Quantifying and Achieving the Capacity of Wireless 1-Hop Network Coding — A Code-Alignment-Based Approach

Chih-Chun Wang, Purdue University

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Joint work with Y. Charlie Hu (Purdue), Ness B. Shroff (The OSU), Dimitrios Koutsonikolas, Abdallah Khreishah.

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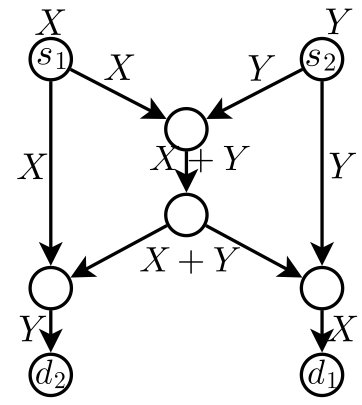
The Benefits of Network Coding

- Network Coding (NC) has been formulated for 10+ years. [Ahlswede *et al.* 98].
- The famous “butterfly" network:



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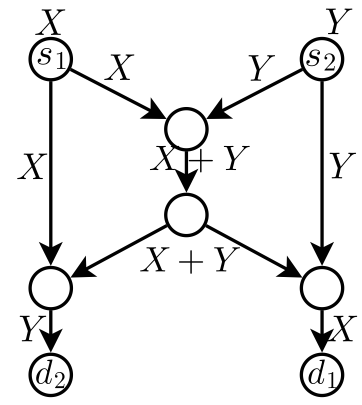

- Many promised advantages:
  - Throughput,
  - Energy and power savings, [Cui *et al*. 08], [Goseling *et al*. 09], etc.
  - Security (cryptography) [Bhattad *et al*. 06], [Ngai *et al*. 09], etc.
  - Error correction [Ahlswede *et al*. 09], [Silva *et al*. 08], etc.
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  - Speed up computation of the min-cuts and min-cut values [Wu *et al*. 06] [Wang *et al*. 09].
  - Storage [Wu 09], P2P [M. Wang *et al*. 07], etc.
  - 40+ papers in ISIT09; 20+ papers in INFOCOM09.

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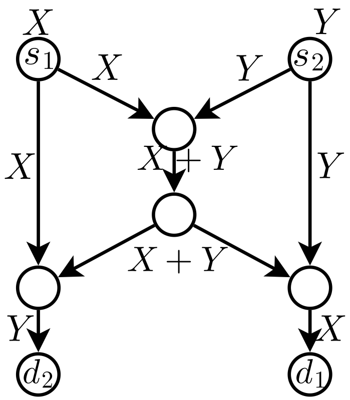

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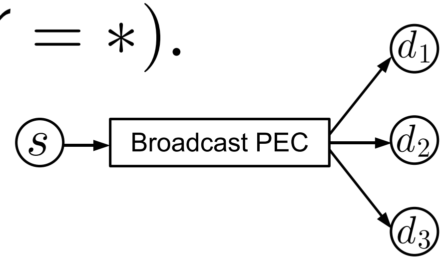
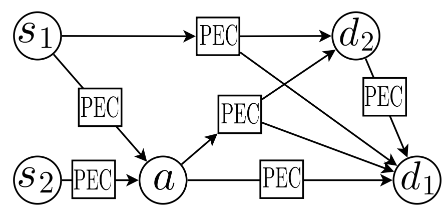
- Many different interpretations of NC:
  - Link-by-Link Forward Error Control [Ghaderi *et al.* 07],
  - Fountain codes [Luby 02],
  - Network-wide multiple-description codes,
  - Cooperative wireless networks [Médard 09],
  - Generalization of the store-&-forward policy [Yeung 06],
  - ...

Yet Another Definition . . .

- A conservative Packet Erasure Channel (PEC) abstraction of packet transmission that is independent from the PHY layer schemes:
  - Input: $X \in \mathsf{GF}(2^b)$ for some sufficiently large b.
  - Output: $Y \in \{X\} \cup \{*\}$ where "$*$" is the erasure symbol.
  - A packet X either arrives perfectly (with the help of CRC), or is considered as erasure and discarded. (No hybrid ARQ).
  - $P(Y = * | X = x) = P(Y = *)$.





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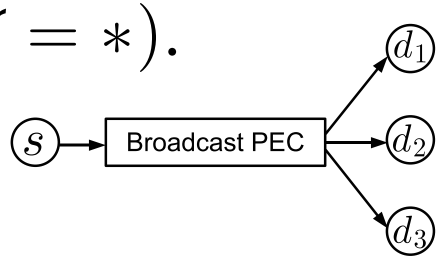

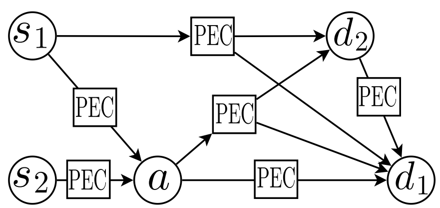

Network coding is ... the network information theory study (especially the achievability part) of a PEC-based network.

The Packet Erasure Channels

- The PEC-based abstraction may be too conservative:
  - Discard the corrupted packets completely, instead of using hybrid ARQ schemes.
  - On the other hand, empirically a wireless mesh network testbed has erasure probability 20%–70%.

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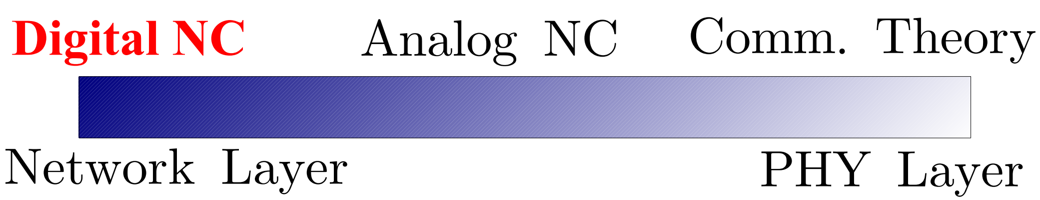
- 

- Wireless Networks (modeled as Packet Erasure Channels) + Throughput Analysis + Digital Network Coding

Wireless 1-Hop Intersession NC

- Intersession NC (INC): Coding over multiple unicast sessions.
- The COPE protocol — 2-hop relay networks [Katti *et al.* 06]

  4 transmissions w/o coding vs. 3 transmissions w. coding

  - r sends $[X + Y]$; d_1 decodes X by subtraction.
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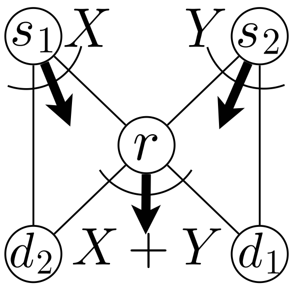

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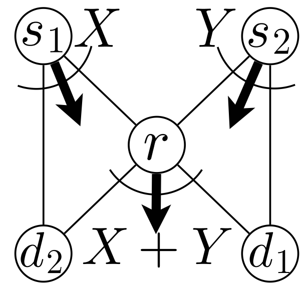

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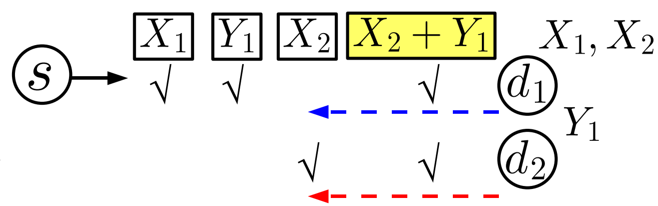

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  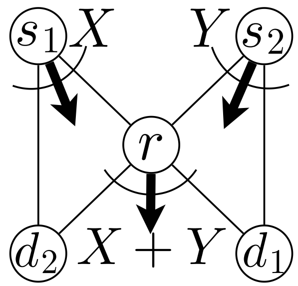

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  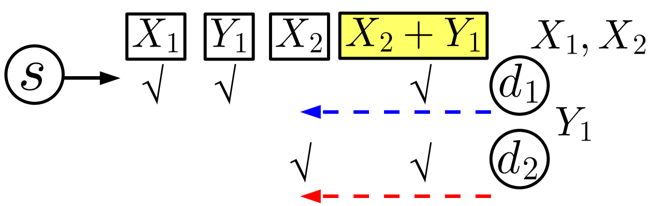

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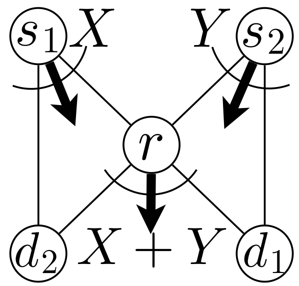

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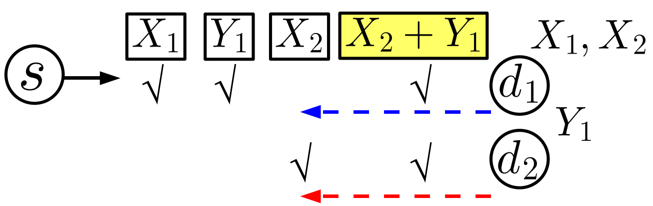
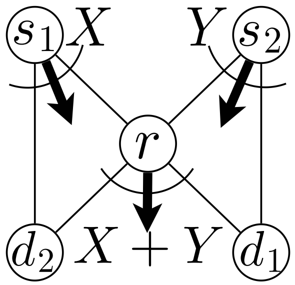


- INC is a hard problem. Ex: Pure random lin. NC does not work.
- Shannon capacities of 1-hop INC remain unknown. It becomes non-trivial for random broadcast PECs and $M>2$ sessions.

Main Theoretical Results

- The benefits of COPE follows from the message side information (MSI).



# of sessions	COPE-like Protocols (Broadcast PECs w. **MSI**)	Gaussian broadcast channels w. **MSI**
M=2	Full capacity region	Full capacity region [Wu 07]
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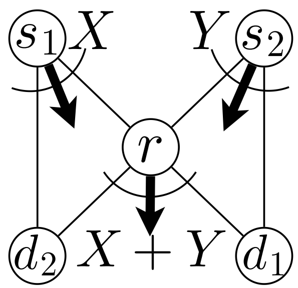

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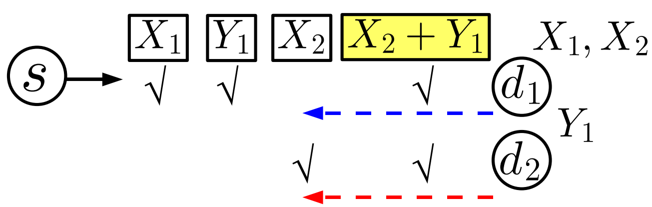
- The benefits of ER follows from the channel output feedback (COF).



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M=2	Full capacity region [Georgiadis et al. 09]	Outer and inner bounds [Ozarow 84]
M=3	Full capacity region	?
General M	(1) Capacity for fair systems; (2) Outer and inner bounds that meet numerically.	?

Part I: Quantifying and achieving the capacity of COPE-like protocols

Part II: Quantifying and achieving the capacity of ER-like protocols

Our Setting

- Memoryless broadcast PECs: Ex: A 1-to-2 PEC is governed by the success probabilities $p_{s\to 12}, p_{s\to 12^c}, p_{s\to 1^c2}, p_{s\to 1^c2^c}$.

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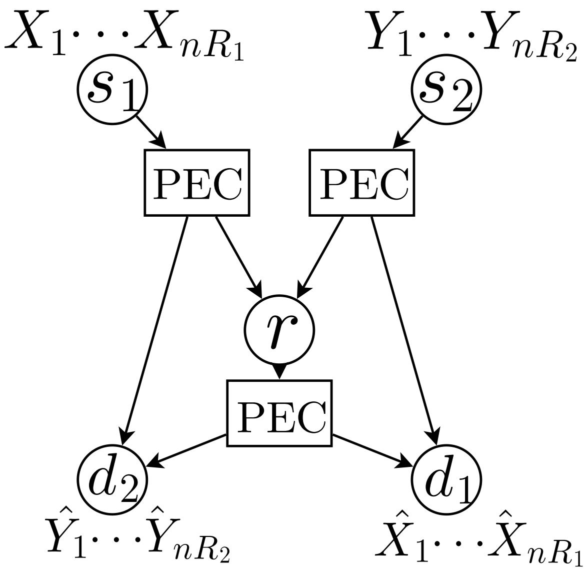

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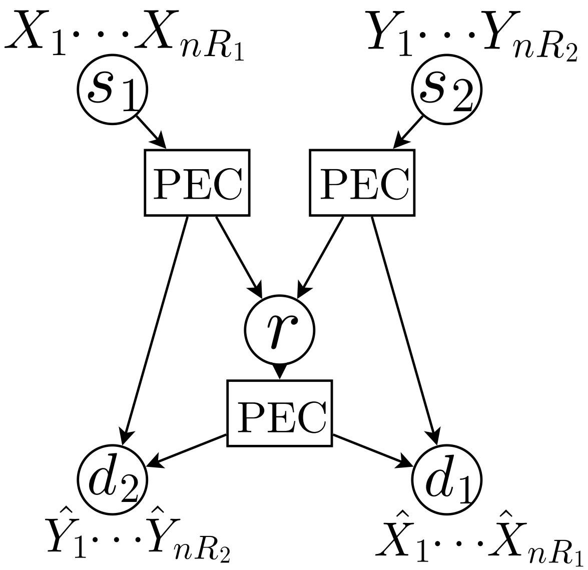

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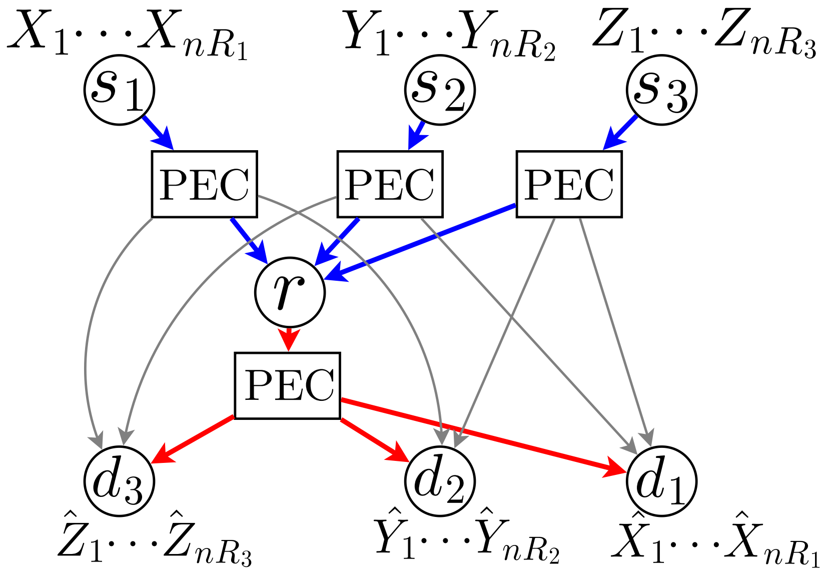

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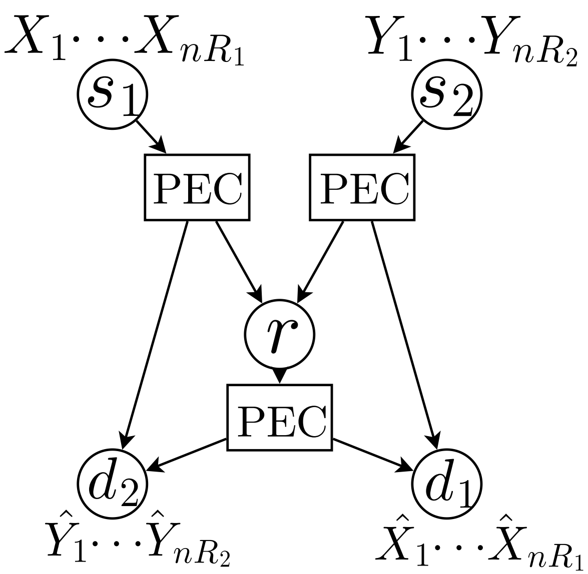

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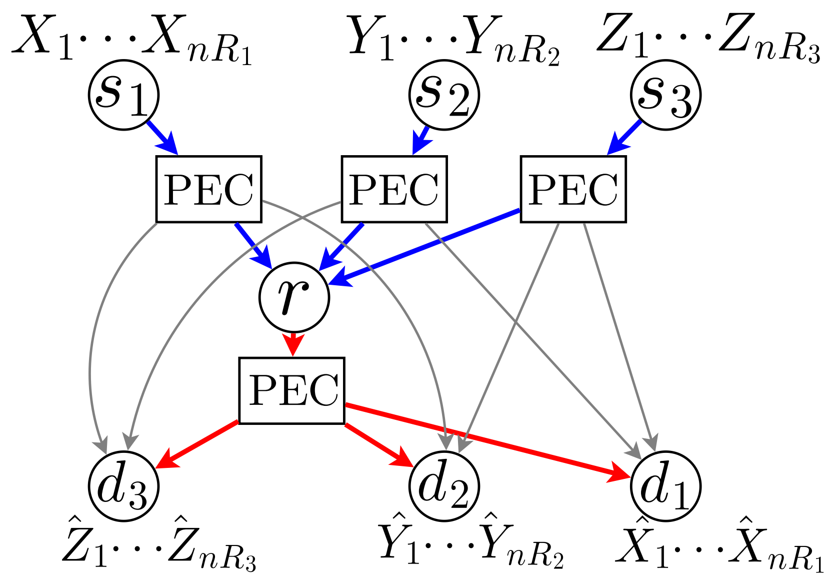

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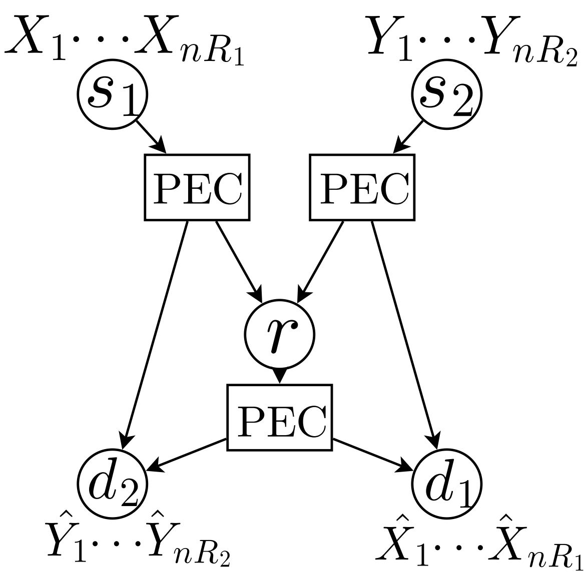

PEC parameters for $M = 2$:

Joint Prob.:

$$p_{s_1\to 2r}, p_{s_1\to 2r^c}, p_{s_1\to 2^c r}, p_{s_1\to 2^c r^c};$$
$$p_{s_2\to 1r}, p_{s_2\to 1r^c}, p_{s_2\to 1^c r}, p_{s_2\to 1^c r^c};$$
$$p_{r\to 12}, p_{r\to 12^c}, p_{r\to 1^c 2}, p_{r\to 1^c 2^c}.$$

Marginal Prob.:

$$p_{r;1} \stackrel{\Delta}{=} p_{r\to 12} + p_{r\to 12^c}$$

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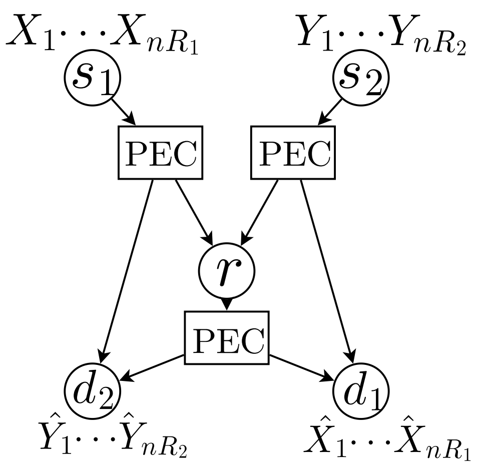

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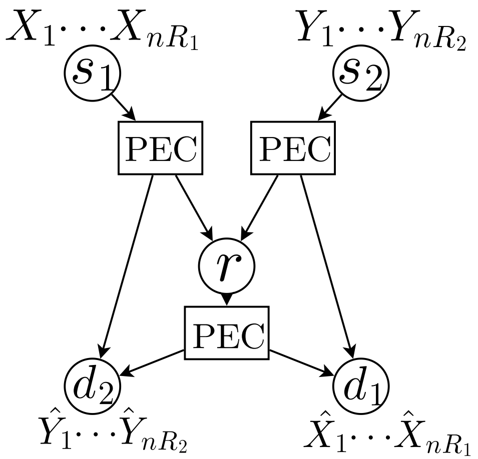

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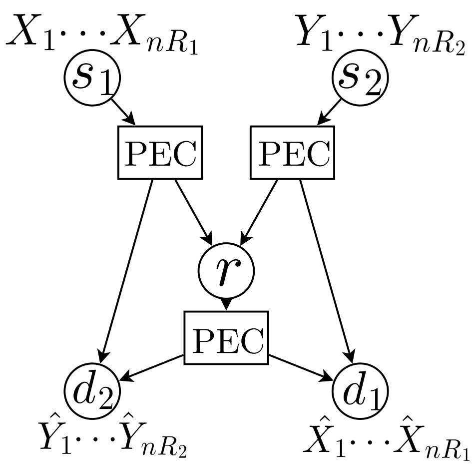

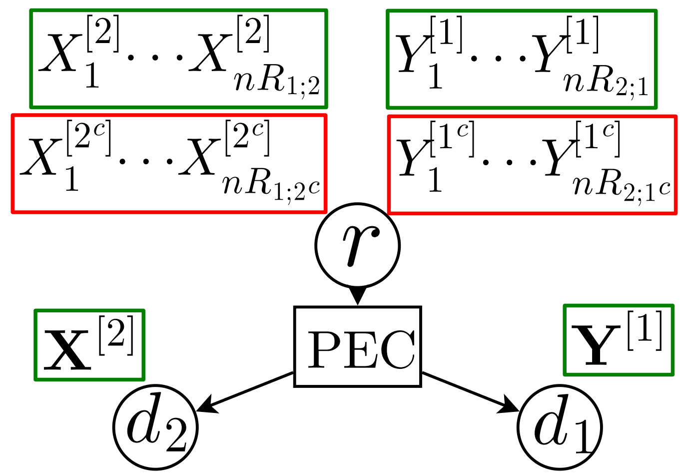

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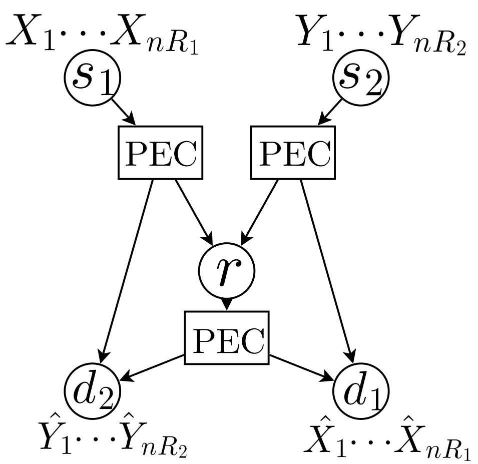
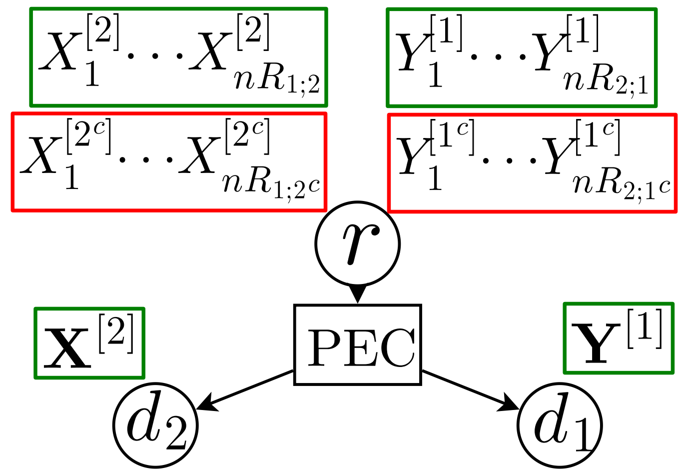
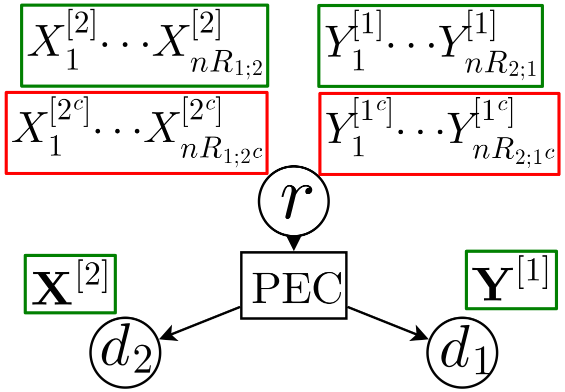




- No feedback is allowed during the transmission of the last n packets by relay r.

