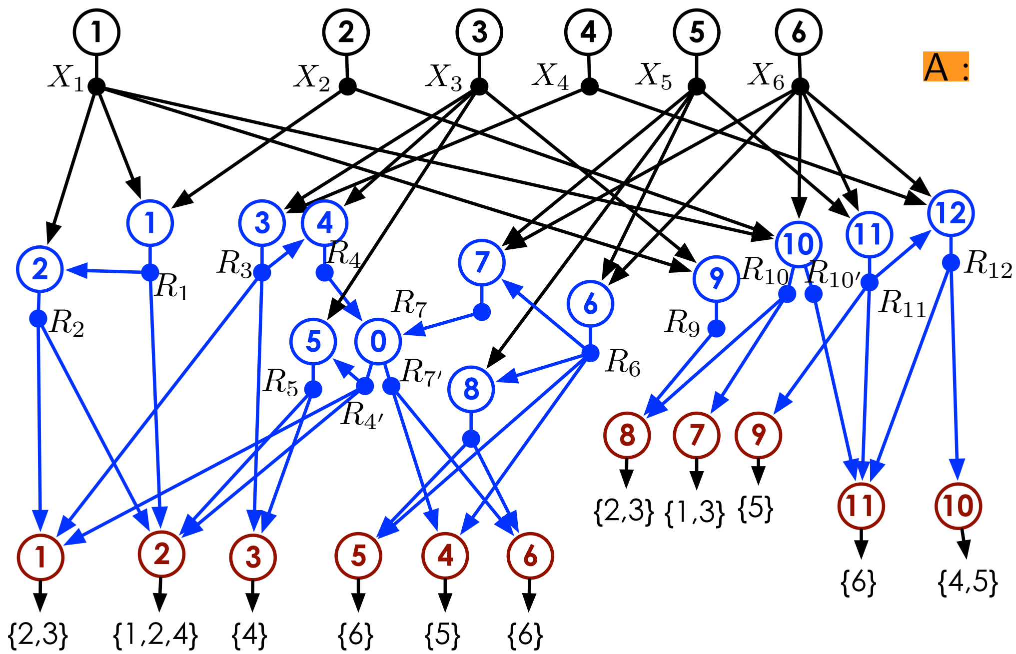
Sink Merge



- rate region of big directly expressible from rate regions of smalls

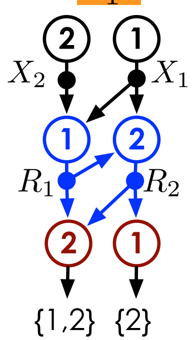
Computationally Enabled Research Agenda – Hierarchy: Combination Operators

A :



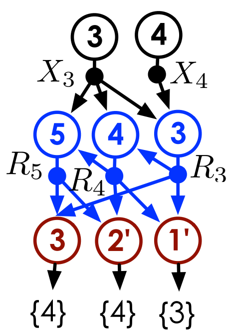
$$
\begin{aligned}
R_i &\geq H(X_2), i = 1, 2 \\
R_1 + R_2 &\geq H(X_1) + H(X_2) \\
R_i &\geq H(X_4), i = 3, 4, 4' \\
R_3 + R_i &\geq H(X_3) + H(X_4), i = 4, 4' \\
R_3 + R_i + R_5 &\geq H(X_3) + 2H(X_4), i = 4, 4' \\
R_6 &\geq H(X_6) \\
R_i + R_8 &\geq H(X_6), i = 7, 7' \\
R_6 + R_i &\geq H(X_5) + H(X_6), i = 7, 7' \\
R_6 + R_i + 2R_8 &\geq H(X_5) + 2H(X_6), i = 7, 7' \\
R_9 &\geq H(X_3) \\
R_{10} &\geq \sum_{i=1}^{3} H(X_i) \\
R_{11} &\geq H(X_5) \\
R_{12} &\geq H(X_4) + H(X_5) \\
R_{10'} + R_{11} + R_{12} &\geq H(X_4) + 2H(X_5) + H(X_6)
\end{aligned}
$$

A_1 :



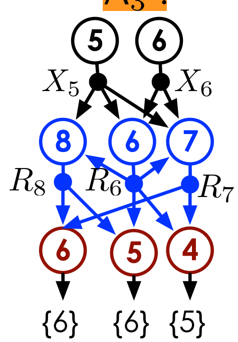
$R_1 \geq H(X_2)$
$R_2 \geq H(X_2)$
$R_1 + R_2 \geq H(X_1) + H(X_2)$