An Intuitive Argument



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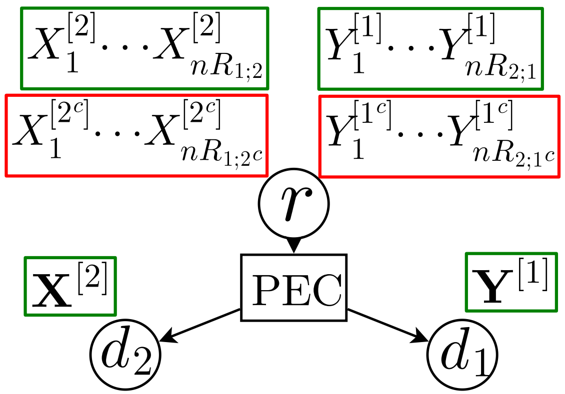

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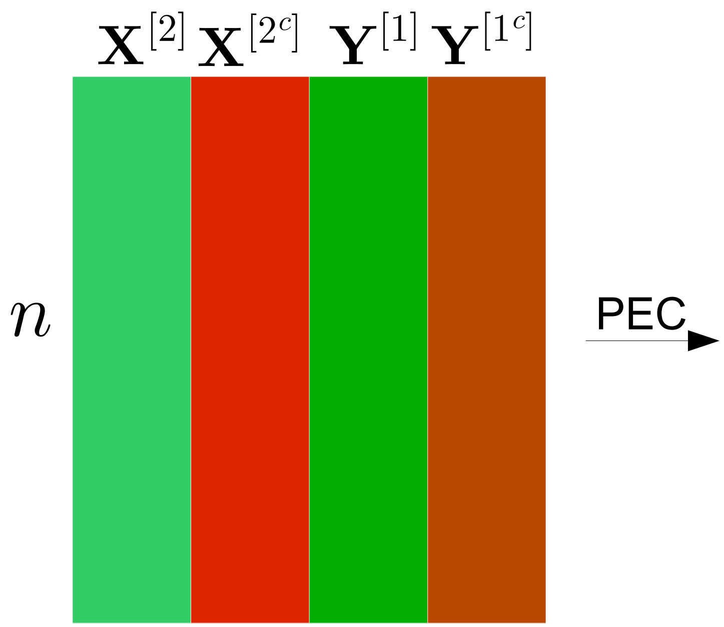

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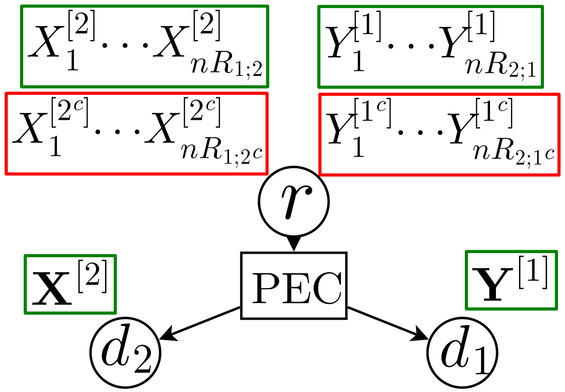

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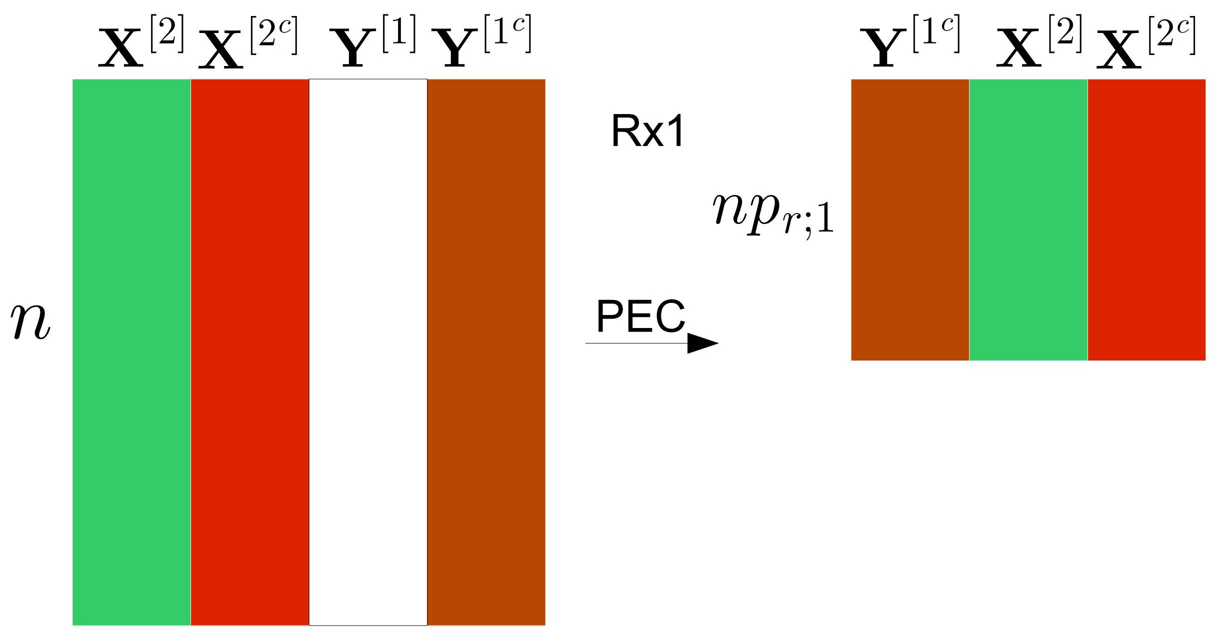

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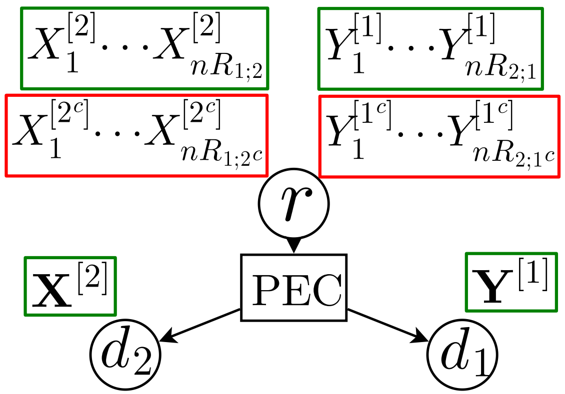

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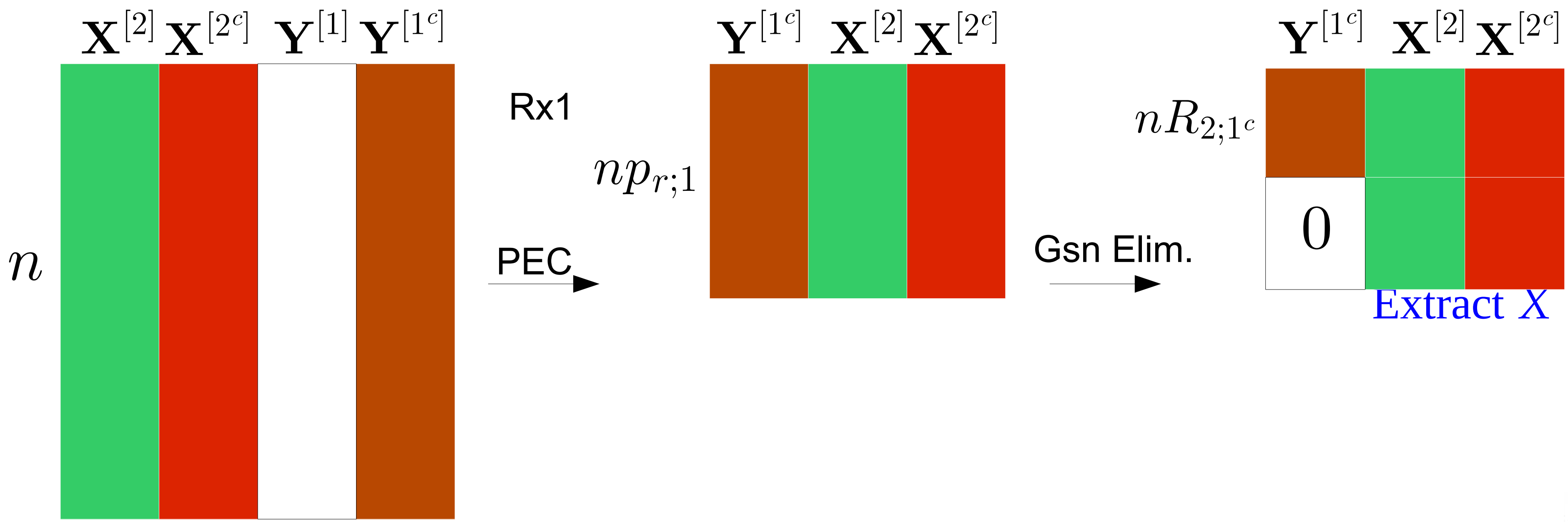

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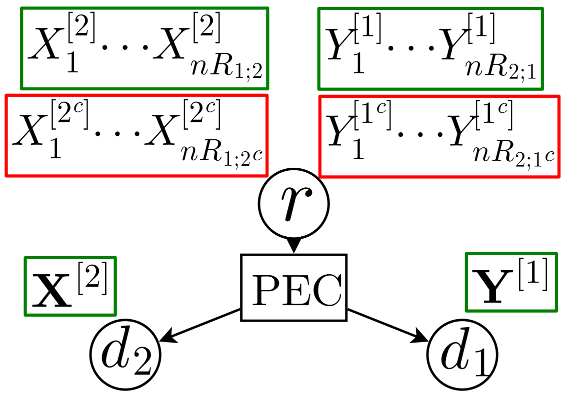

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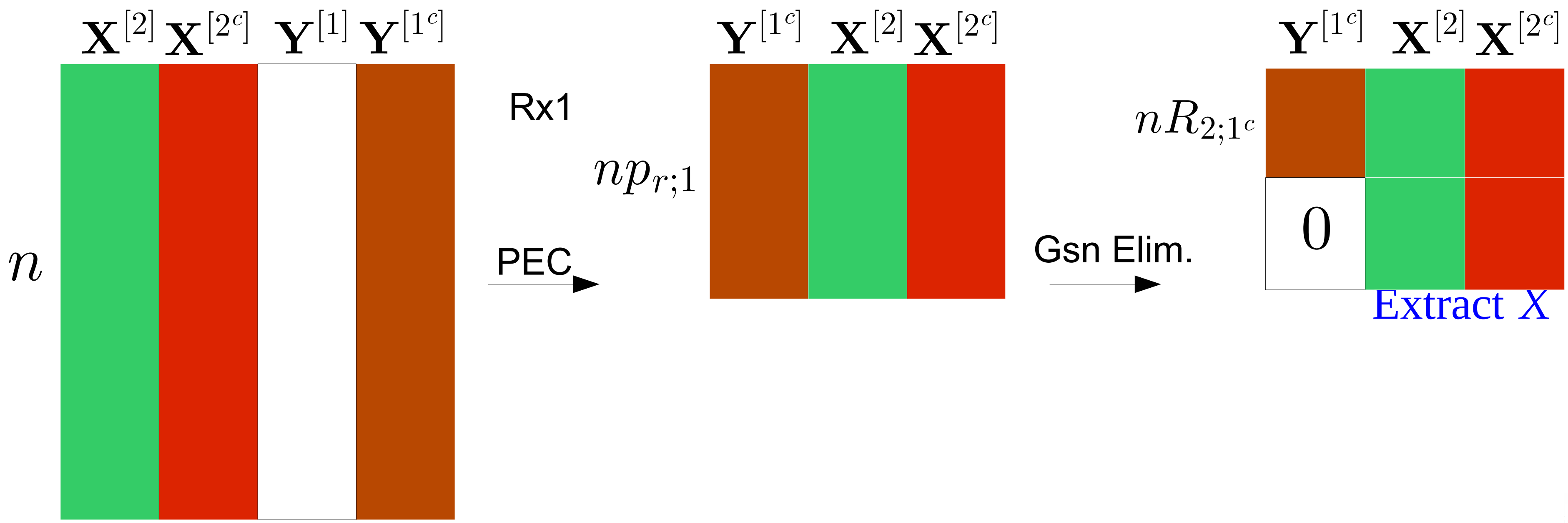

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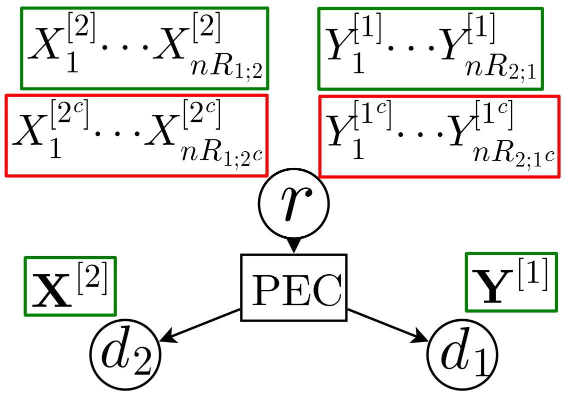

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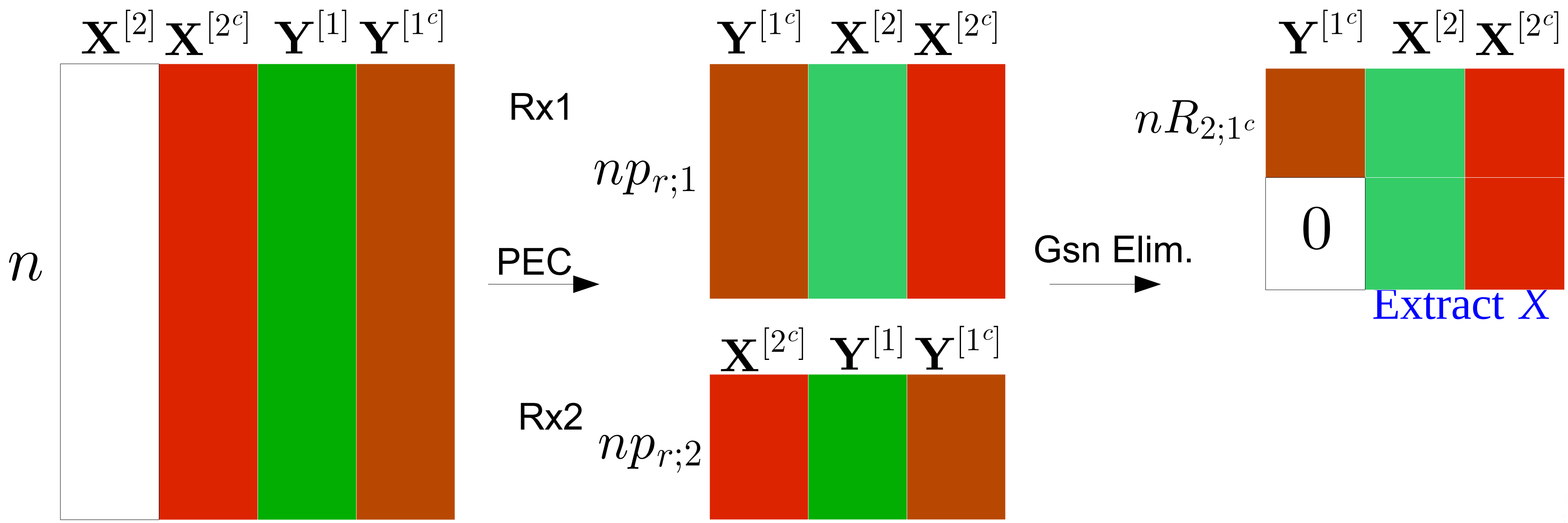

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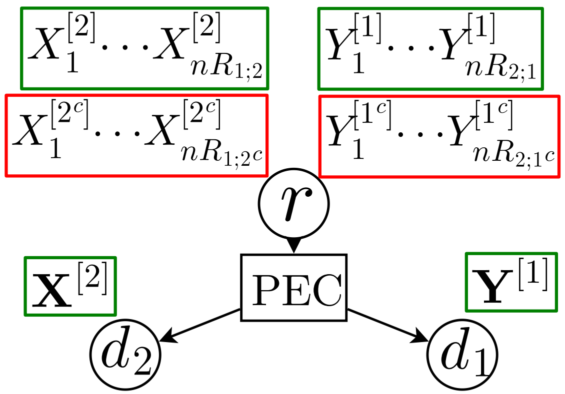
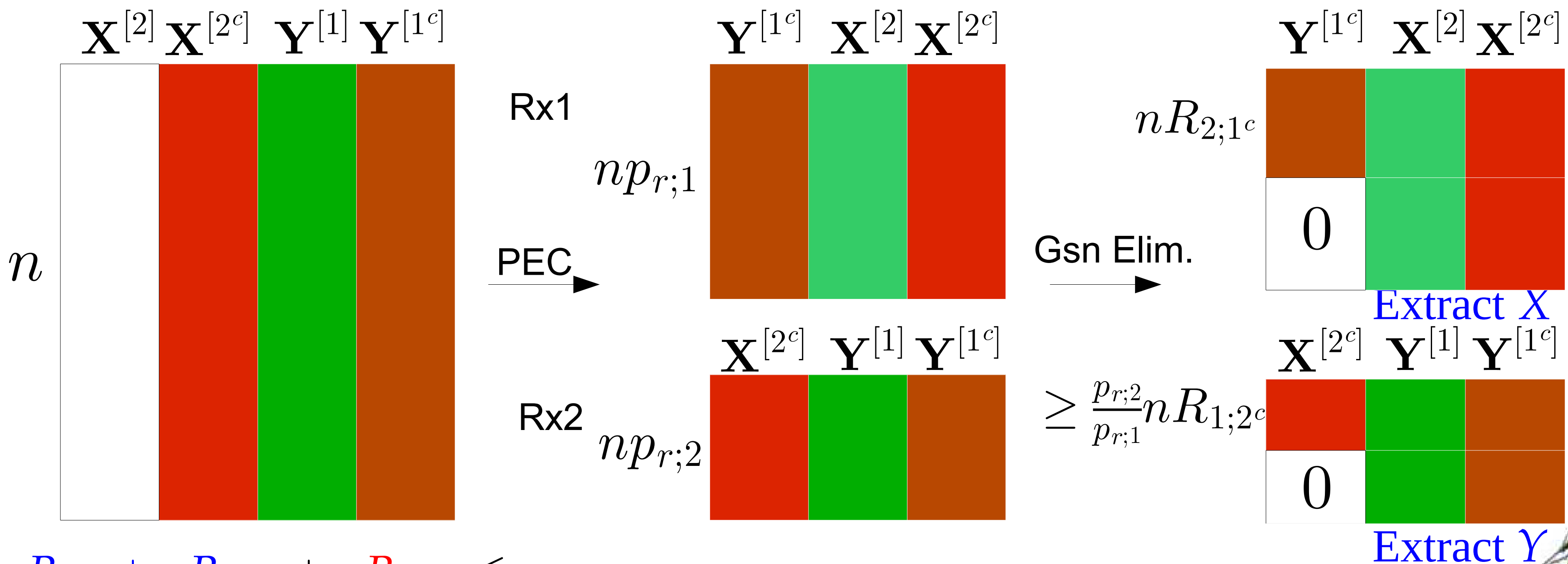


Without loss of generality, assume:

$p_{r;1} > p_{r;2}$.

For the sake of illustration, we first focus on linear codes.



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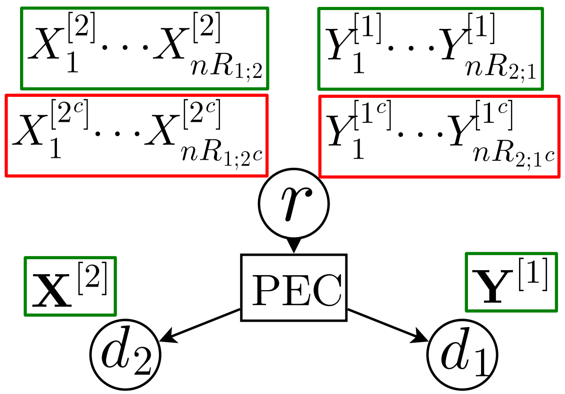

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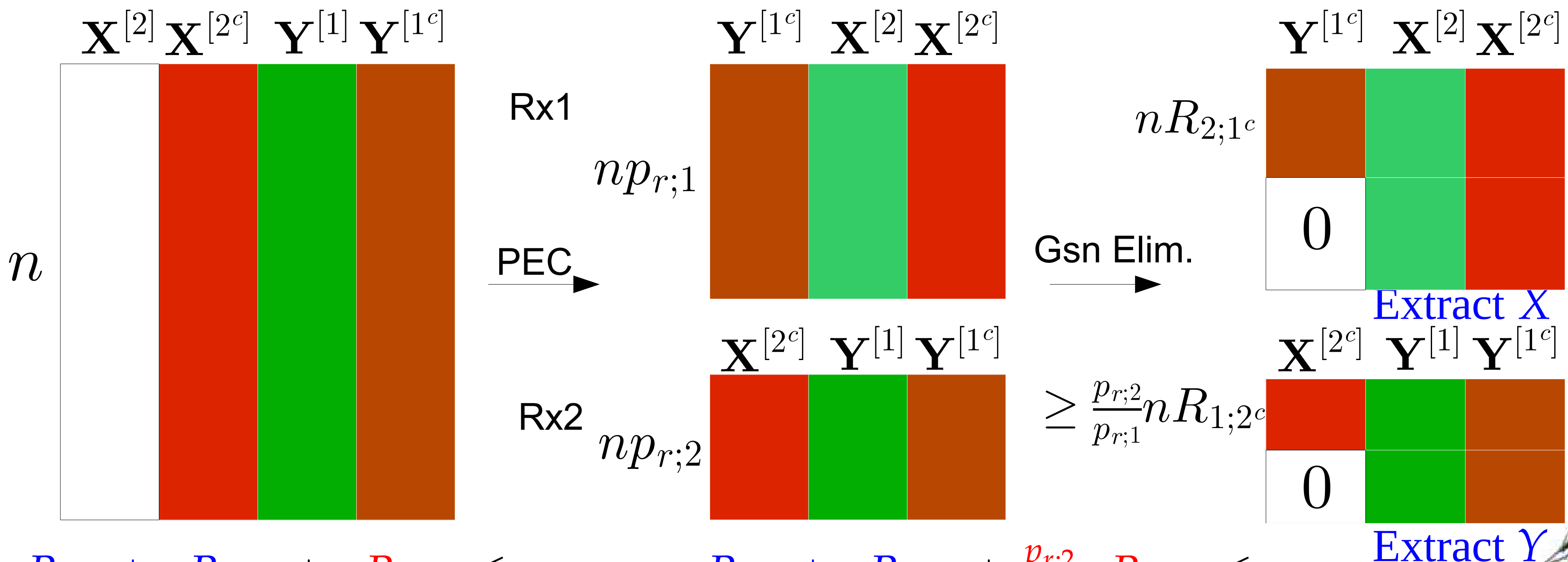

$$nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}, \quad nR_{2;1} + nR_{2;1^c} + \frac{p_{r;2}}{p_{r;1}} nR_{1;2^c} \leq np_{r;2}.$$

Continued

- The cap. outer bound, $M = 2$ users:

  main info $+$ minimal interference $\leq$ the overall available slots

  d_1's perspective: $nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}$

  d_2's perspective: $nR_{2;1} + nR_{2;1^c} + \frac{p_{r;2}}{p_{r;1}} nR_{1;2^c} \leq np_{r;2}$.