A_2 :



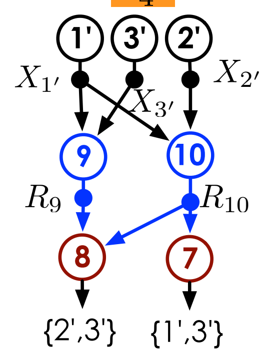
$R_i \geq H(X_4), i = 3, 4$
$R_3 + R_4 \geq H(X_3) + H(X_4)$
$R_3 + R_4 + 2R_5 \geq H(X_3) + 2H(X_4)$

A_3 :



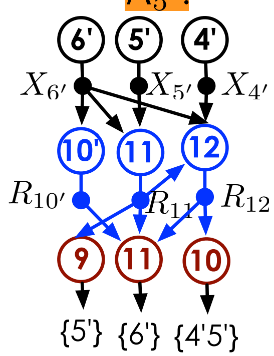
$R_6 \geq H(X_6)$
$R_7 + R_8 \geq H(X_6)$
$R_6 + R_7 \geq H(X_5) + H(X_6)$
$R_6 + R_7 + 2R_8 \geq H(X_5) + 2H(X_6)$

A_4 :



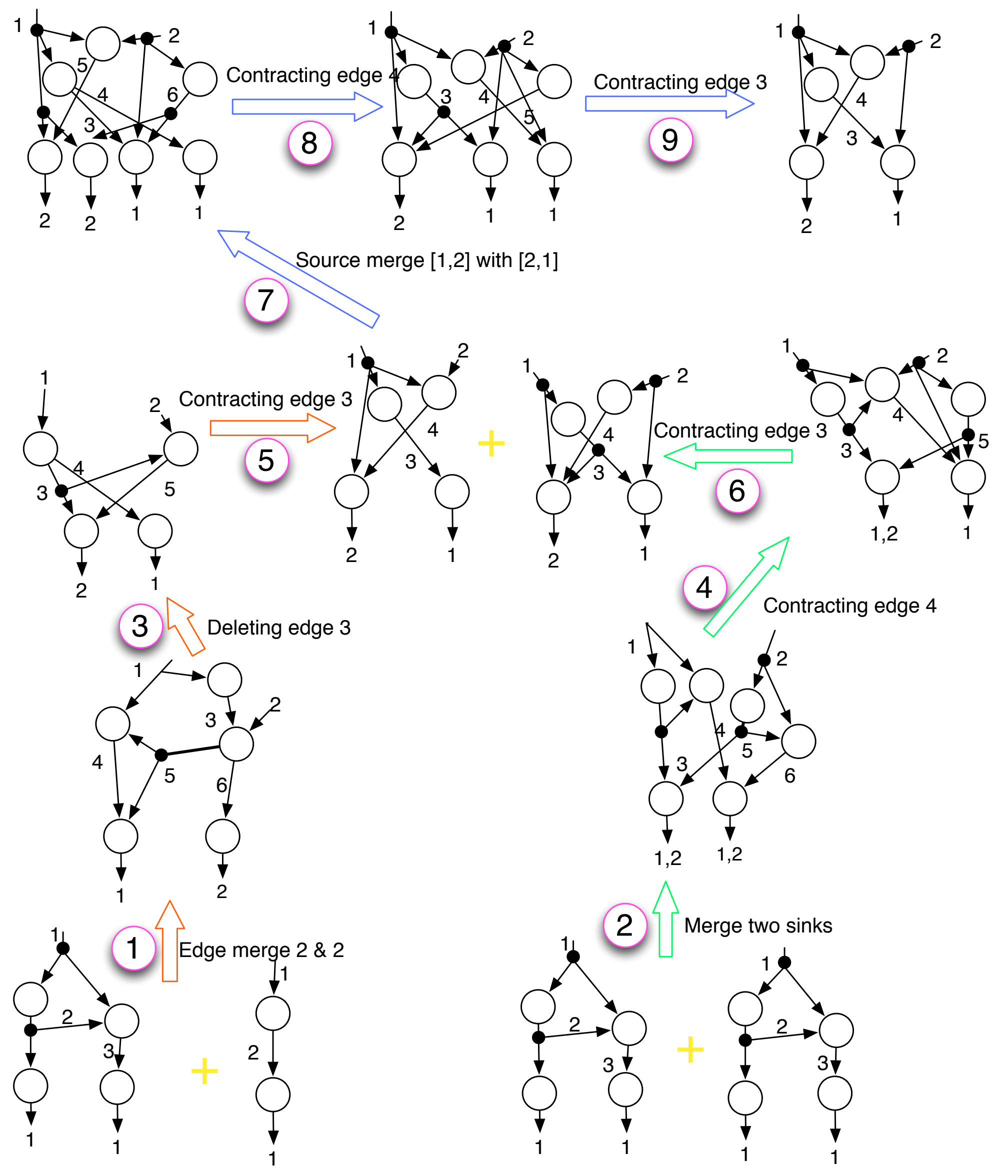
$R_9 \geq H(X_{3'})$
$R_{10} \geq H(X_{1'}) + H(X_{2'}) + H(X_{3'})$