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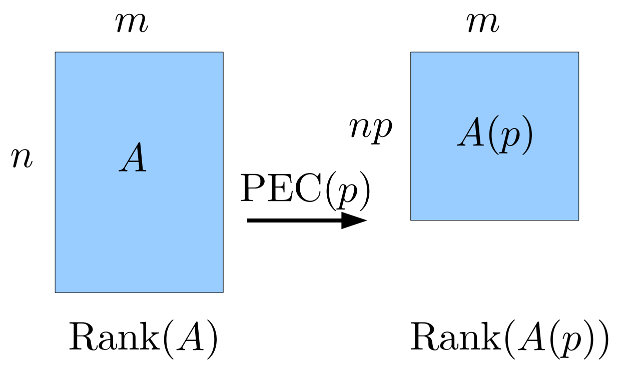
  The concavity of information transmission when using linear codes.



Continued

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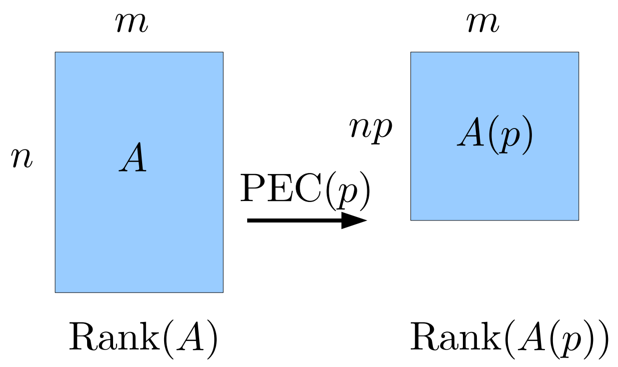
At least p fraction of $\text{Rank}(A)$ basis vectors of A will be passed to $A(p)$.

$\Rightarrow \text{Rank}(A(p)) \geq p \cdot \text{Rank}(A)$.



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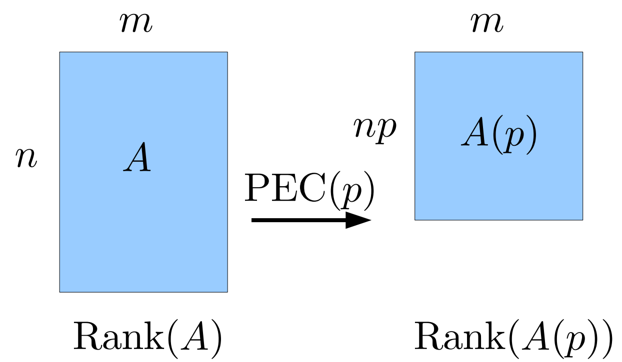

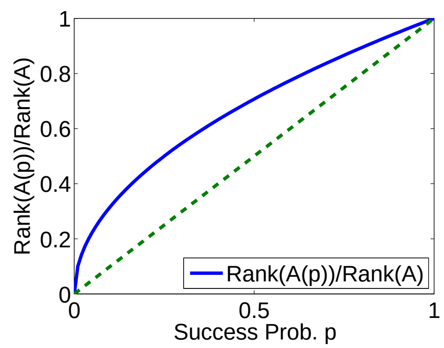

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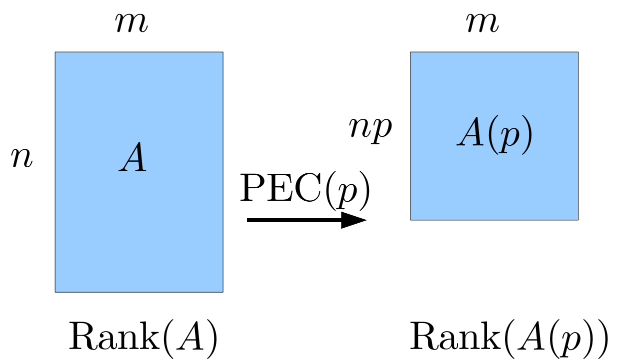

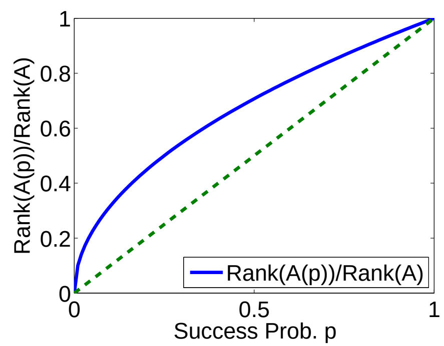

- When focusing on the mutual info. instead, the info. concavity argument can be generalized for non-linear codes [Wang, ISIT 10].

Cap. 2-User Brdcst PEC w. MSI

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d_1's perspective: $nR_{1;2} + nR_{1;2^c} + nR_{2;1^c} \leq np_{r;1}$

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The achievability: A 2-Stage coding scheme.

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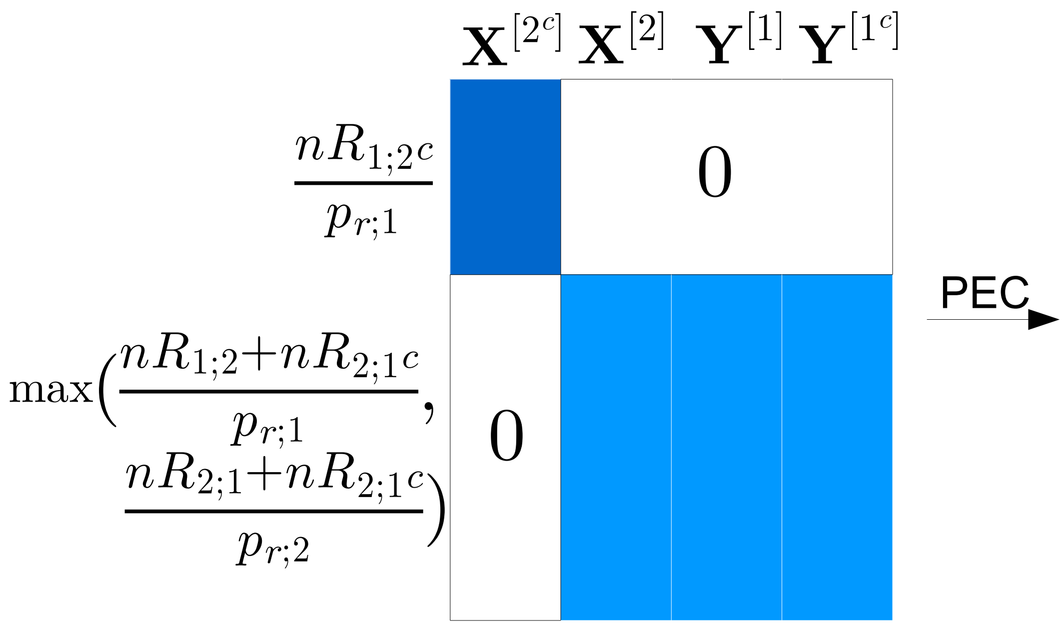

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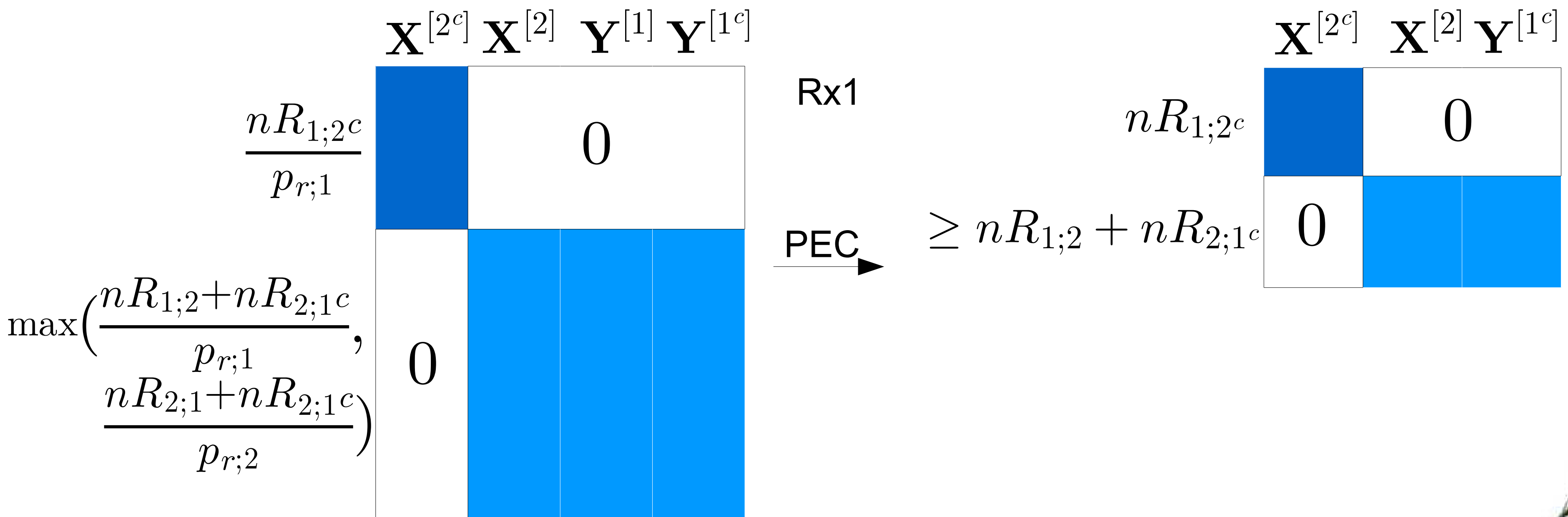

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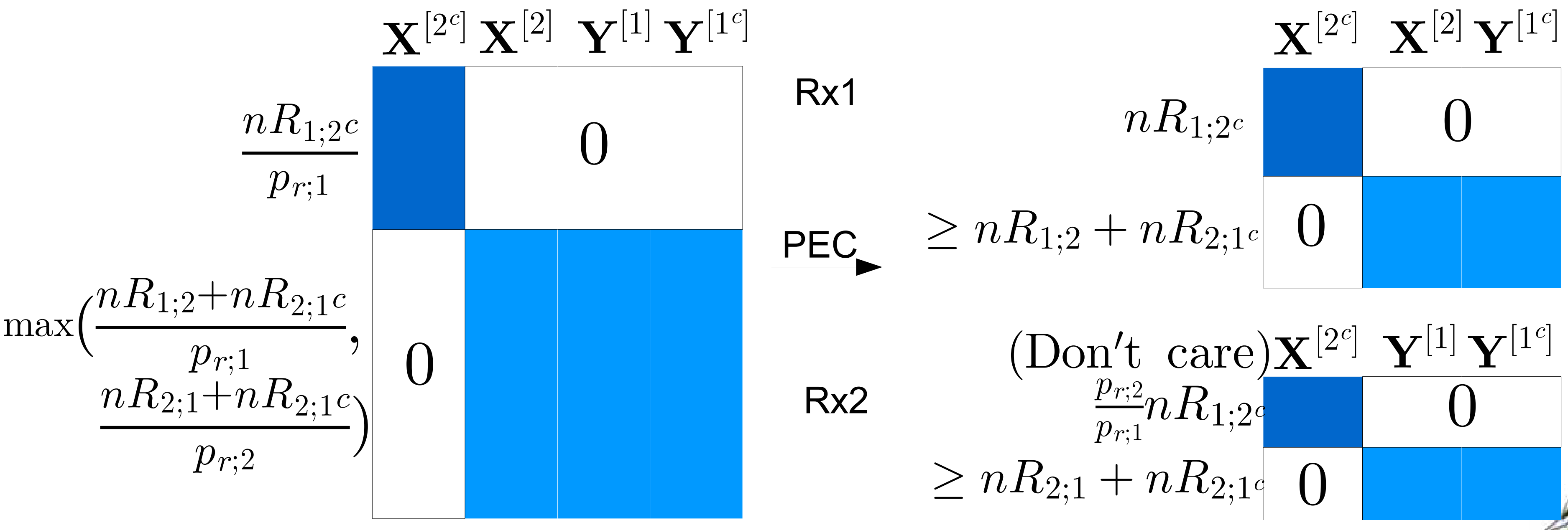

Combine it with $s_i \to r$ coding

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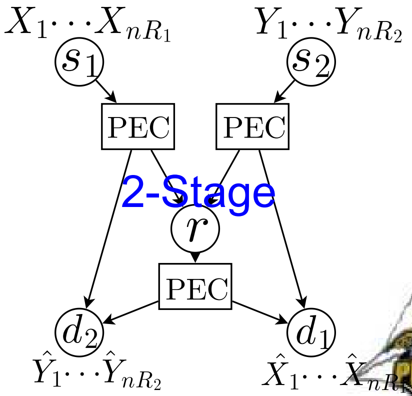
Given that $R_1 = R_{1;2} + R_{1;2^c}$, maximizing R_2 is equivalent to allocating the smallest R_1 to $R_{1;2^c}$. I.e., the stronger overhearing the better.



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- By s_1 performing random linear NC, we max. the overhearing

$$\min nR_{1;2^c} = (nR_1 - np_{s_1;2})^+$$

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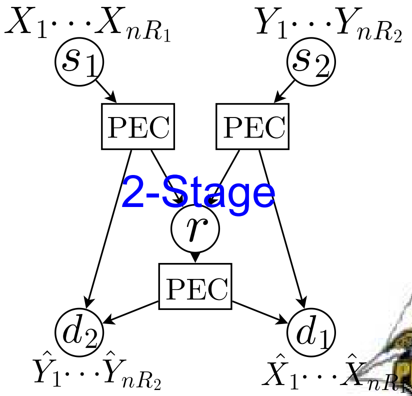

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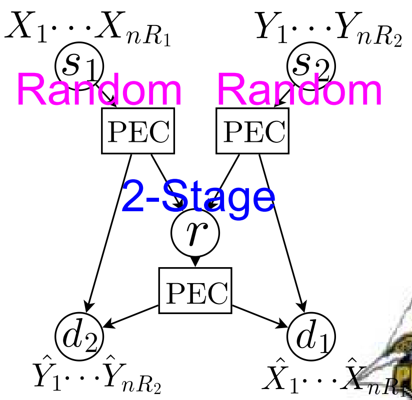

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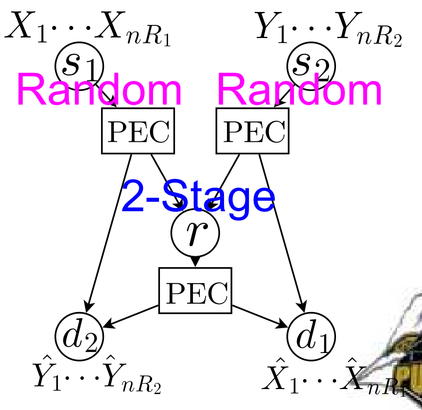
- Therefore:

$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

$$R_2 \leq p_{r;2} - \frac{p_{r;2}}{p_{r;1}} (R_1 - p_{s_1;2})^+$$



The Capacity Regions (Cont'd)

- After combining the $s_i \longrightarrow r$ coding:

$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

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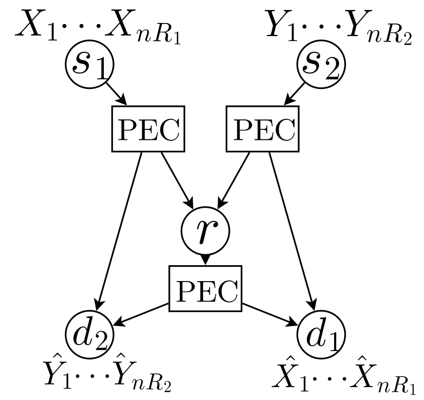

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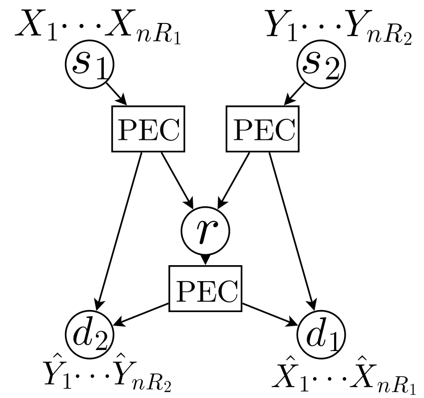

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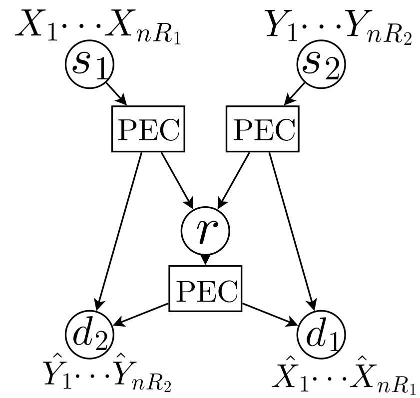
$$R_1 \leq p_{r;1} - (R_2 - p_{s_2;1})^+$$

$$R_2 \leq p_{r;2} - \frac{p_{r;2}}{p_{r;1}}(R_1 - p_{s_1;2})^+$$

- To ensure that s_i can convey all the info. to r, we must also have $R_1 \leq p_{s_1;r}$ and $R_2 \leq p_{s_2;r}$.

The Capacity Regions (Cont'd)



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- To ensure that s_i can convey all the info. to r, we must also have $R_1 \le p_{s_1;r}$ and $R_2 \le p_{s_2;r}$.
- Final Results:

$$R_1 \le \min\left(p_{s_1;r}, p_{r;1} - (R_2 - p_{s_2;1})^+\right)$$

$$R_2 \le \min\left(p_{s_2;r}, p_{r;2} - \frac{p_{r;2}}{p_{r;1}}(R_1 - p_{s_1;2})^+\right)$$

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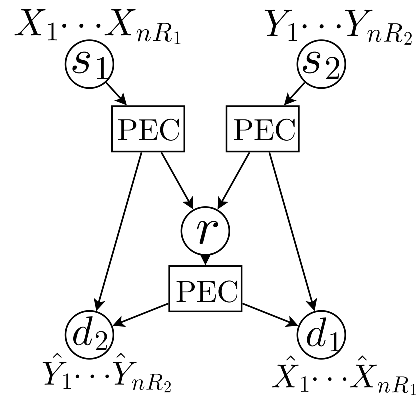

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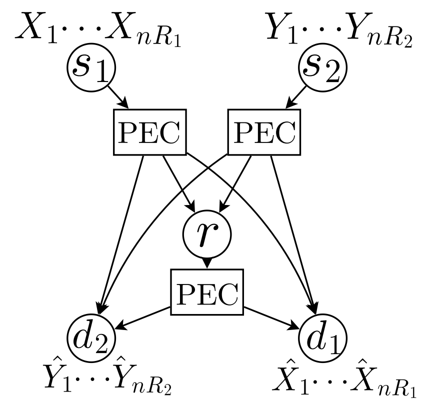

- Can be combined with opportunistic routing (jump over 2 hops):

The Capacity Regions (Cont'd)

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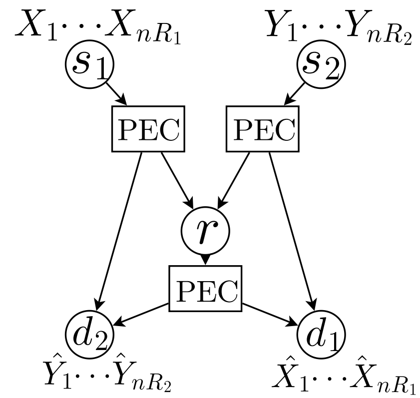

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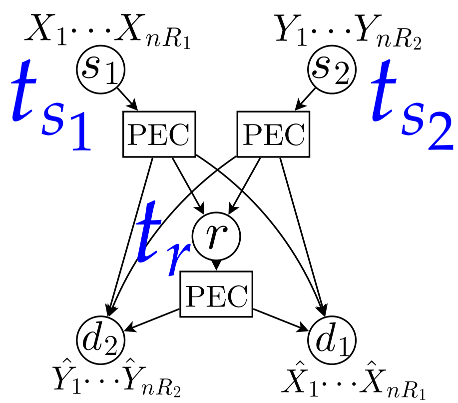
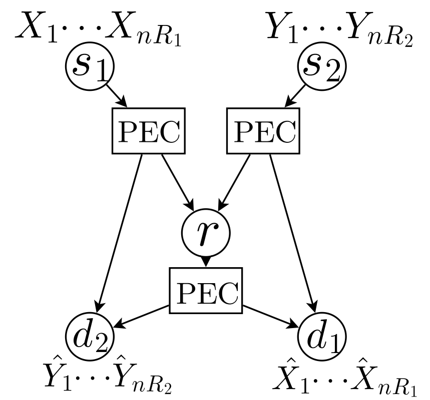


- Can be combined with opportunistic routing (jump over 2 hops):
- Can be combined with cross-layer optimization: Each round of $3n$ packets $\Rightarrow$ variable scheduling t_{s_1}, t_{s_2}, t_r.

The Capacity Regions (Cont'd)

- After combining the $s_i \to r$ coding:



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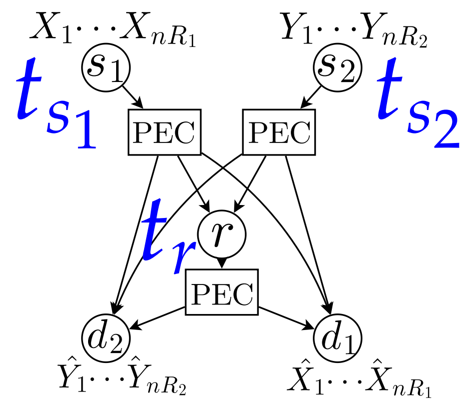

$s \to r$ avail. slots min inter.