A_5 :



$R_{11} \geq H(X_{5'})$
$R_{12} \geq H(X_{4'}) + H(X_{5'})$
$R_{10'} + R_{11} + R_{12} \geq H(X_{4'}) + 2H(X_{5'}) + H(X_{6'})$

Computationally Enabled Research Agenda – Hierarchy: Operator Concatenation



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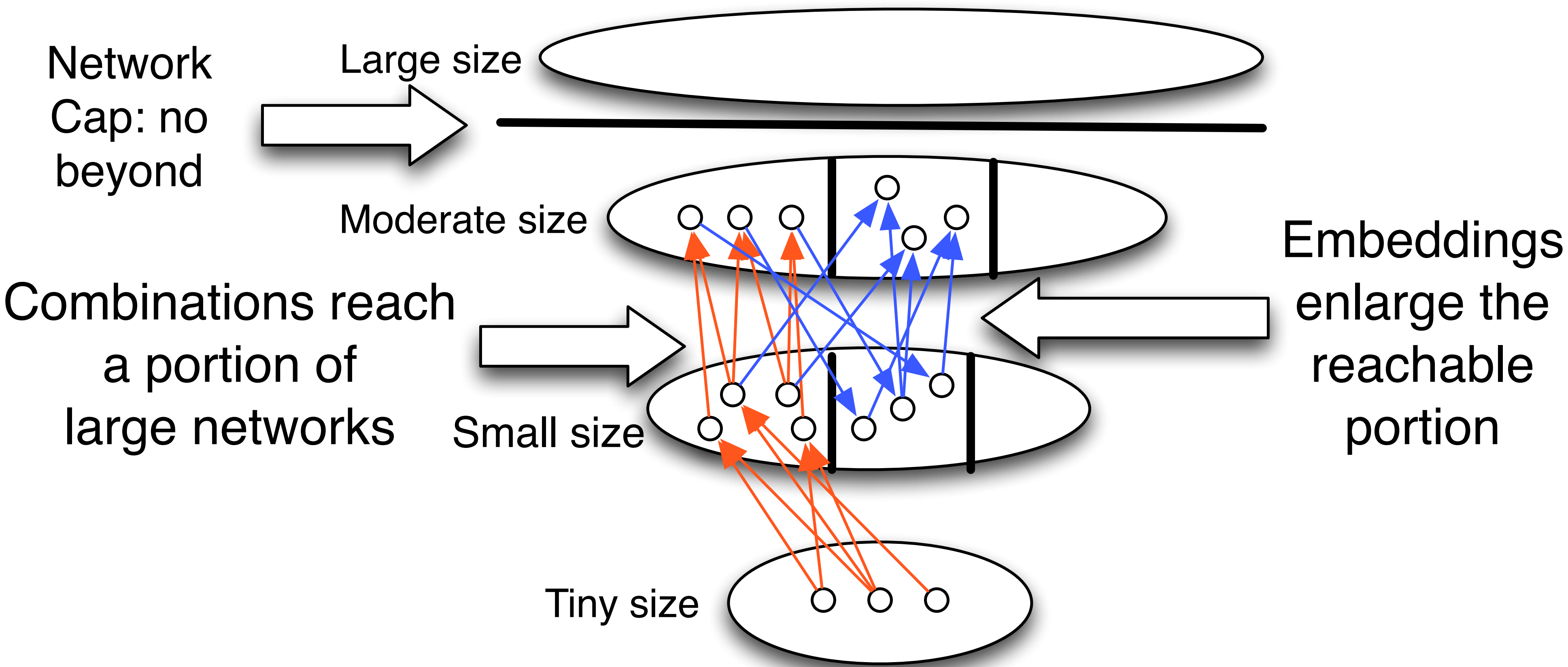

Use operators *together* to get RR for big networks. Partial Network Closure.