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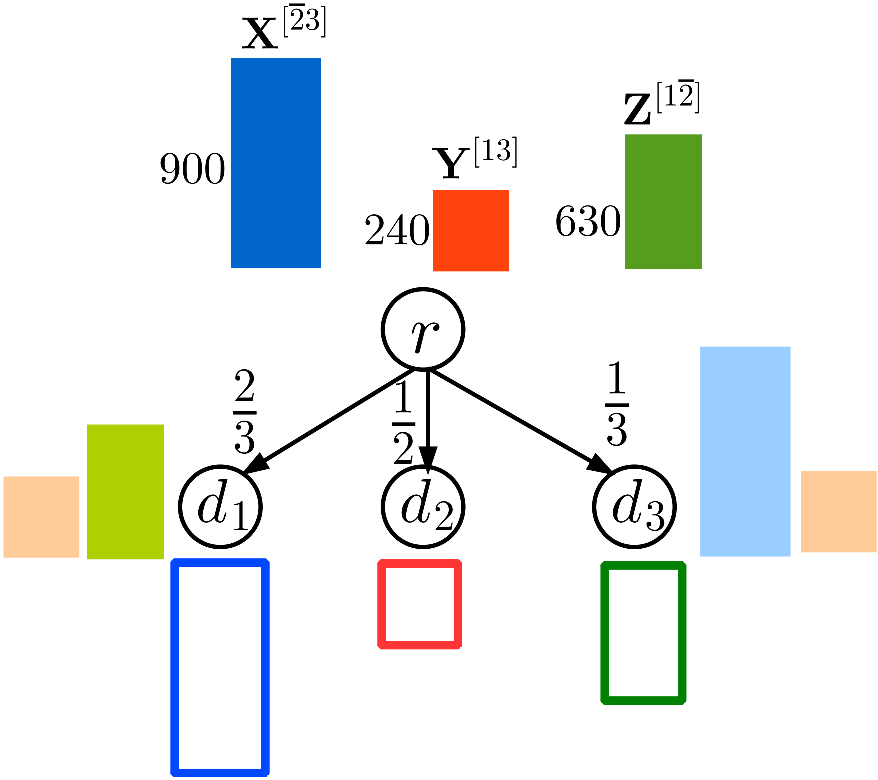
An Illustrative Example of $M = 3$



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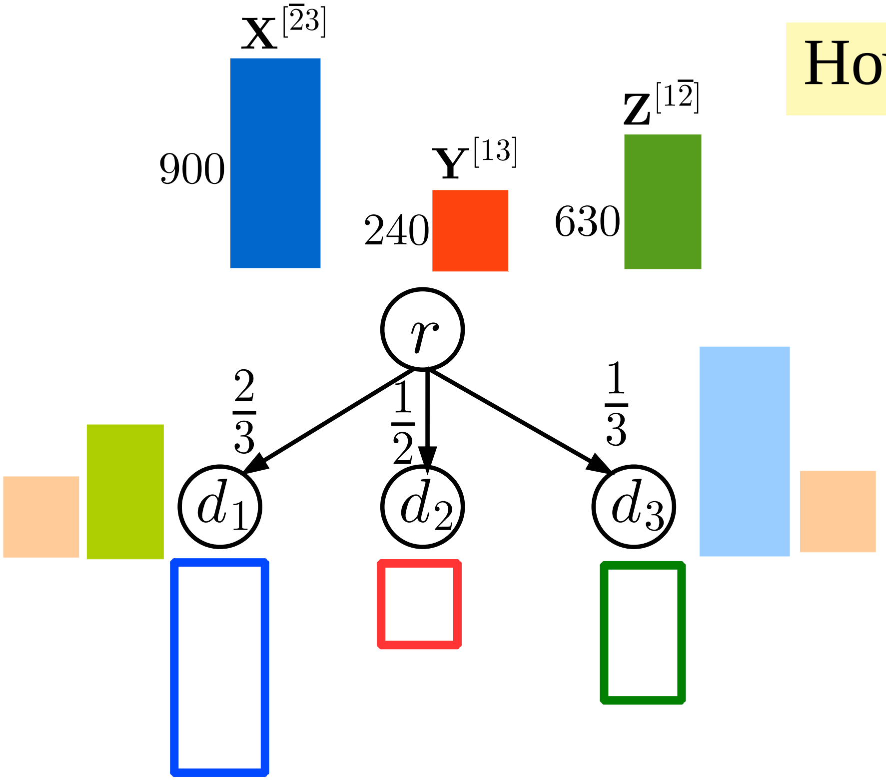

How many time slots to finish transmission?

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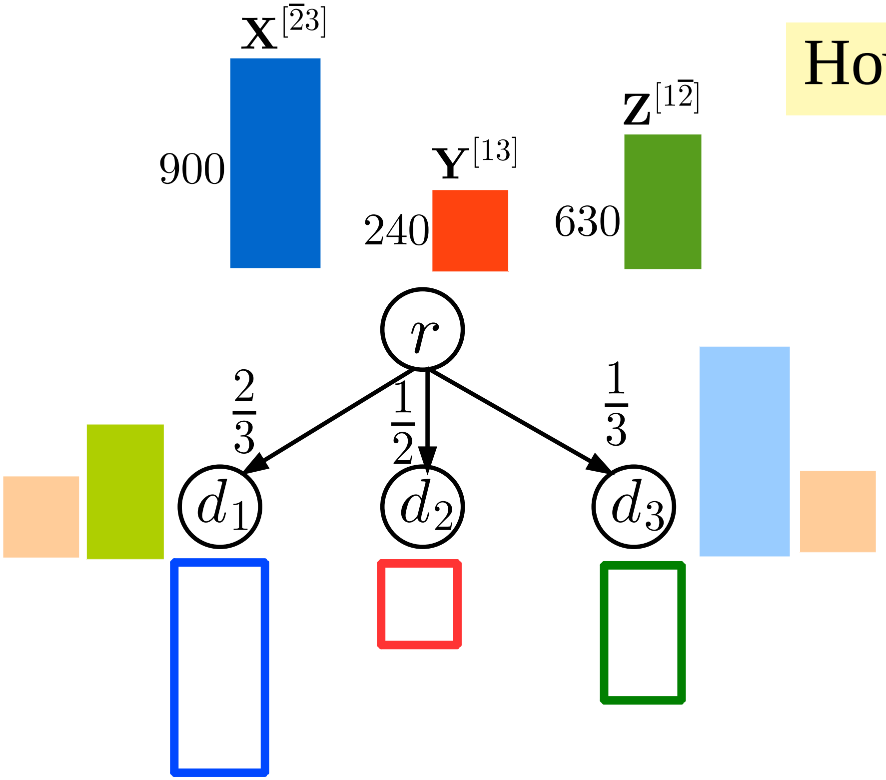

How many time slots to finish transmission?

Solution 1 — Time sharing:

$$\frac{900}{2/3} + \frac{240}{1/2} + \frac{630}{1/3} = 3720$$

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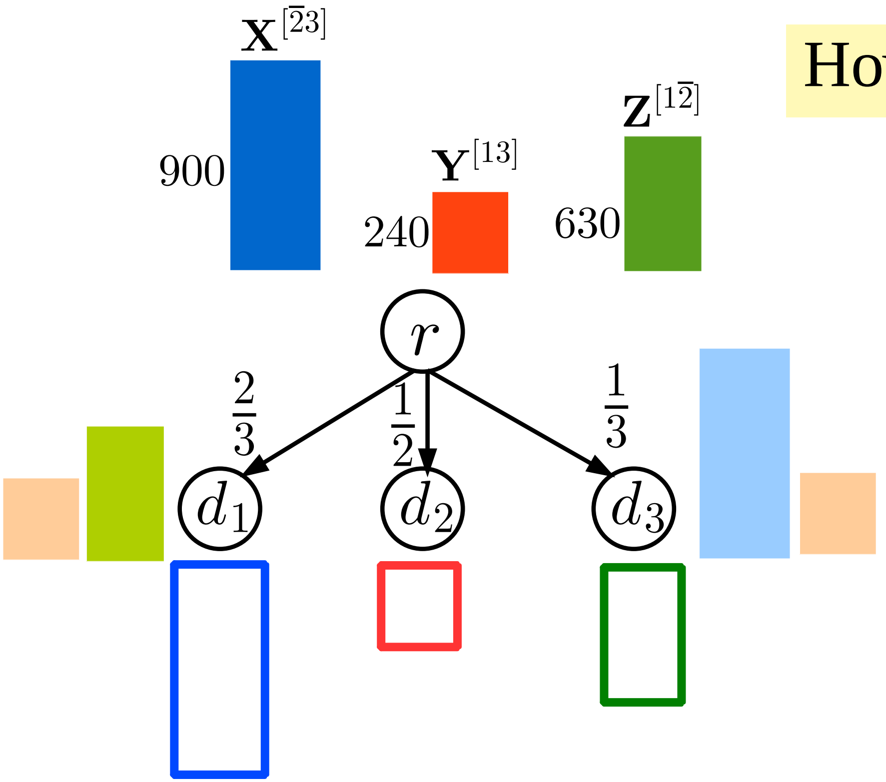

How many time slots to finish transmission?

Solution 1 — Time sharing:

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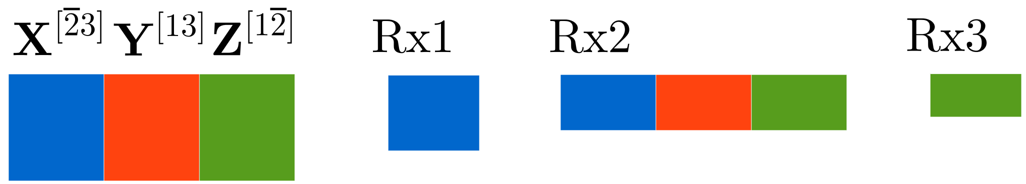
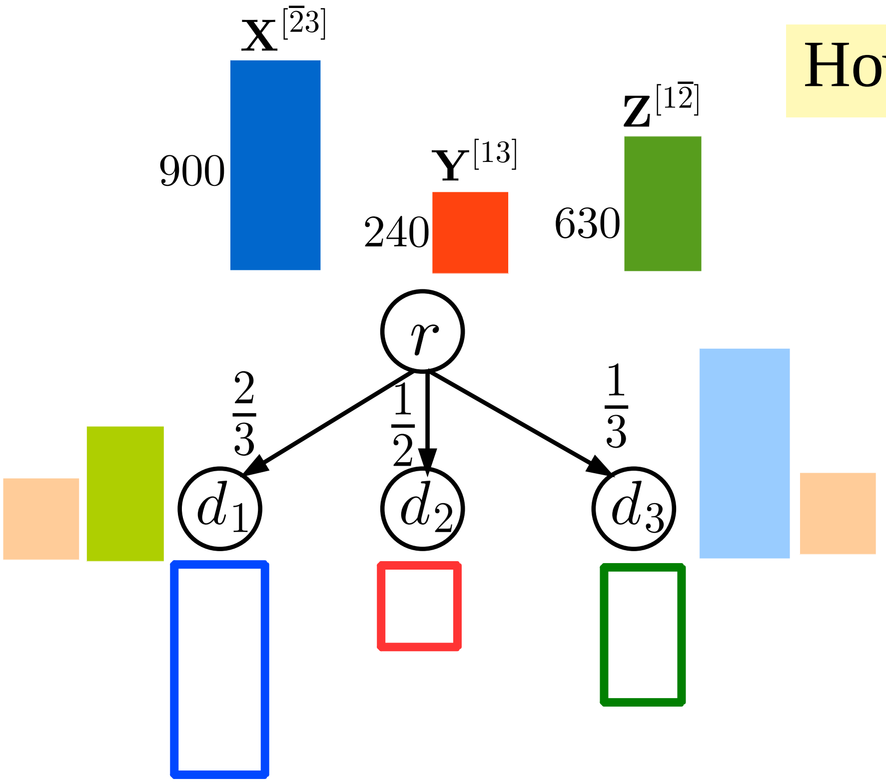
Solution 2 — Random mixing:



$$\max\left(\frac{900}{2/3}, \frac{900+240+630}{1/2}, \frac{630}{1/3}\right) = 3540$$

An Illustrative Example of $M = 3$



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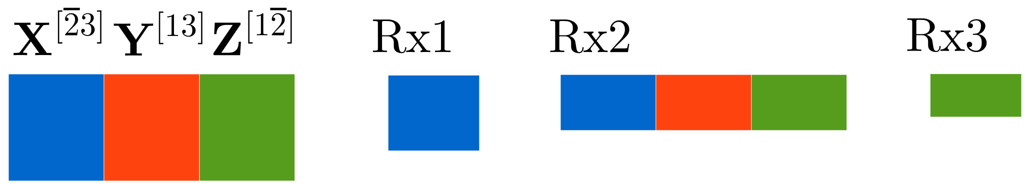
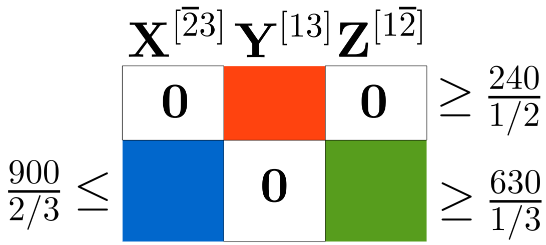
Solution 2 — Random mixing:



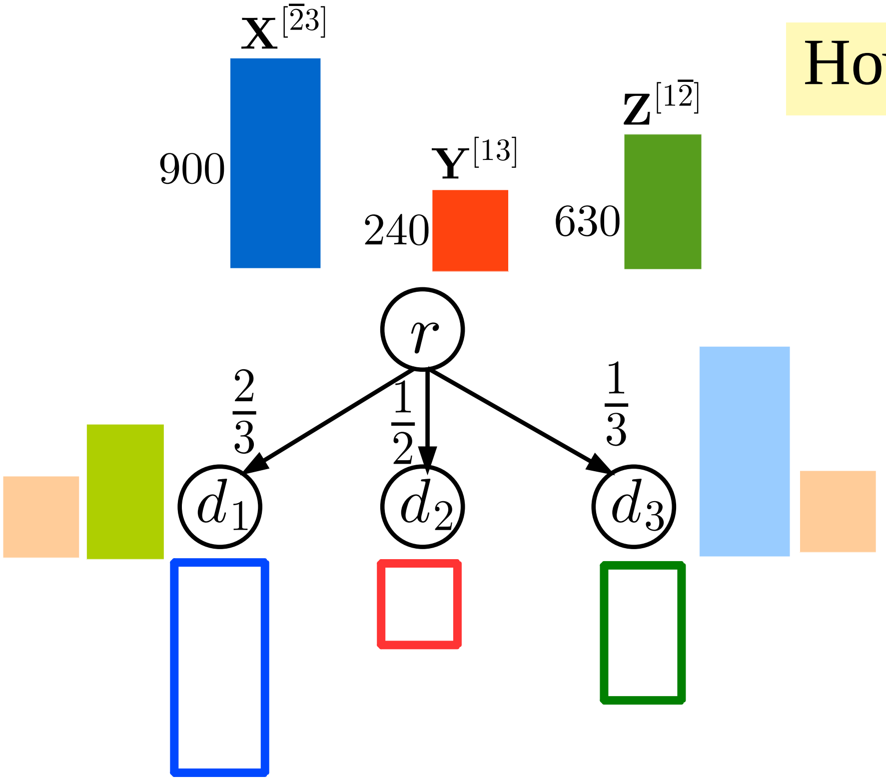
$$\max\left(\frac{900}{2/3}, \frac{900+240+630}{1/2}, \frac{630}{1/3}\right) = 3540$$

Solution 3 — A 2-staged scheme:



$$\frac{240}{1/2} + \max\left(\frac{900}{2/3}, \frac{630}{1/3}\right) = 2370$$

An Illustrative Example of $M = 3$

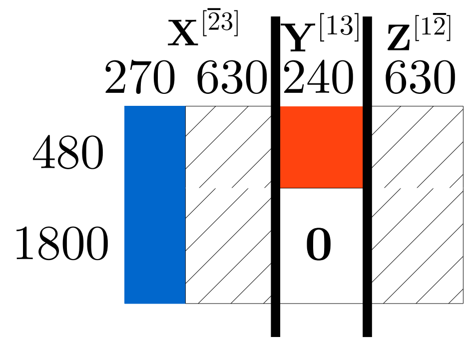


How many time slots to finish transmission?

Solution 1 — Time sharing:

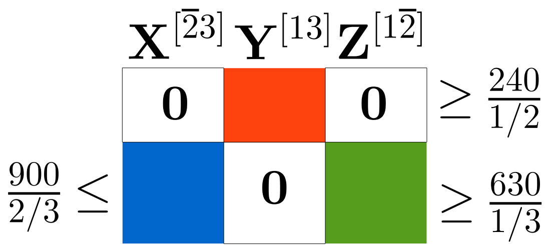
$$\frac{900}{2/3} + \frac{240}{1/2} + \frac{630}{1/3} = 3720$$

Solution 4 — Code Alignment :



Totally 2280 time slots.

Solution 3 — A 2-staged scheme:



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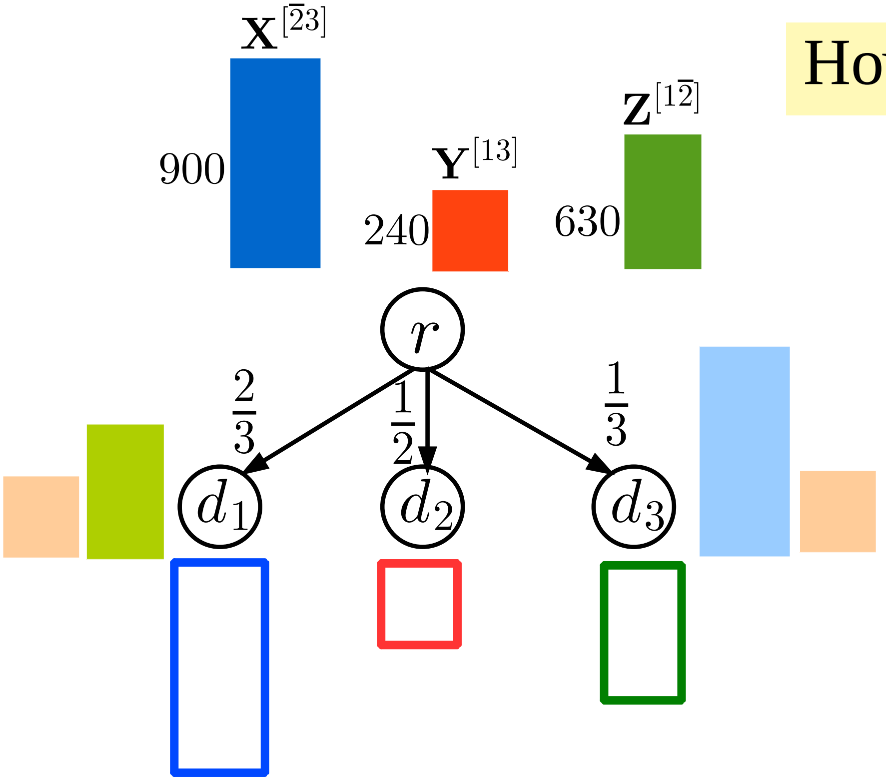

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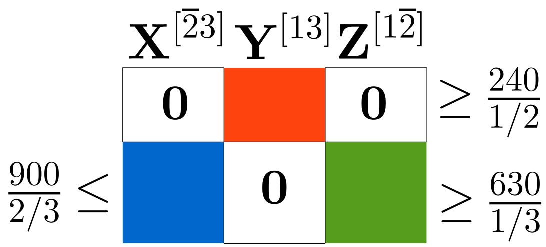

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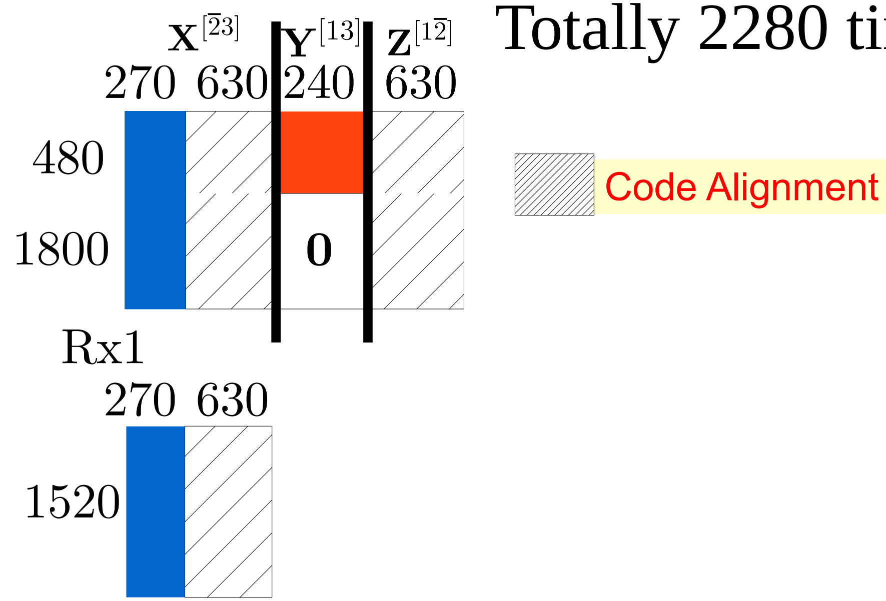

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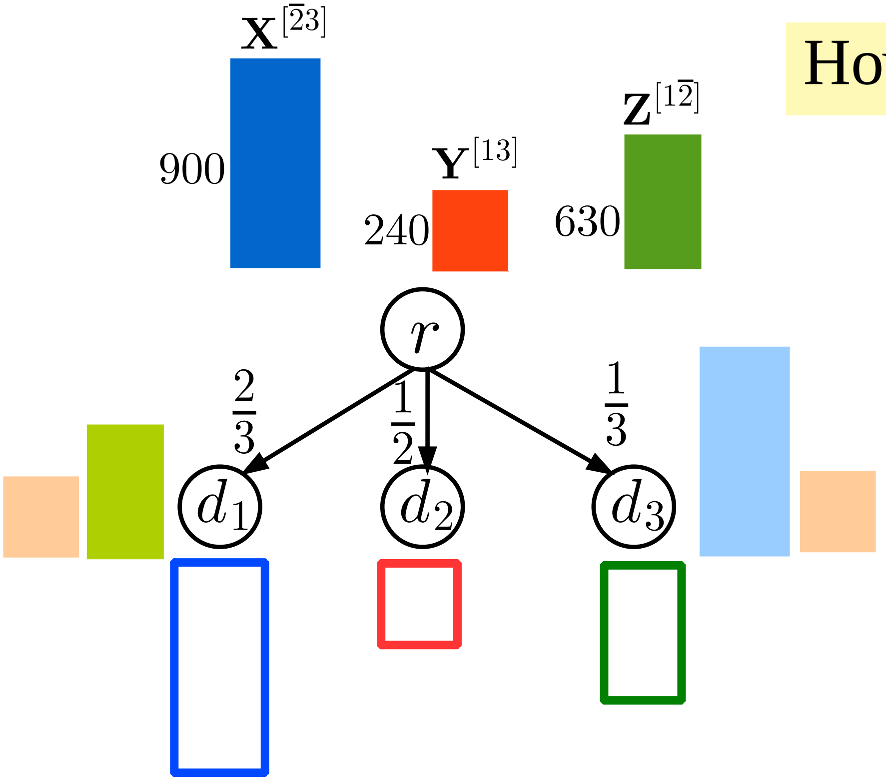

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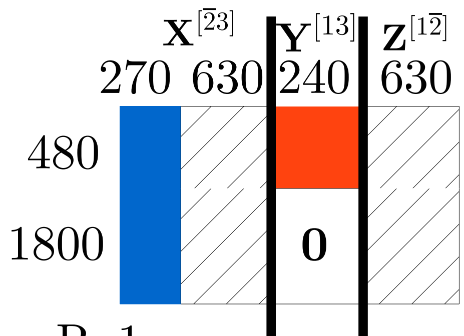
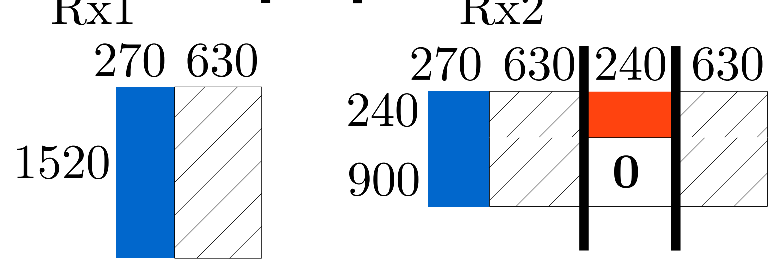
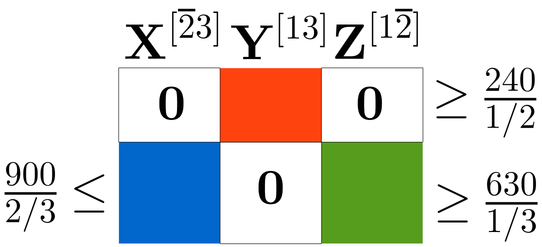


Code Alignment

Instead of decoding X and Z, we decode only $X + Z$.



Solution 3 — A 2-staged scheme:



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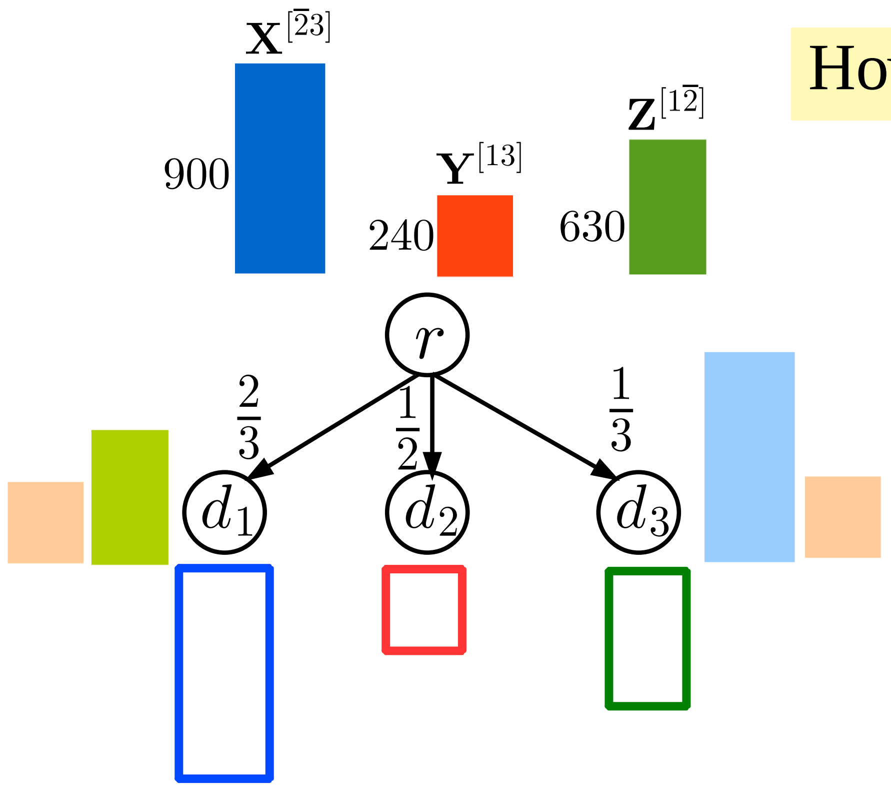

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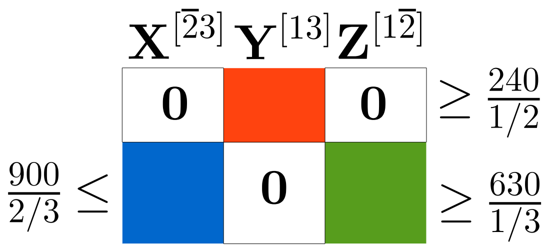

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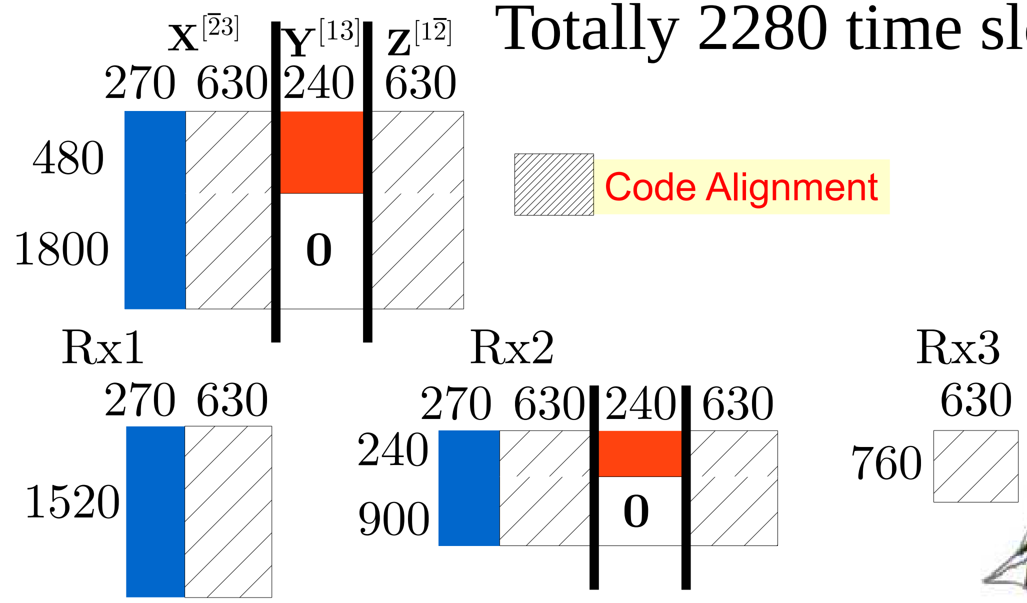
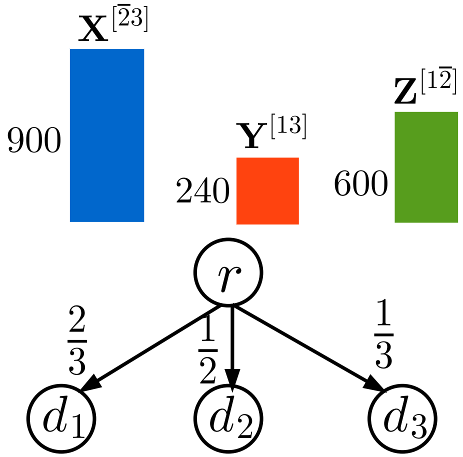
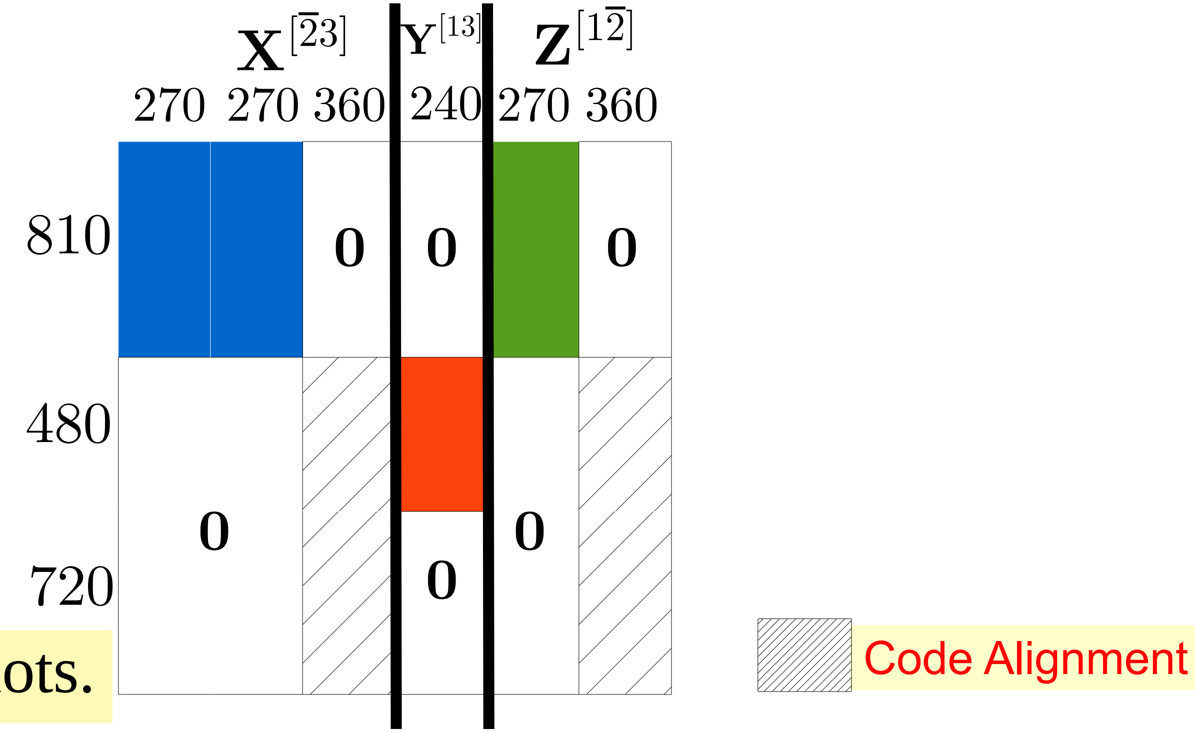
Solution 4 — Code Alignment:

Totally 2280 time slots.



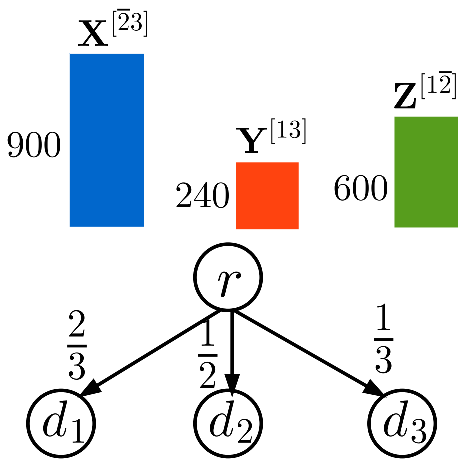
Solution 5: A Hybrid Scheme

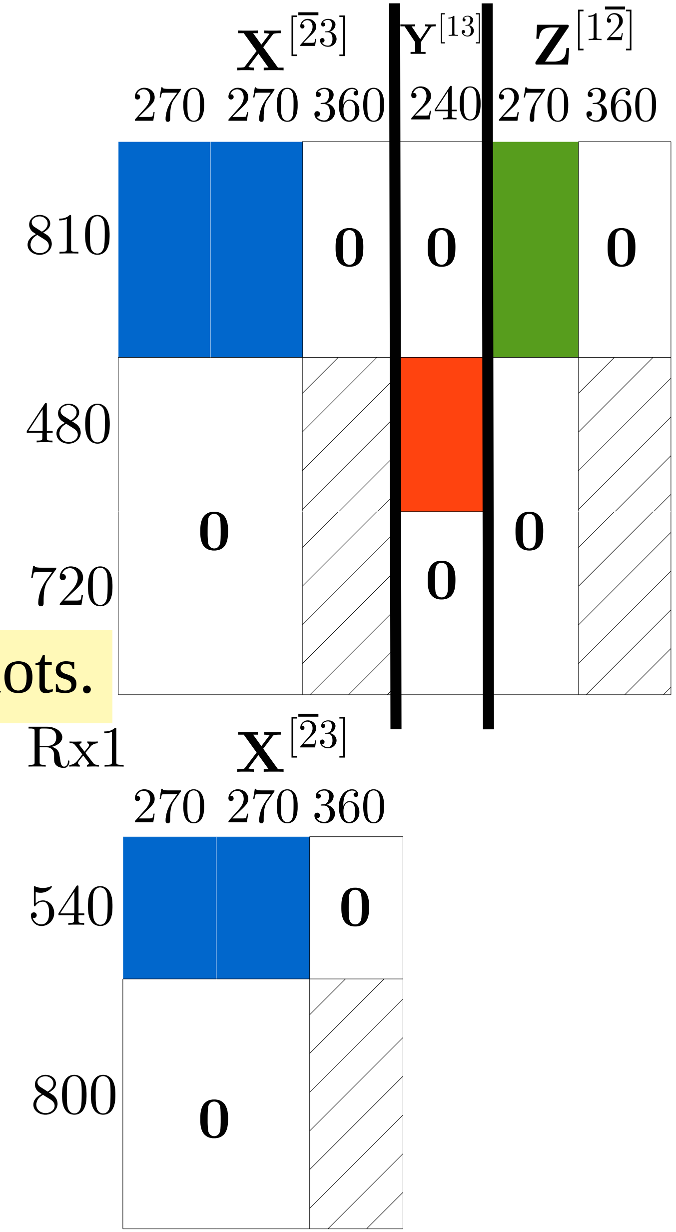




Totally 2010 time slots.

Solution 5: A Hybrid Scheme

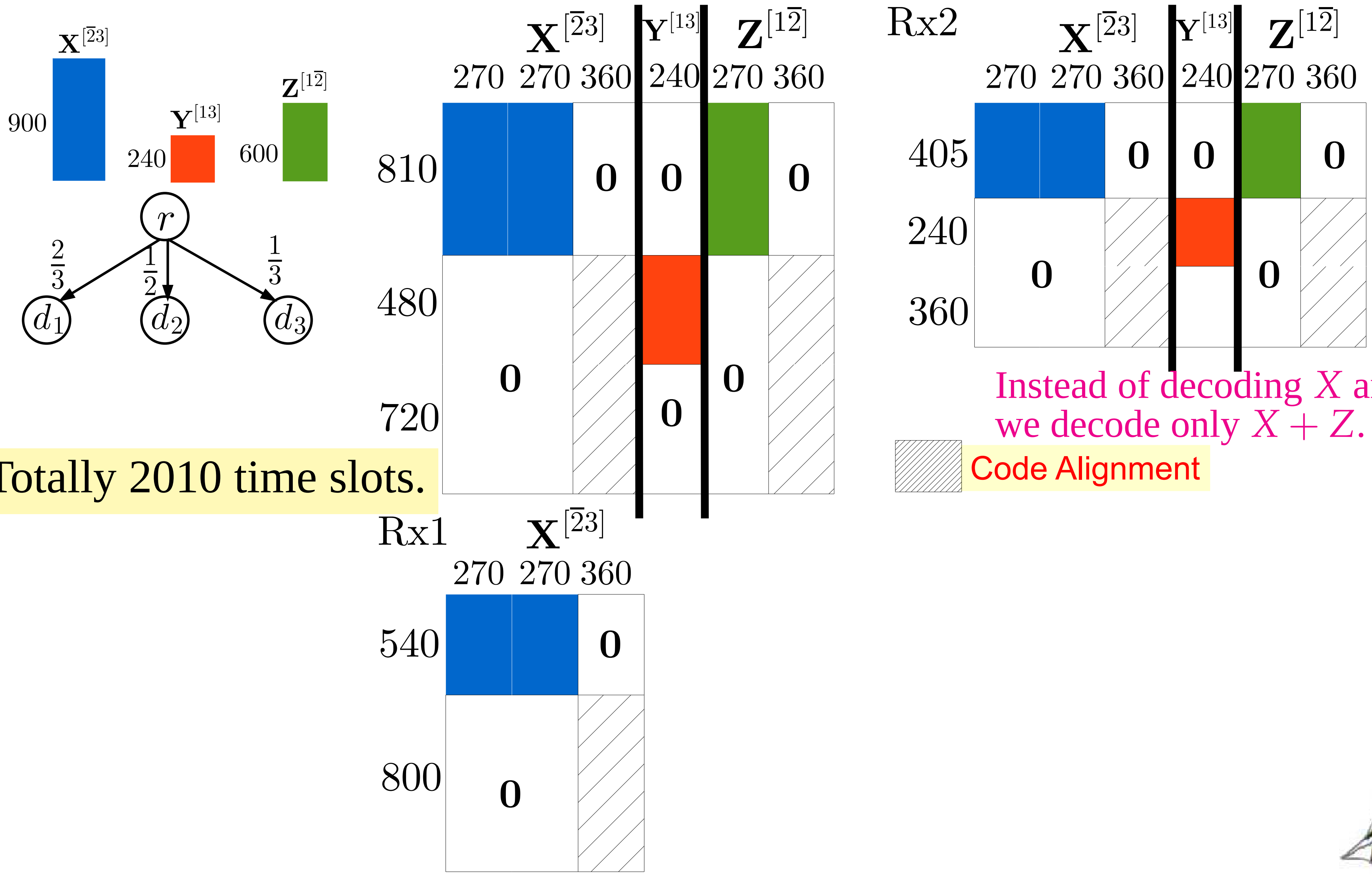




Totally 2010 time slots.

Code Alignment

Solution 5: A Hybrid Scheme

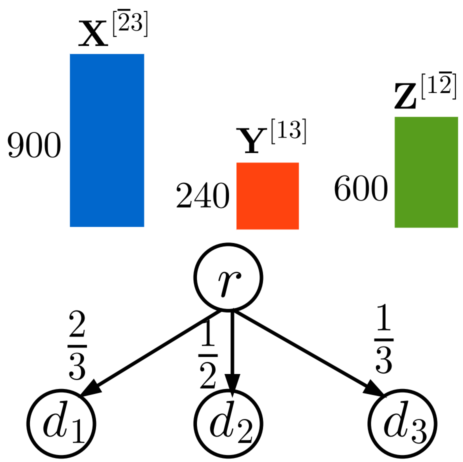


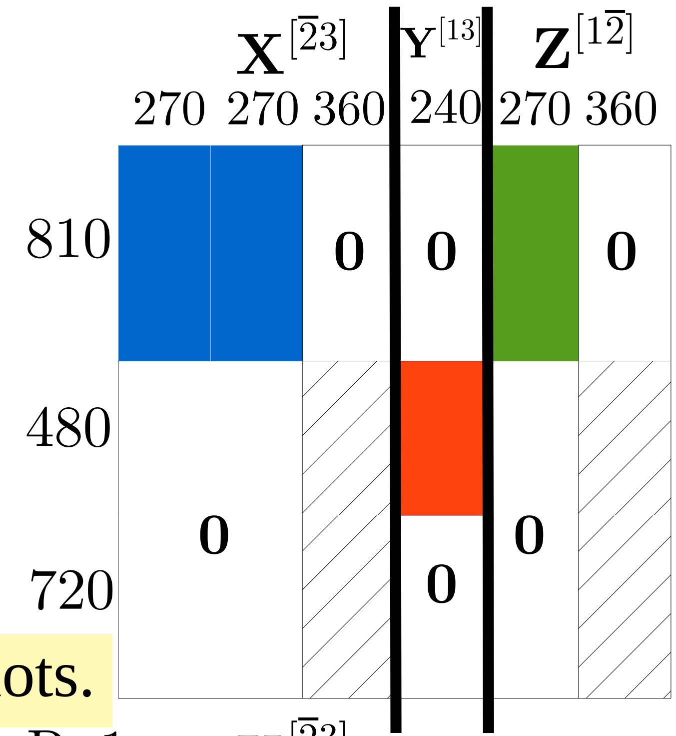
Totally 2010 time slots.

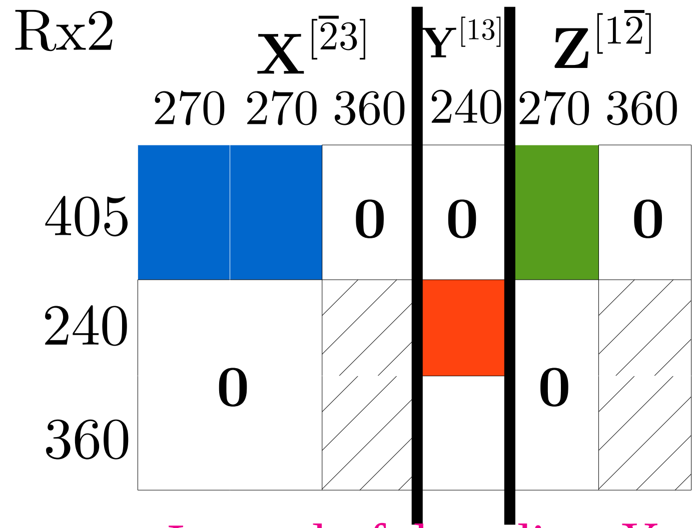
Instead of decoding X and Z, we decode only $X + Z$.

Code Alignment

Solution 5: A Hybrid Scheme







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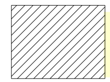
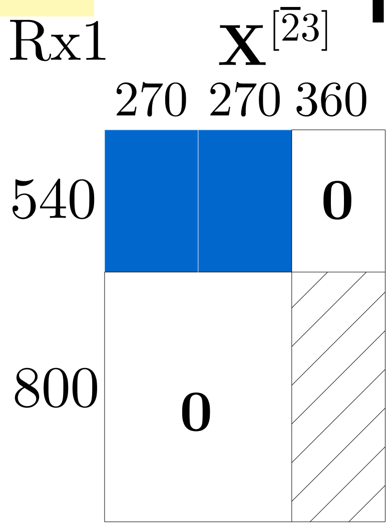
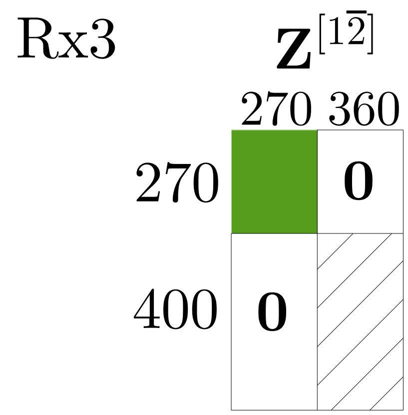
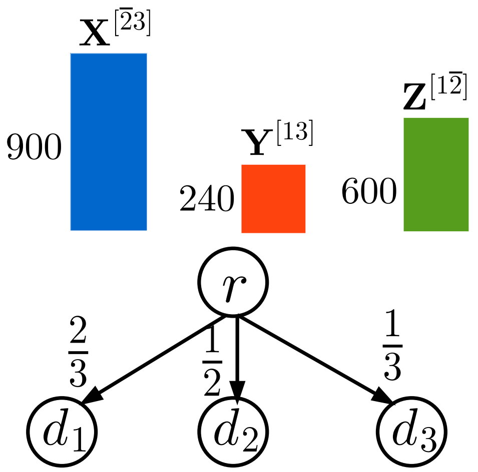

Code Alignment

Totally 2010 time slots.





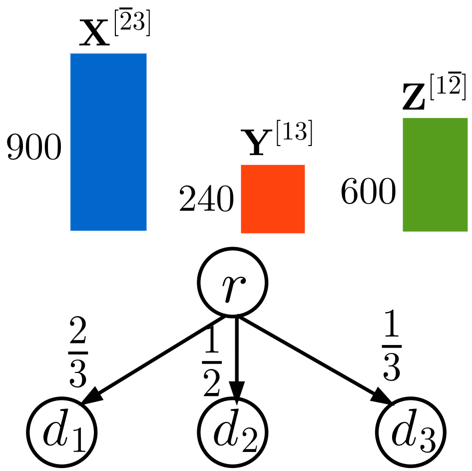
Is The Hybrid Scheme Optimal?



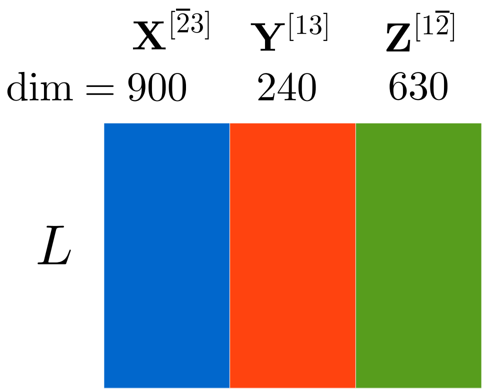
Can we finish tx in $<$2010 slots?

Is The Hybrid Scheme Optimal?

Can we finish tx in <2010 slots?



Any network code.