Computationally Enabled Research Agenda – Hierarchy: Operator Concatenation

Start with the single $(1,1)$, single $(2,1)$, and the four $(1,2)$ networks; These 6 tiny networks can generate new 11635 networks w/ small cap!

size\cap	combination operators only			embedding and combinations		
	(3,3)	(3,4)	(4,4)	(3,3)	(3,4)	(4,4)
(1,3)	4	4	4	4	4	4
(1,4)	0	10	10	0	10	10
(2,2)	3	3	3	8	15	16
(2,3)	13	16	16	30	131	155
(2,4)	0	97	101	0	516	648
(3,2)	2	3	2	4	10	11
(3,3)	24	24	24	42	353	833
(3,4)	0	135	135	0	2361	5481
(4,2)	0	0	3	0	0	3
(4,3)	0	0	17	0	0	44
(4,4)	0	0	253	0	0	4430
all	46	292	568	88	3400	11635

Computationally Enabled Research Agenda – Hierarchy: Operator Concatenation

With the increase of cap size, number of new networks increases!

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Embedding operations are important in the process!

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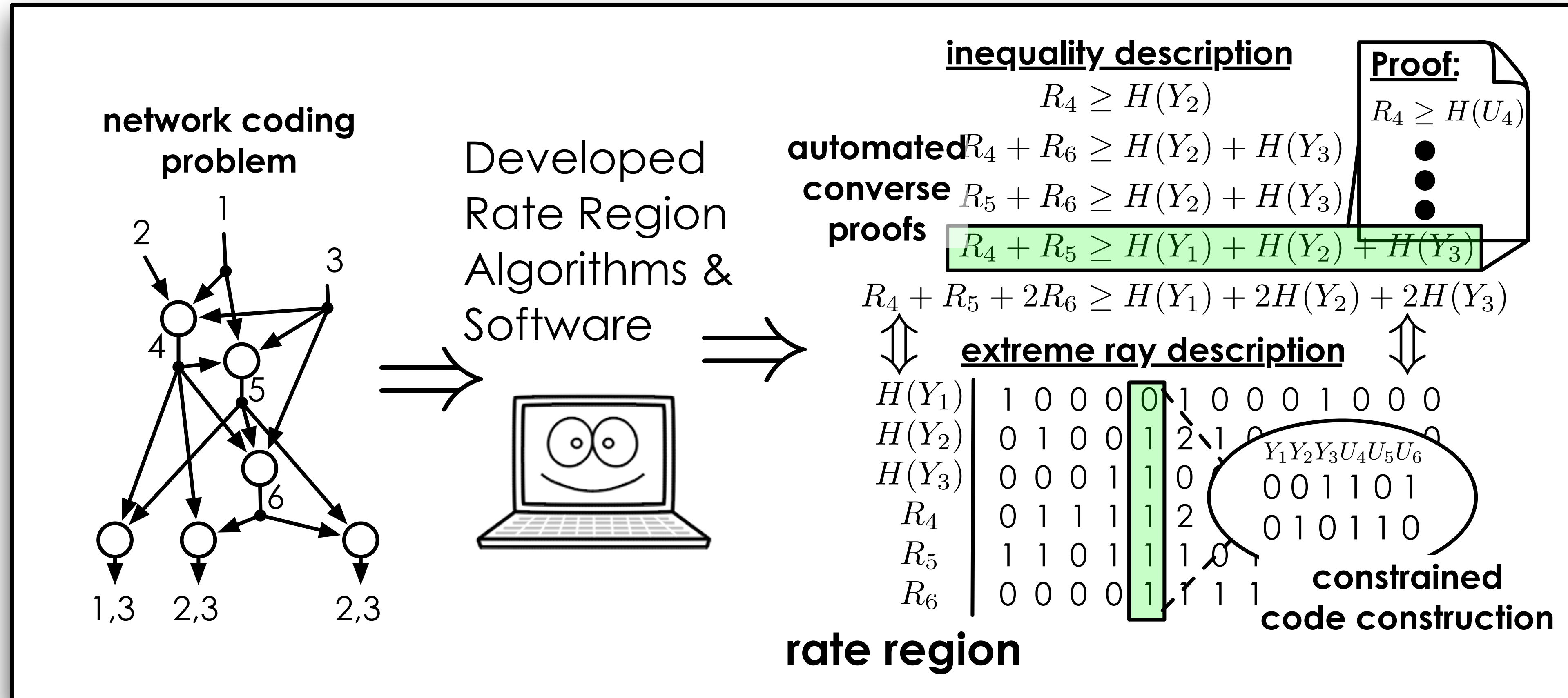
Propose a new rate region combination operator for connecting multiple sinks to multiple sources based on common information