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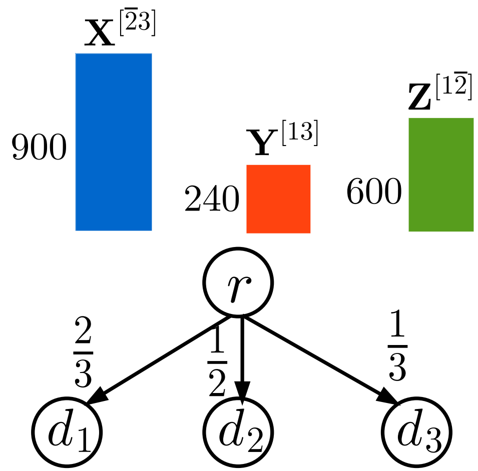
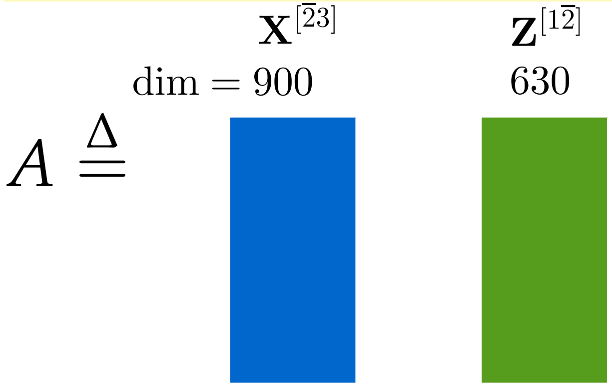
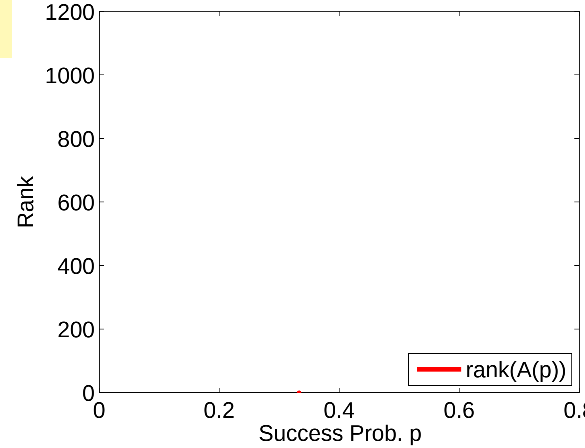
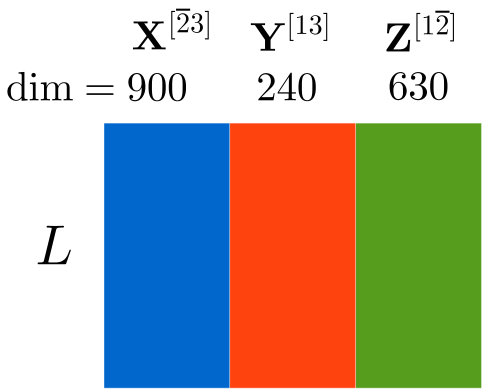


Can we finish tx in <2010





Any network code.



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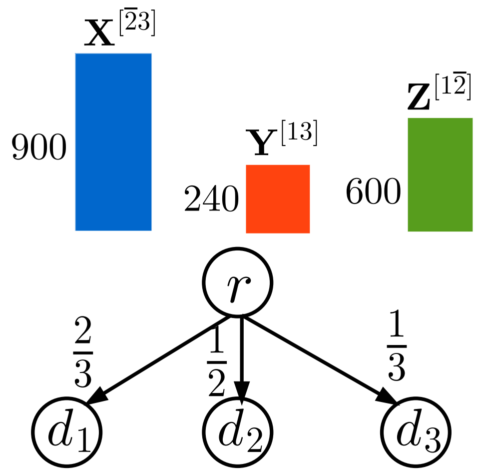
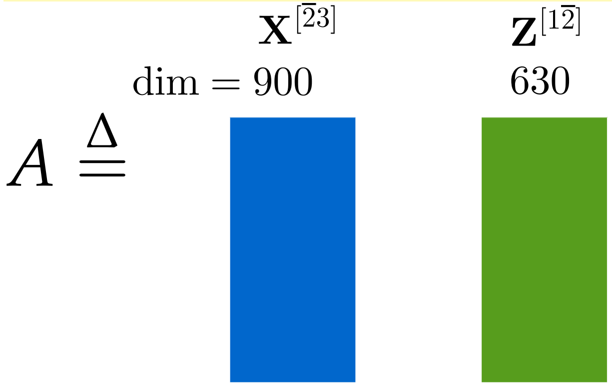
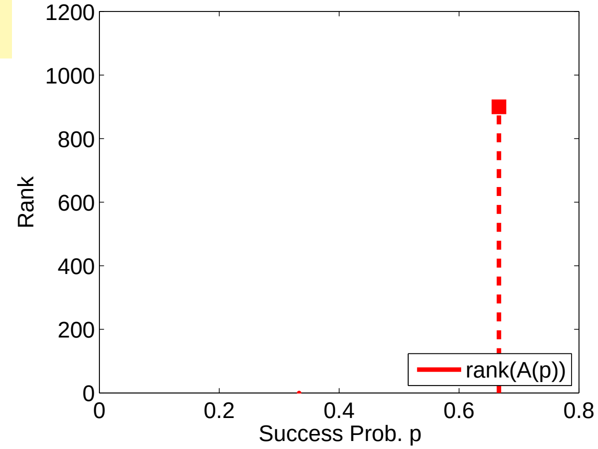


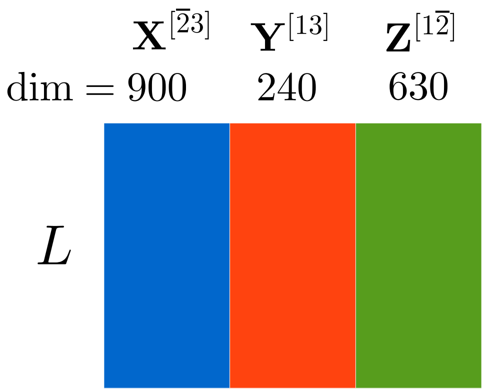
Can we finish tx in <2010





Any network code.

Decodability at $d_1 \Rightarrow \mathrm{Rank}(A(2/3)) \geq 900$.



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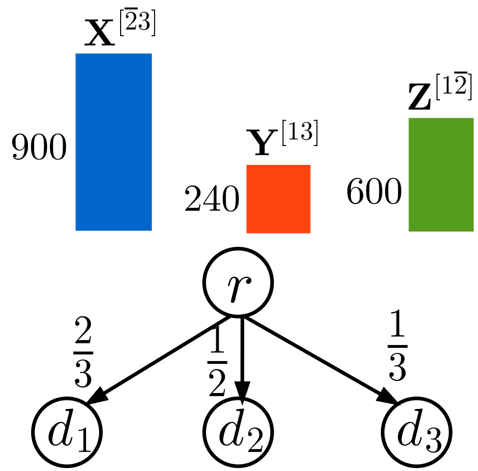
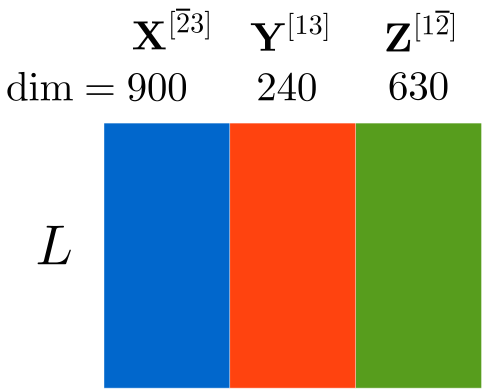
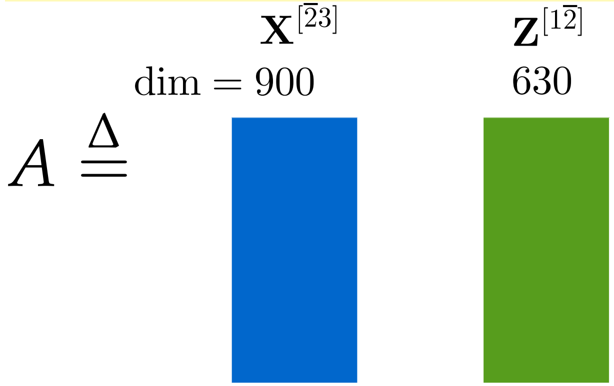
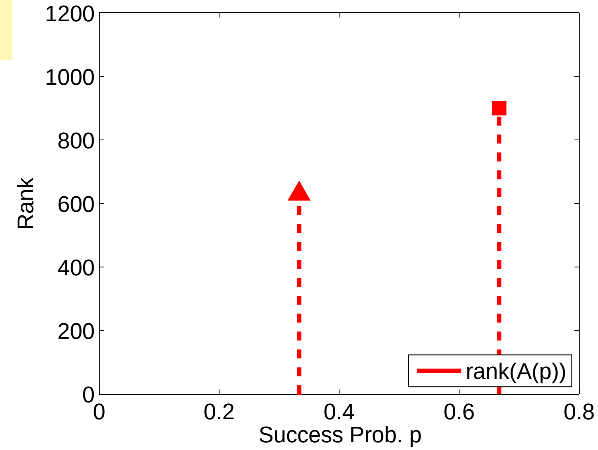


Any network code.



Can we finish tx in <2010

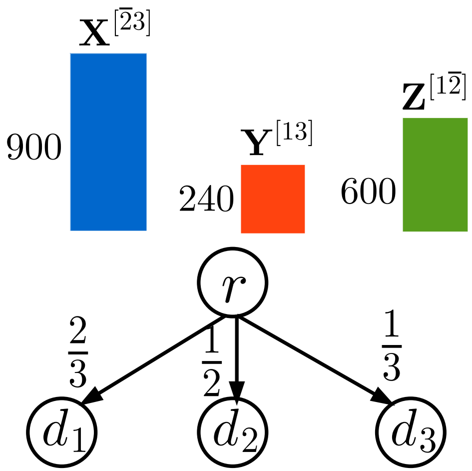




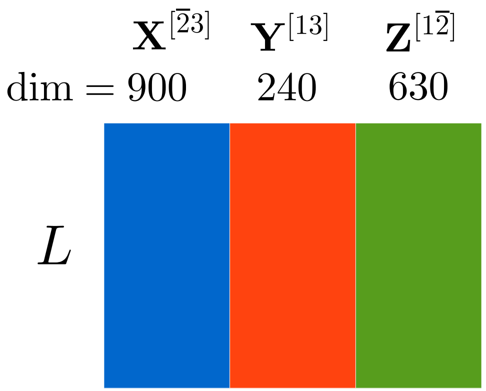
Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900.$

Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630.$

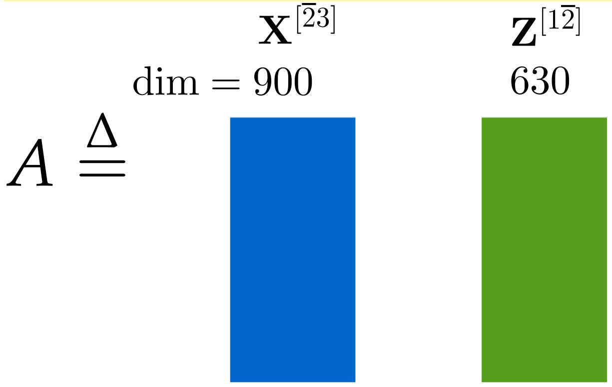
Is The Hybrid Scheme Optimal?

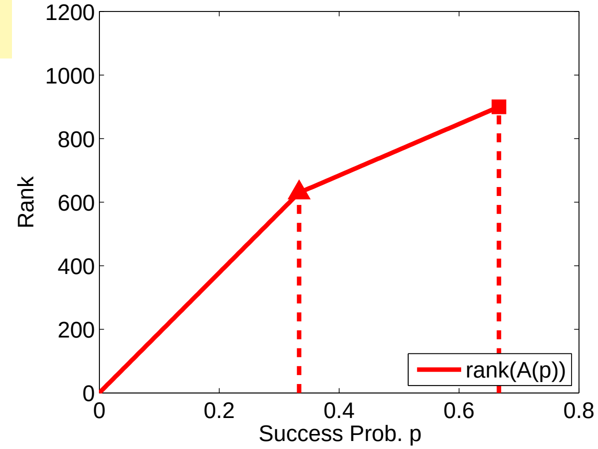


Any network code.



Can we finish tx in <2010



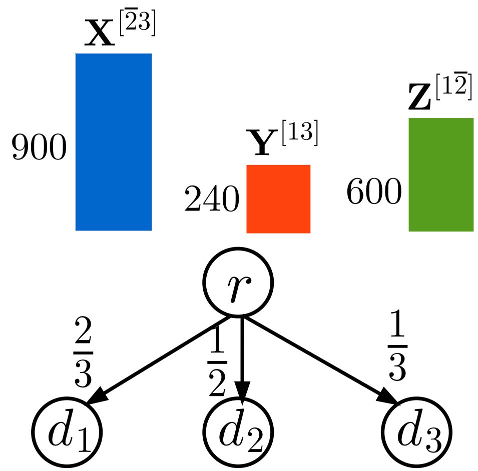


Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

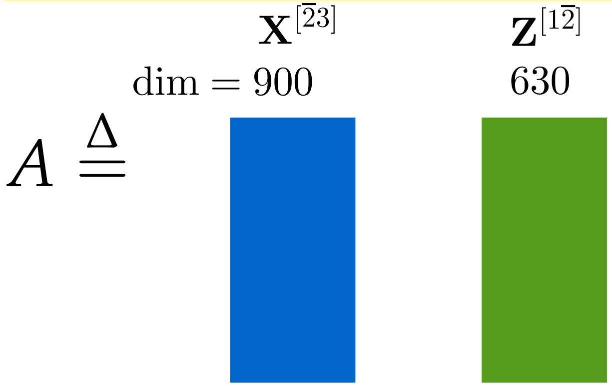
Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630$.

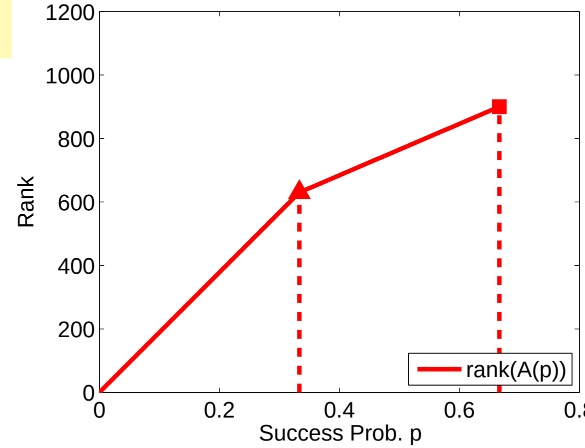
Concavity of information transmission.

Is The Hybrid Scheme Optimal?

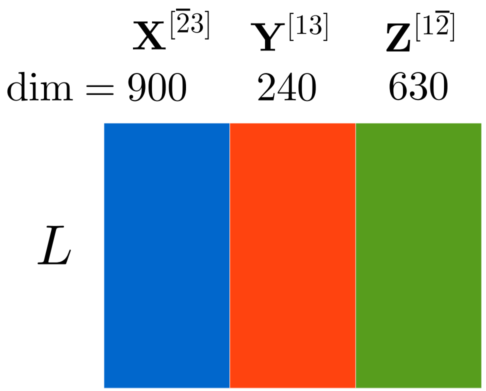


Can we finish tx in <2010





Any network code.



Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

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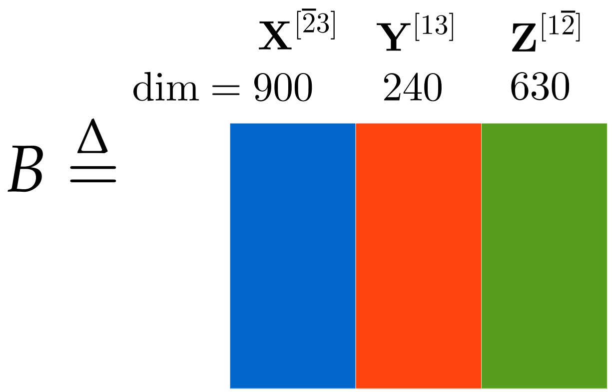
Concavity of information transmission.



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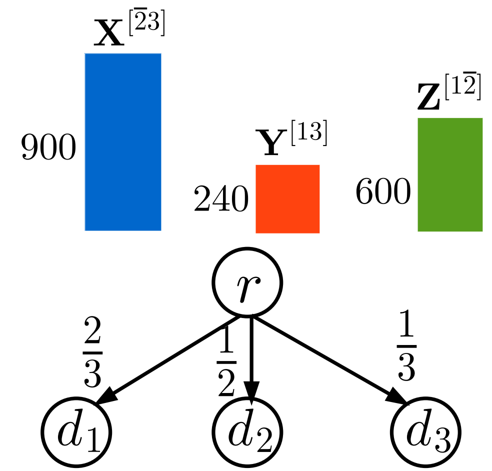
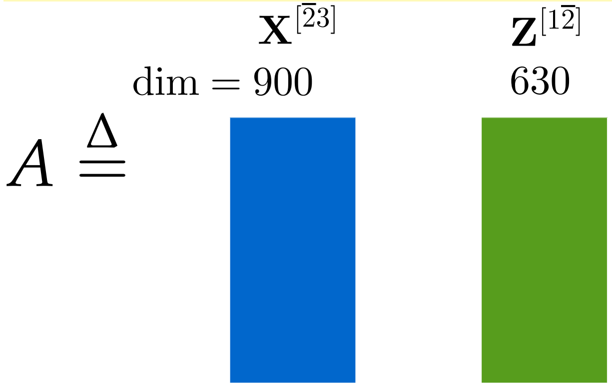
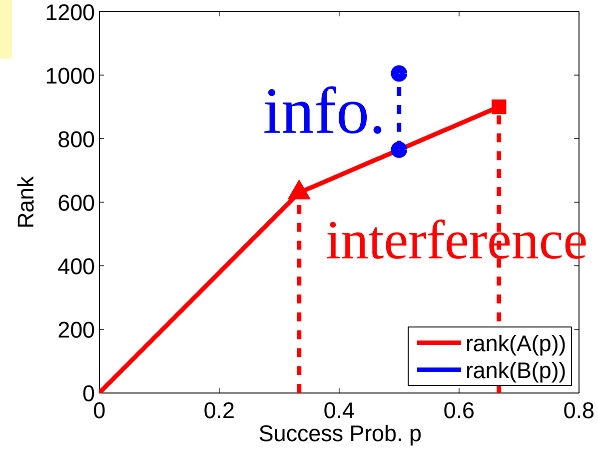
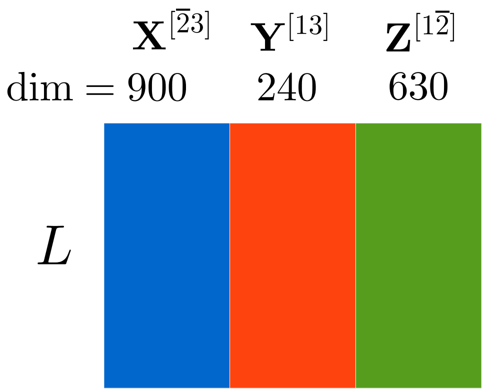


Can we finish tx in <2010





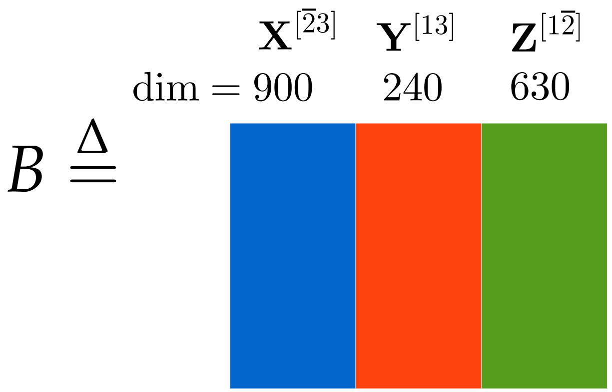
Any network code.



Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630$.

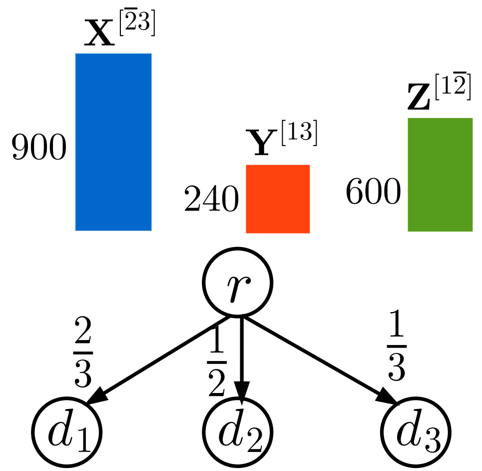
Concavity of information transmission.



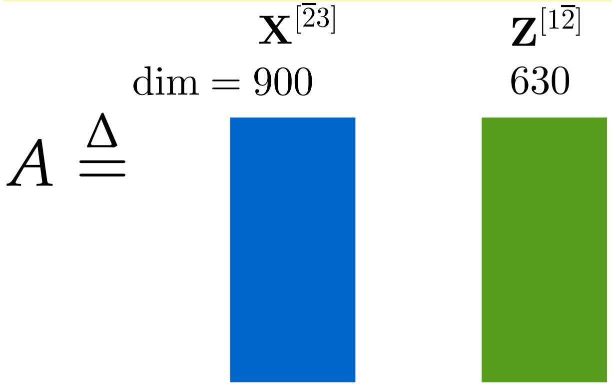
info. interference.

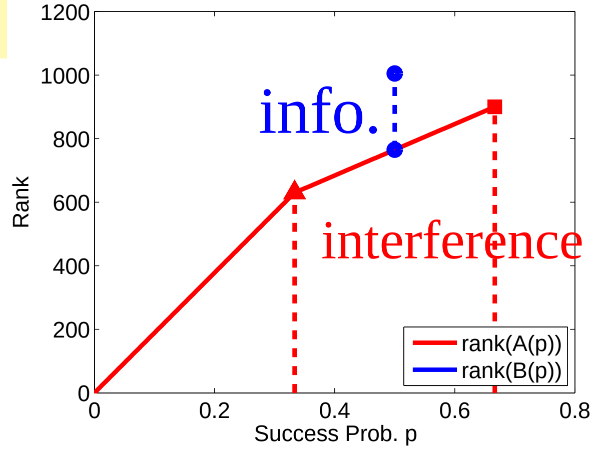
Decodability at $d_2 \Rightarrow \text{Rank}(B(1/2)) \geq 240 + \text{Rank}(A(1/2)) = 1005$.

Is The Hybrid Scheme Optimal?

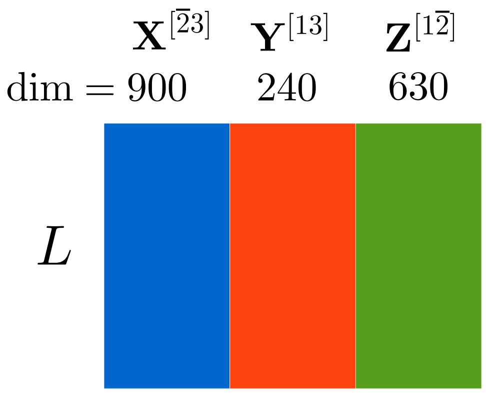


Can we finish tx in <2010





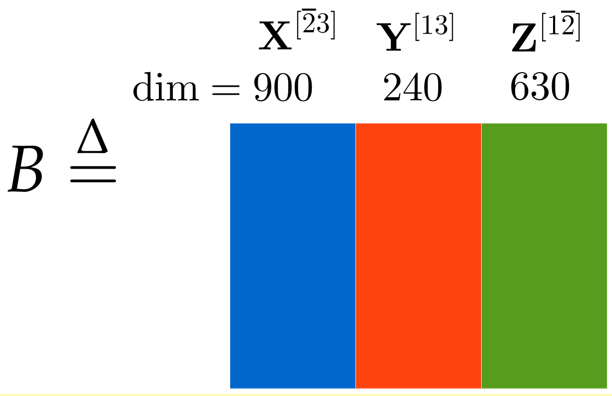
Any network code.



Decodability at $d_1 \Rightarrow \text{Rank}(A(2/3)) \geq 900$.

Decodability at $d_3 \Rightarrow \text{Rank}(A(1/3)) \geq 630$.

Concavity of information transmission.



Total time slots at d_2:

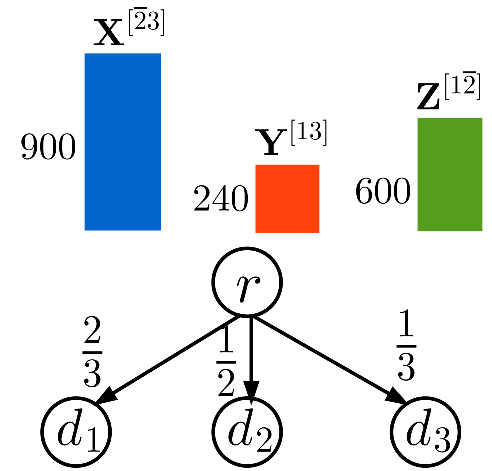
$L \cdot \frac{1}{2} \geq \text{Rank}(B(1/2)) \geq 1005$

$\Rightarrow L \geq 2010$.

info. interference.

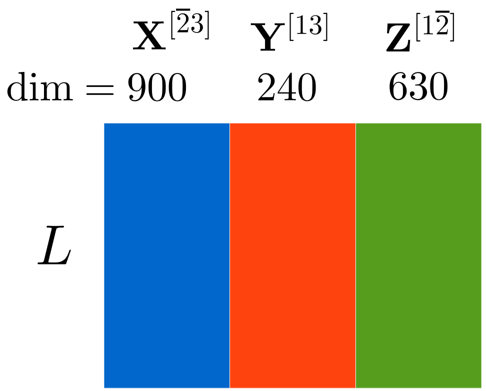
Decodability at $d_2 \Rightarrow \text{Rank}(B(1/2)) \geq 240 + \text{Rank}(A(1/2)) = 1005$.

Is The Hybrid Scheme Optimal?



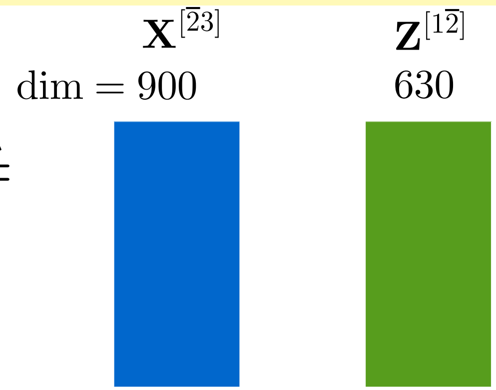
Any network code.

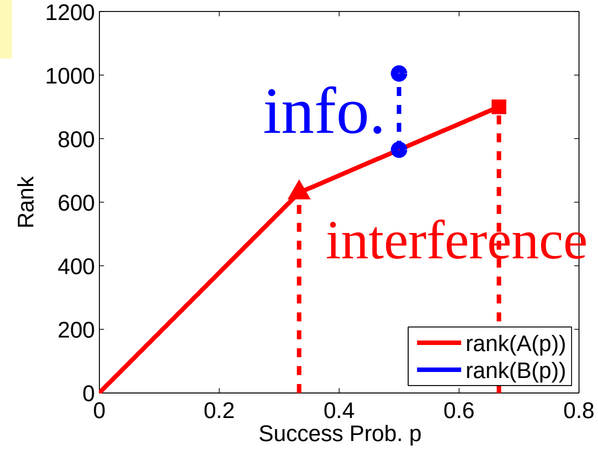
L



Can we finish tx in <2010

$A \triangleq$





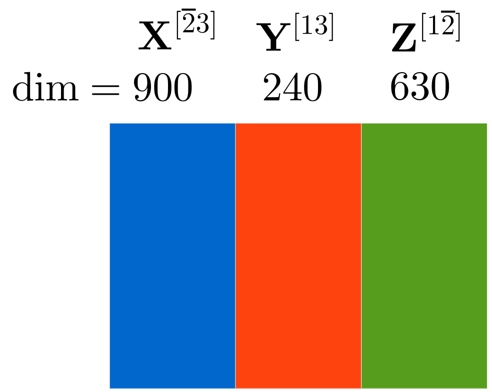
Decodability at $d_1 \Rightarrow$ Ran

The same arguments hold for non-linear codes as well.

Decodability at $d_3 \Rightarrow \mathrm{Rank}(A(1/3)) \geq 630.$

Concavity of information transmission.

$B \triangleq$



Total time slots at d_2:

$L \cdot \frac{1}{2} \geq \mathrm{Rank}(B(1/2)) \geq 1005$

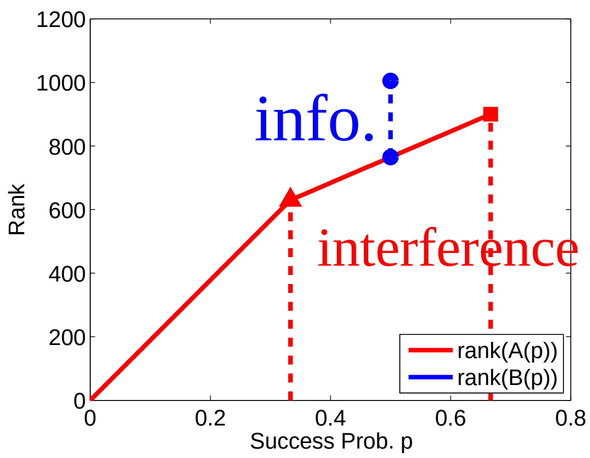
$\Rightarrow L \geq 2010.$

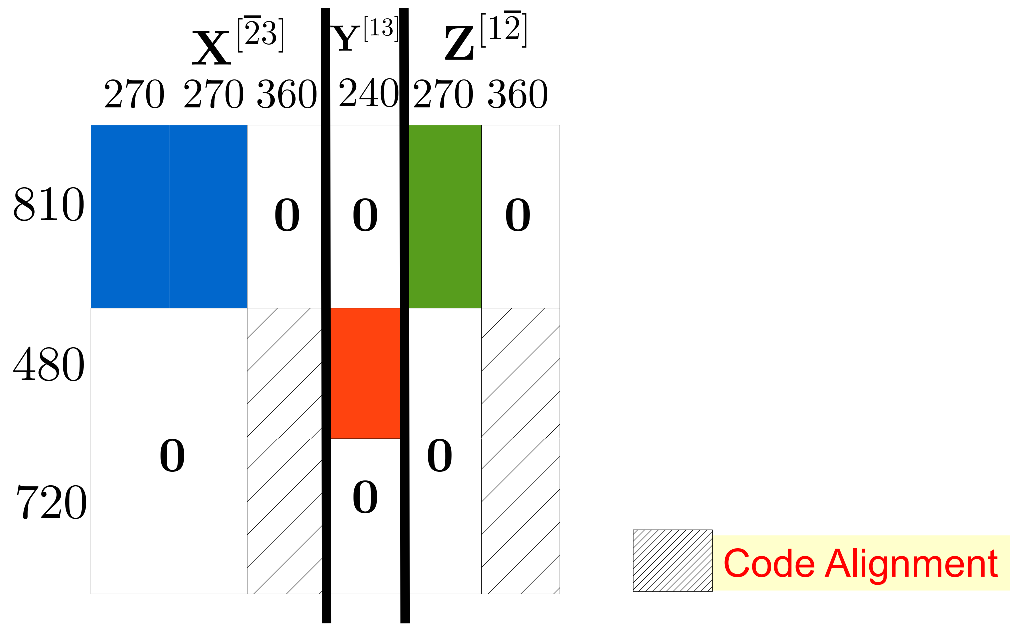
info. interference.

Decodability at $d_2 \Rightarrow \mathrm{Rank}(B(1/2)) \geq 240 + \mathrm{Rank}(A(1/2)) = 1005.$

M-Session Cap. Region

- Outer bound: Interference quantification + information concavity
- Inner bound: Hybrid schemes with stage-based approaches + code alignment.





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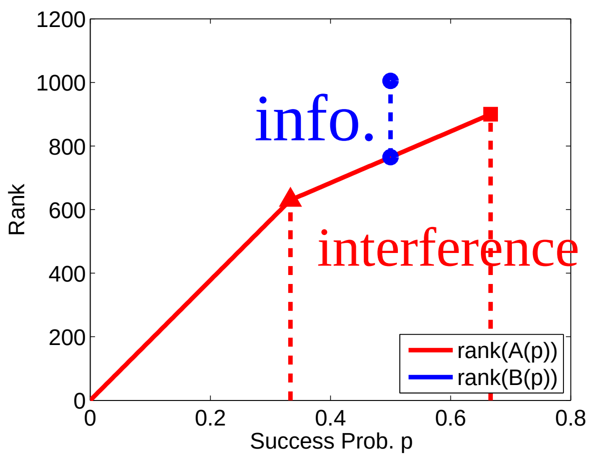

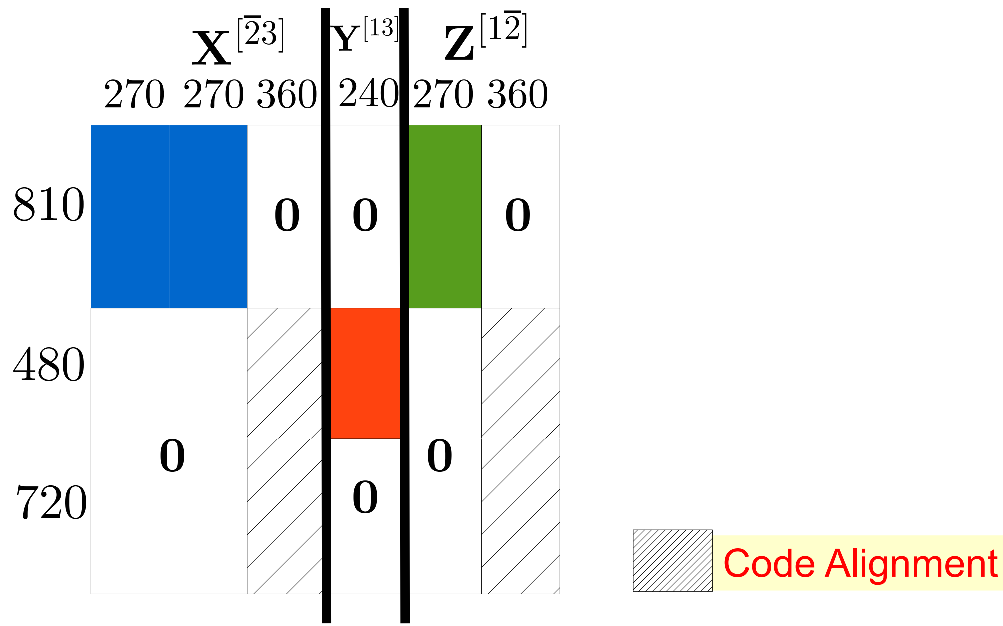

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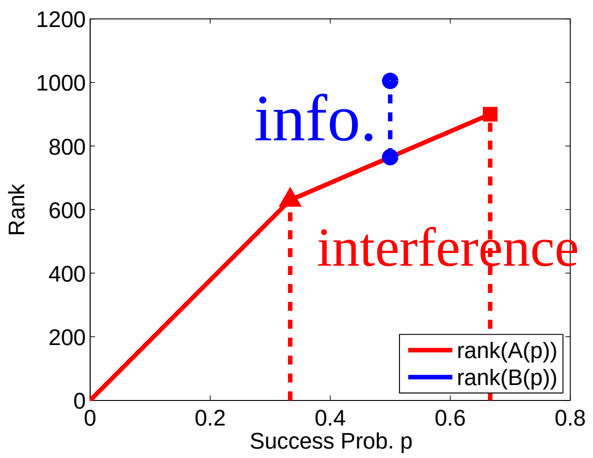

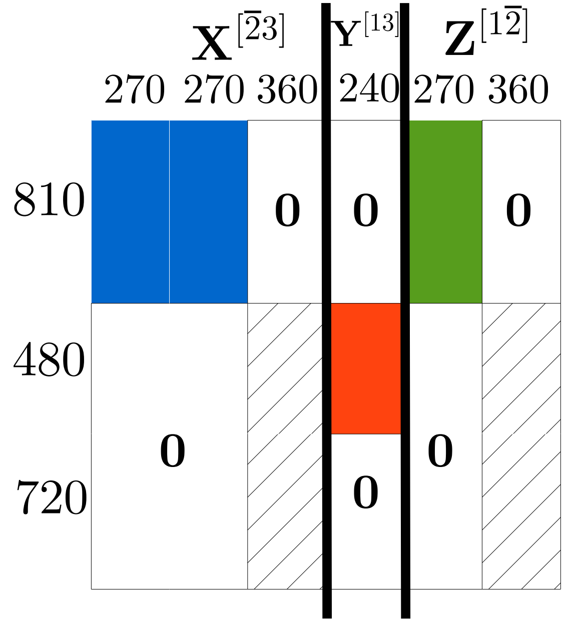

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$$R_1 \leq \min(p_{s_1;r}, p_{r;d_1} - \max\left((R_2 - p_{s_2;d_1})^+ + (R_3 - p_{s_3;d_1\cup d_2})^+, (R_2 - p_{s_2;d_1\cup d_3})^+ + (R_3 - p_{s_3;d_1})^+\right)),$$

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$$\left(\frac{p_{r;d_2}}{p_{r;d_1}} - \frac{p_{r;d_2} - p_{r;d_3}}{p_{r;d_1} - p_{r;d_3}}\right)(R_1 - p_{s_1;d_2\cup d_3})^+ + \frac{p_{r;d_2} - p_{r;d_3}}{p_{r;d_1} - p_{r;d_3}}(R_1 - p_{s_1;d_2})^+$$
$$+\left(1 - \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}\right)(R_3 - p_{s_3;d_1\cup d_2})^+ + \frac{p_{r;d_1} - p_{r;d_2}}{p_{r;d_1} - p_{r;d_3}}(R_3 - p_{s_3;d_2})^+,$$
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[illegible]

The capacity region is governed by linear inequalities.

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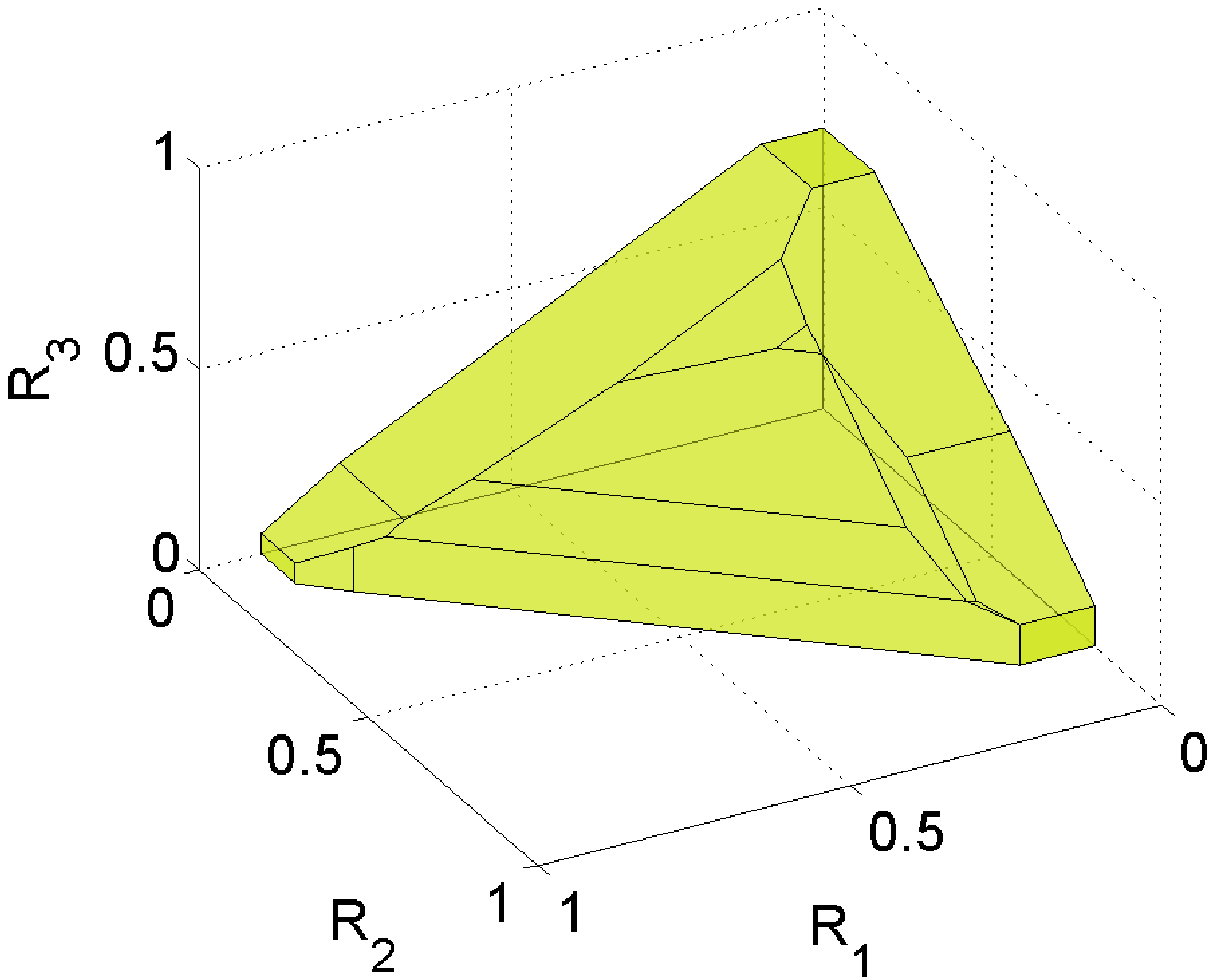
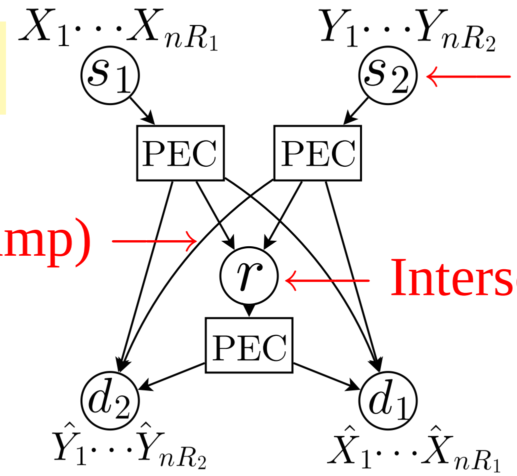
A 3-User Cap. Illustration



The Throughput Improvements

Competing technologies:



Cross-layer (time allocation).

Opp. Routing (direct-jump)

Intersession NC (Mixing at r).

The Throughput Improvements

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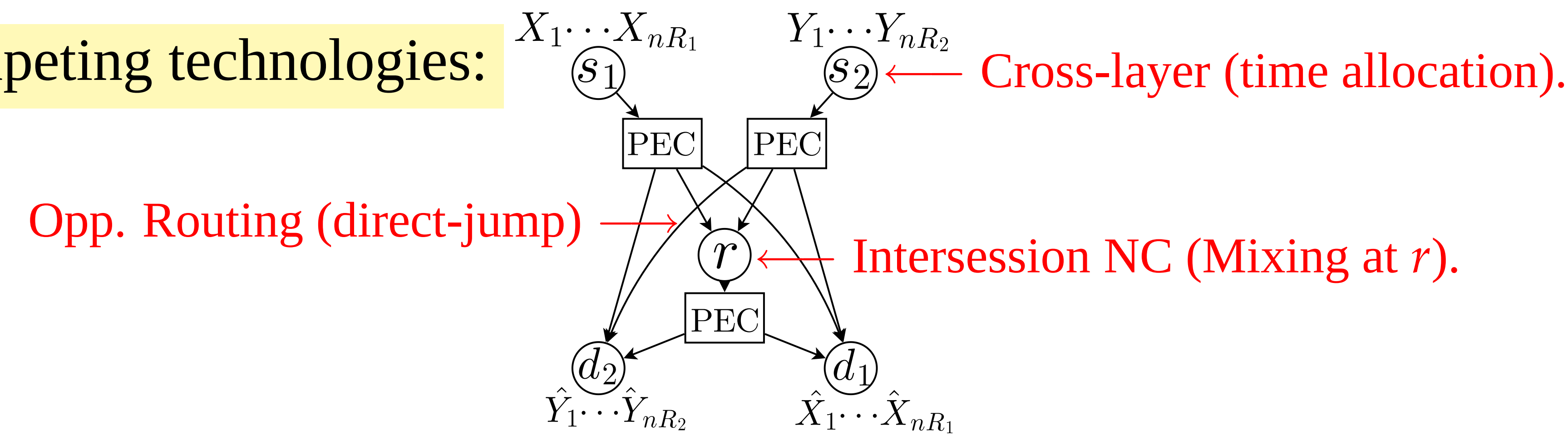
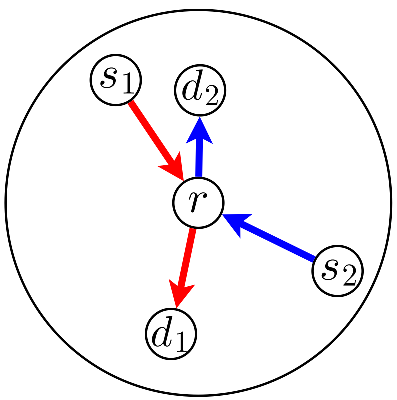
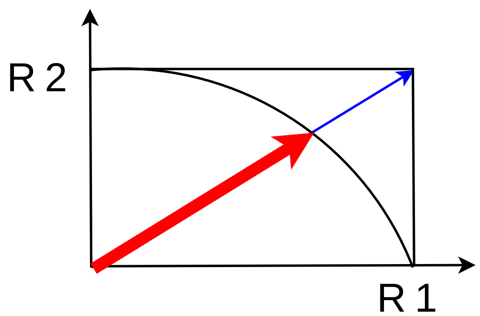
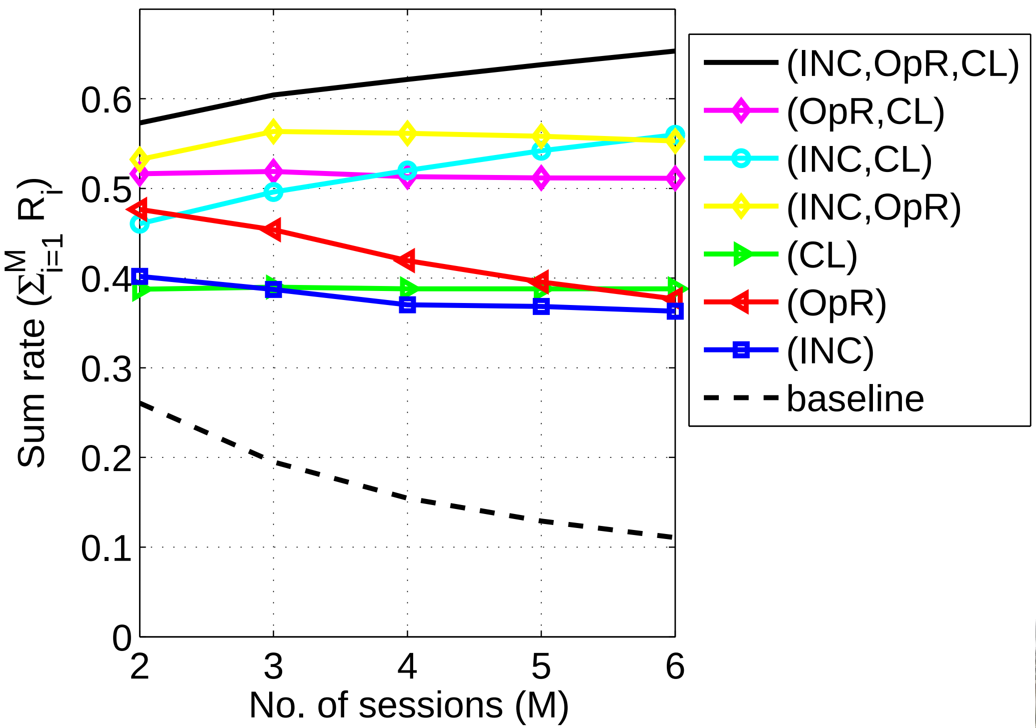


2-hop random networks, Rayleigh fading, proportional fairness.