What is a network coding capacity region?



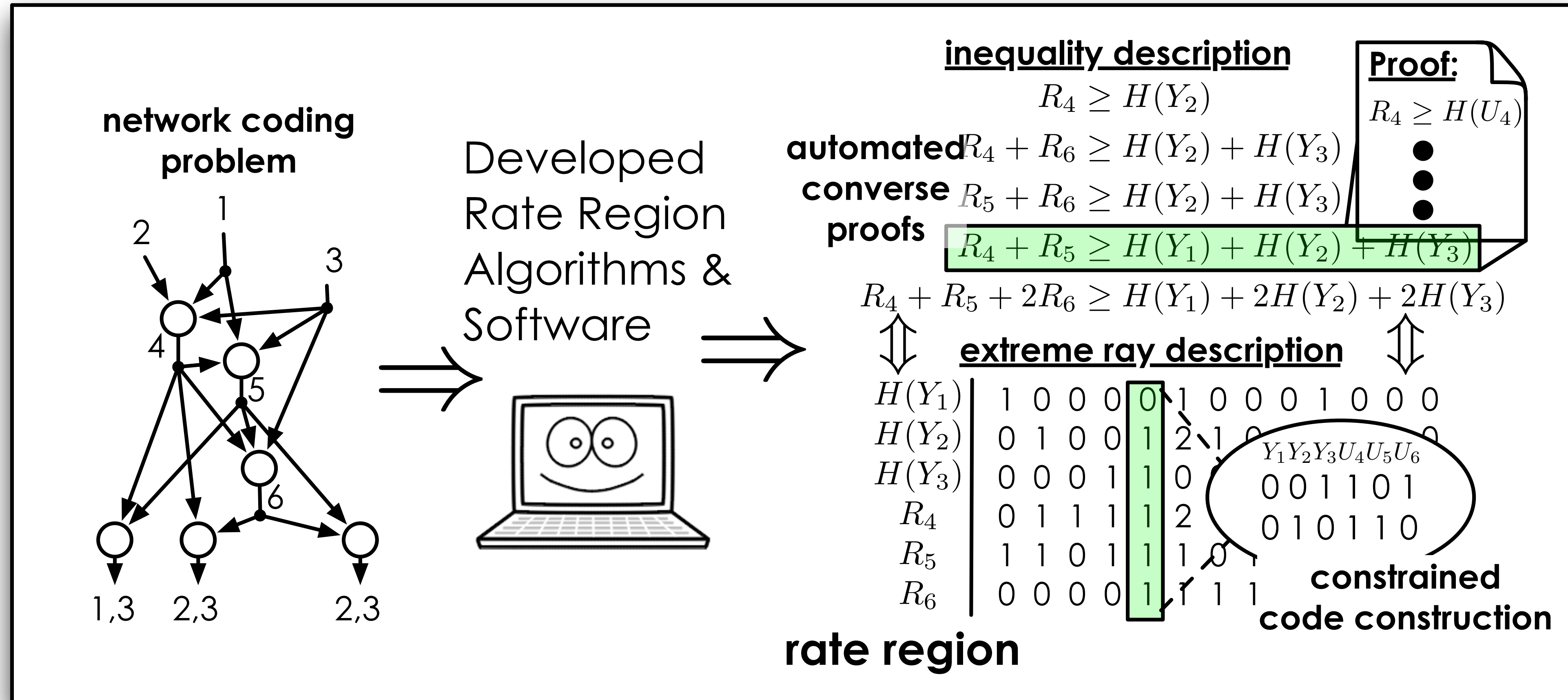
Substitutes outer/inner bounds to Γ_N^* into Yan, Yeung, Zhang '12

$$\mathcal{R}_* = \mathrm{proj}_{R_{\mathcal{E}}, [H(Y_s)|s \in \mathcal{S}]}(\overline{\mathrm{con}(\Gamma_{\mathrm{N}}^* \cap \mathcal{L}_{123})} \cap \mathcal{L}_{45}) \tag{7}$$

where $\mathcal{L}_{123} := \left\{ \mathbf{h} \middle| h_{Y_{\mathcal{S}}} = \Sigma_{s \in \mathcal{S}} h_{Y_s},\ h_{X_{\mathsf{Out}(i)}|X_{\mathrm{In}(i)}} = 0 \right\}$ and

$\mathcal{L}_{45} := \{(\mathbf{h}^T, \mathbf{R}^T)^T \in \mathbb{R}_+^{2^N - 1 + |\mathcal{E}|} : R_e \geq h_{U_e}, e \in \mathcal{E},\ h_{Y_{\beta(t)}|U_{\mathsf{In}(t)}} = 0\}$

What is a network coding capacity region?



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Notions of Symmetry – Formalize via Groups

1. Symmetries of $\Gamma_N \cap \mathcal{L}_{\mathsf{A}}$ where $\mathcal{L}_{\mathsf{A}} = \mathcal{L}_{123} \cap \mathcal{L}_{45}$
   (a) Symmetries of polyhedral cones
   (b) Symmetries of Γ_N
   (c) Symmetries of $\Gamma_N \cap \mathcal{L}_{\mathsf{A}}$
   (d) Application – reduces the complexity of proving the converse
2. Symmetries between different network coding problem instances
3. Symmetries among network codes
   (a) Symmetries among linear codes
   (b) Application to proving matched inner bound
   (c) Symmetries among nonlinear codes

Our Related Journal Submissions & Software

[1] Congduan Li, Steven Weber, and John Walsh, "On Multi-source Networks: Enumeration, Rate Region Computation, and Hierarchy," *IEEE Trans. Inform. Theory*, vol. 63, no. 11, pp. 7283–7303, Nov. 2017. [Online]. Available: https://doi.org/10.1109/TIT.2017.2745620

[2] Congduan Li, Steven Weber, and John MacLaren Walsh, "On Multilevel Diversity Coding Systems," *IEEE Trans. Inform. Theory*, vol. 63, no. 1, pp. 230–251, Jan. 2017. [Online]. Available: https://doi.org/10.1109/TIT.2016.2628791

[3] Jayant Apte and John MacLaren Walsh, "Explicit Polyhedral Bounds on Network Coding Rate Regions via Entropy Function Region: Algorithms, Symmetry, and Computation," *IEEE Trans. Inform. Theory*, 2016, Submitted July 22, 2016, revised July 6, 2017. [Online]. Available: http://arxiv.org/abs/1607.06833

[4] ——, "Constrained Linear Representability of Polymatroids and Algorithms for Computing Achievability Proofs in Network Coding," *IEEE Trans. Inform. Theory*, Submitted May 15, 2016. [Online]. Available: http://arxiv.org/abs/1605.04598

[5] Yunshu Liu and John MacLaren Walsh, "Mapping the Region of Entropic Vectors with Support Enumeration & Information Geometry," *IEEE Trans. Inform. Theory*, Submitted December 08, 2015. [Online]. Available: http://arxiv.org/abs/1512.03324

[6] Congduan Li and John MacLaren Walsh, "Network Coding Rate Region Calculation." [Online]. Available: http://www.ece.drexel.edu/walsh/aspitrg/software.html

[7] ——, "Network Enumeration and Hierarchy." [Online]. Available: http://www.ece.drexel.edu/walsh/aspitrg/software.html

[8] Jayant Apte and John MacLaren Walsh, "Information Theoretic Achievability Prover." [Online]. Available: https://github.com/jayant91089/itap

[9] ——, "Information Theoretic Converse Prover." [Online]. Available: https://github.com/jayant91089/itcp