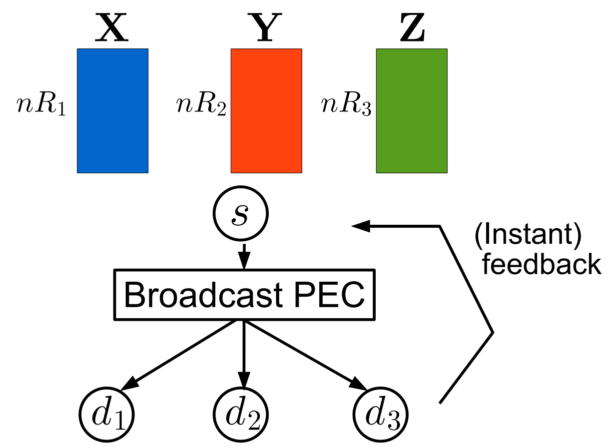


Part I: Quantifying and achieving the capacity of COPE-like protocols

Part II: Quantifying and achieving the capacity of ER-like protocols

1-Hop Cellular (AP) Networks

- 1-hop access point networks. M dest.
- M can be large, say ≈ 20.
  (For 2-hop relay networks $M \leq 6$).
- Each session has nR_i packets.
- The source s uses the channel n times.



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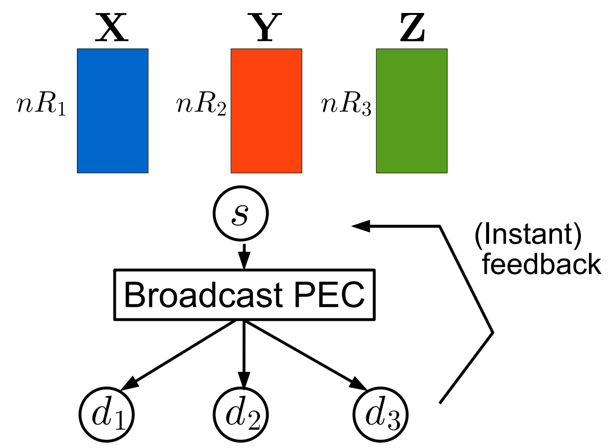

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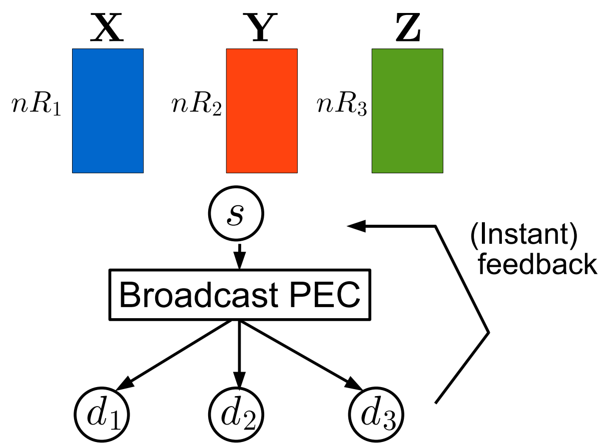
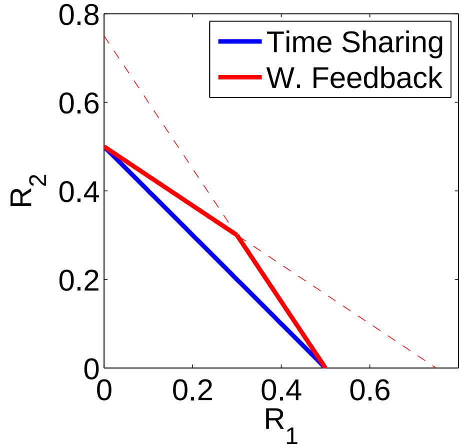
- For $M = 2$, w. feedback, the capacity is [Georgiadis *et al.* 09].

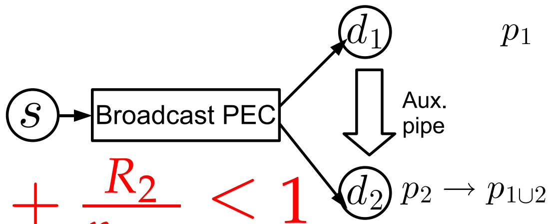




$$\begin{cases} \frac{R_1}{p_{1\cup 2}} + \frac{R_2}{p_2} \leq 1 \\ \frac{R_1}{p_1} + \frac{R_2}{p_{1\cup 2}} \leq 1 \end{cases}$$

Georgiadis' Proof

- Outer bound [Ozarow *et al.* 84]: Introduce auxiliary pipes to convert it into *physically degraded channels*, for which feedback does not increase the capacity [El Gamal 78].



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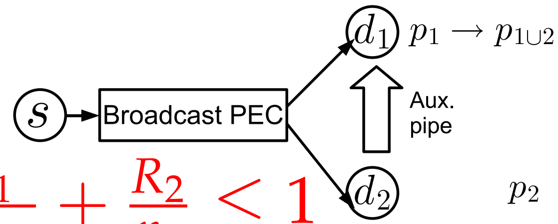

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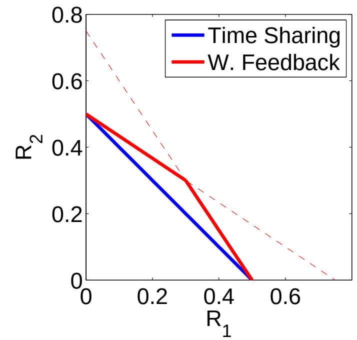

The cap. of the original CH with feedback
$\prec$ The cap. of the new physically degraded CH with feedback
$=$ The cap. of the new physically degraded CH without feedback

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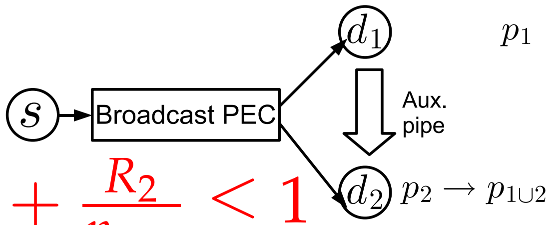

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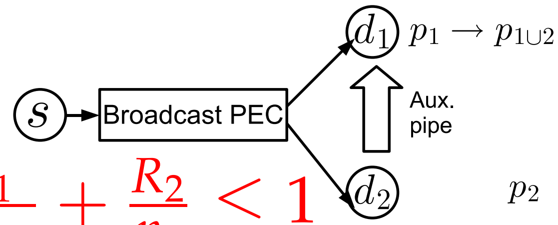

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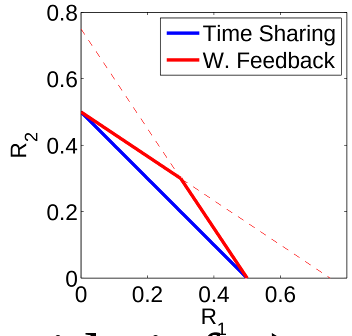

- Inner bound: A 2-phase approach. (Creating its own side info.)

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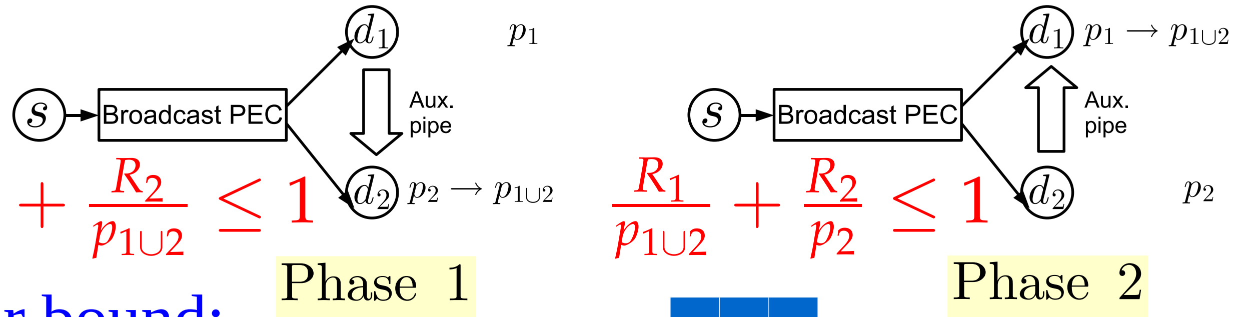

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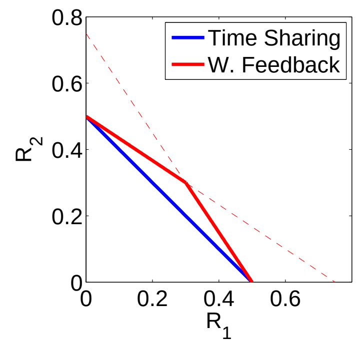

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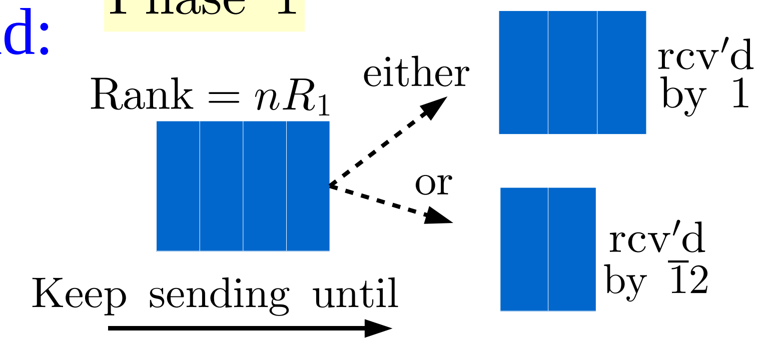

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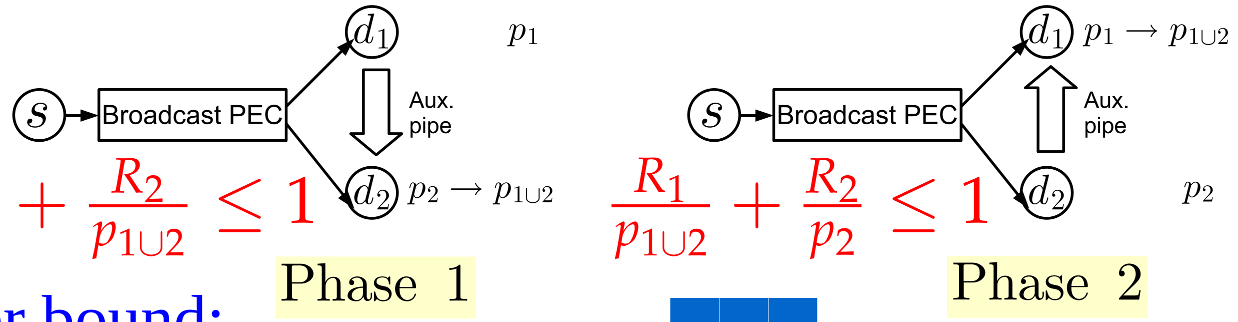

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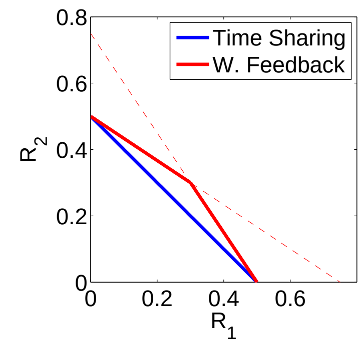

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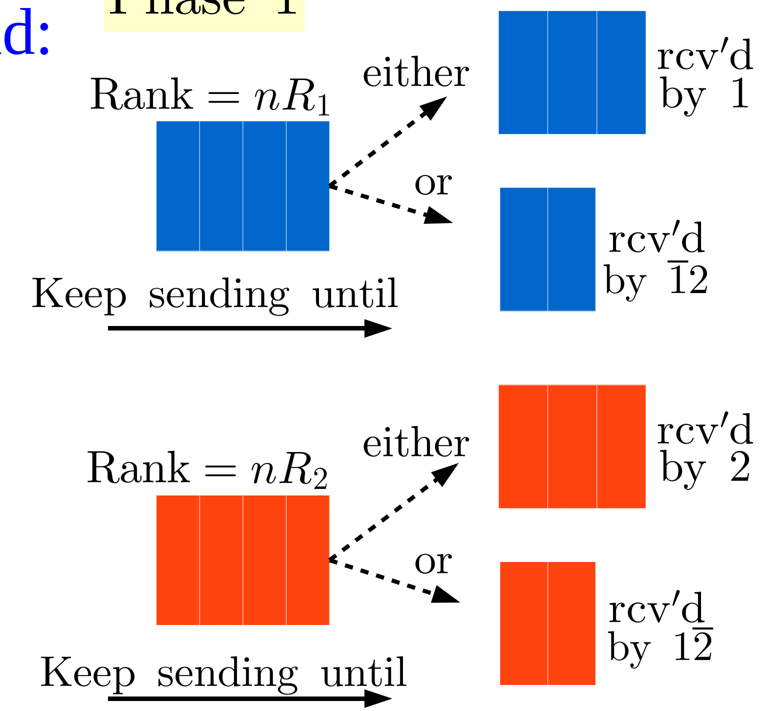

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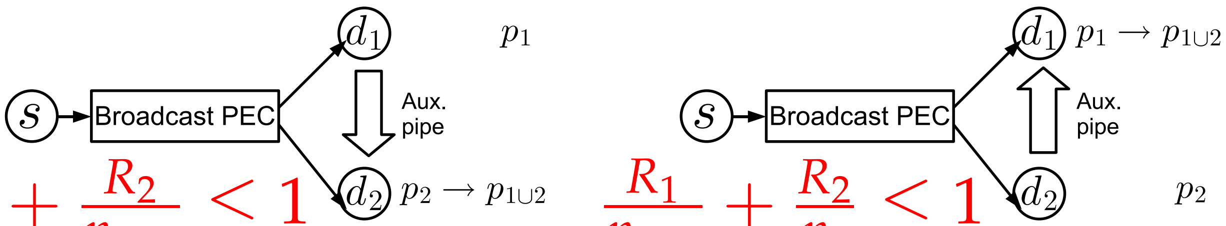

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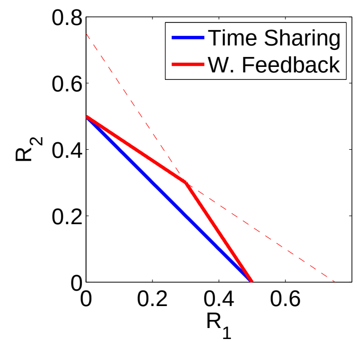

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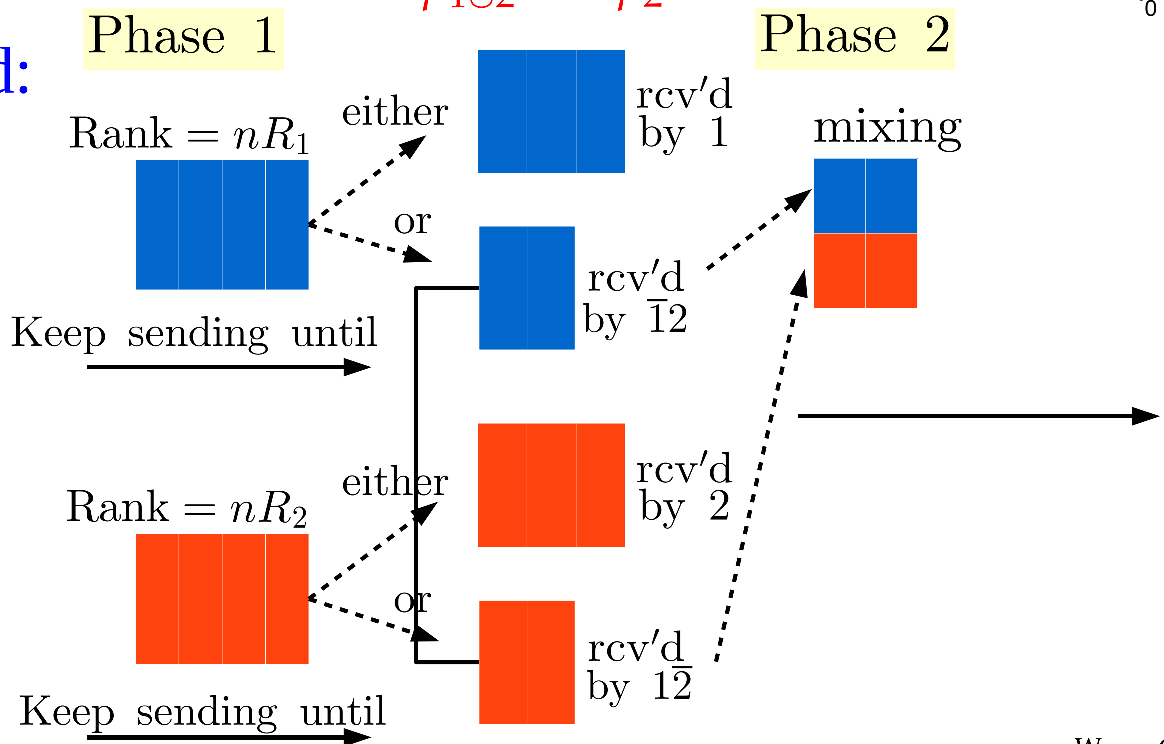

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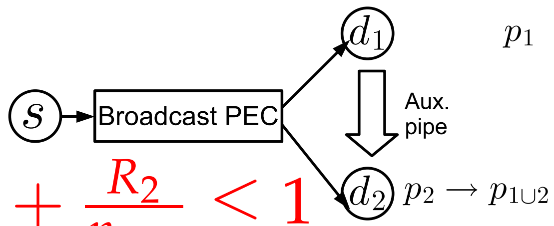

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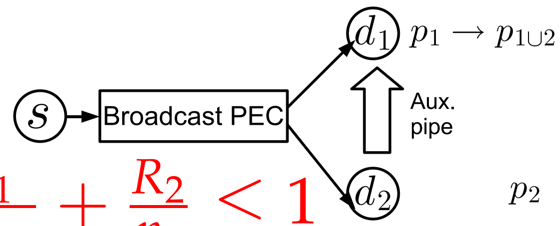

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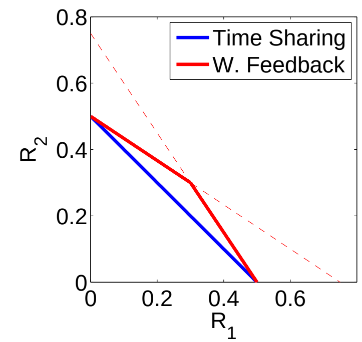

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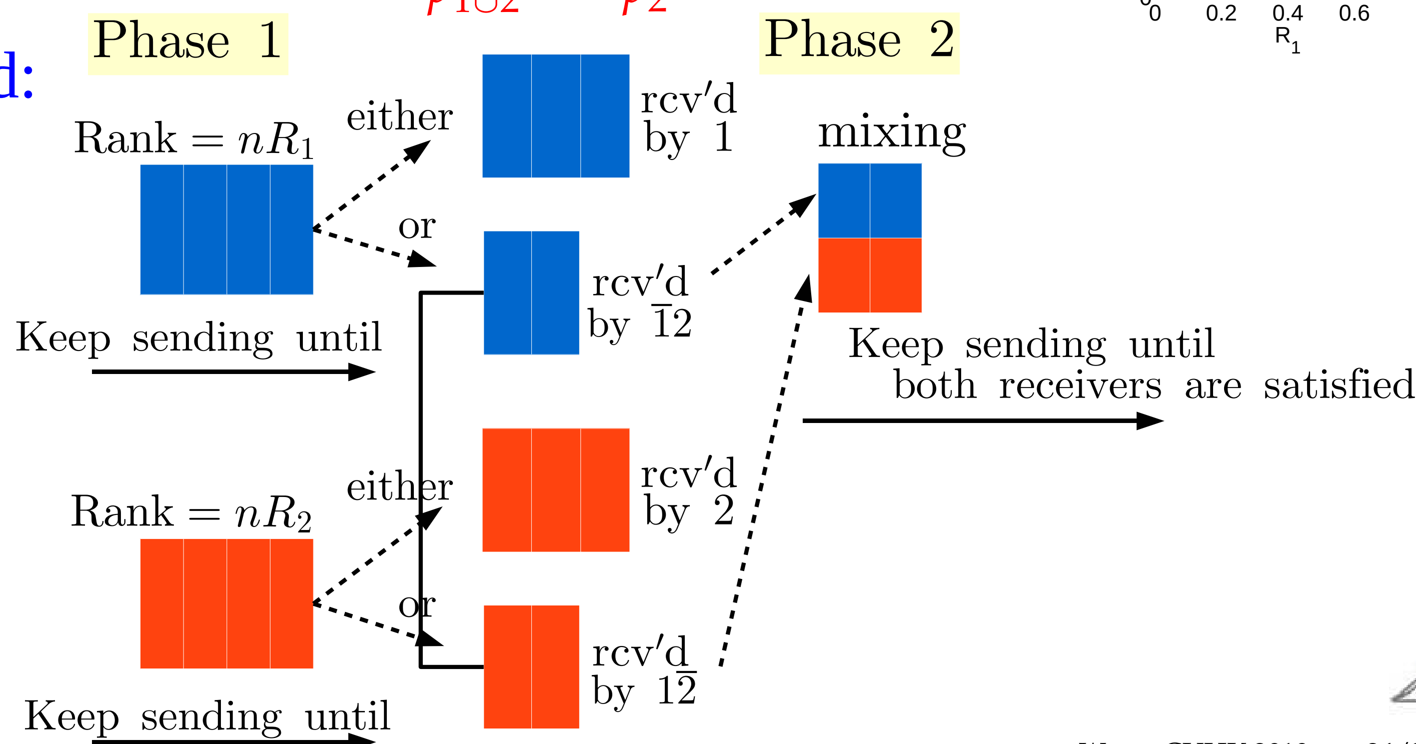

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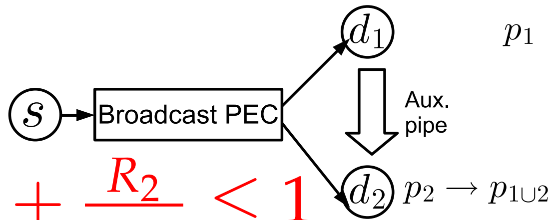

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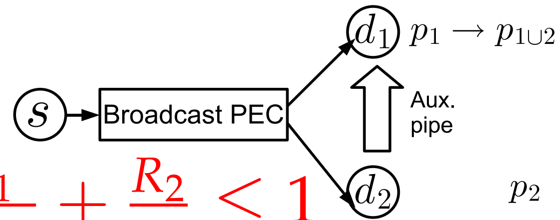

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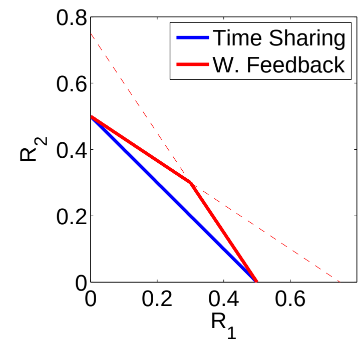

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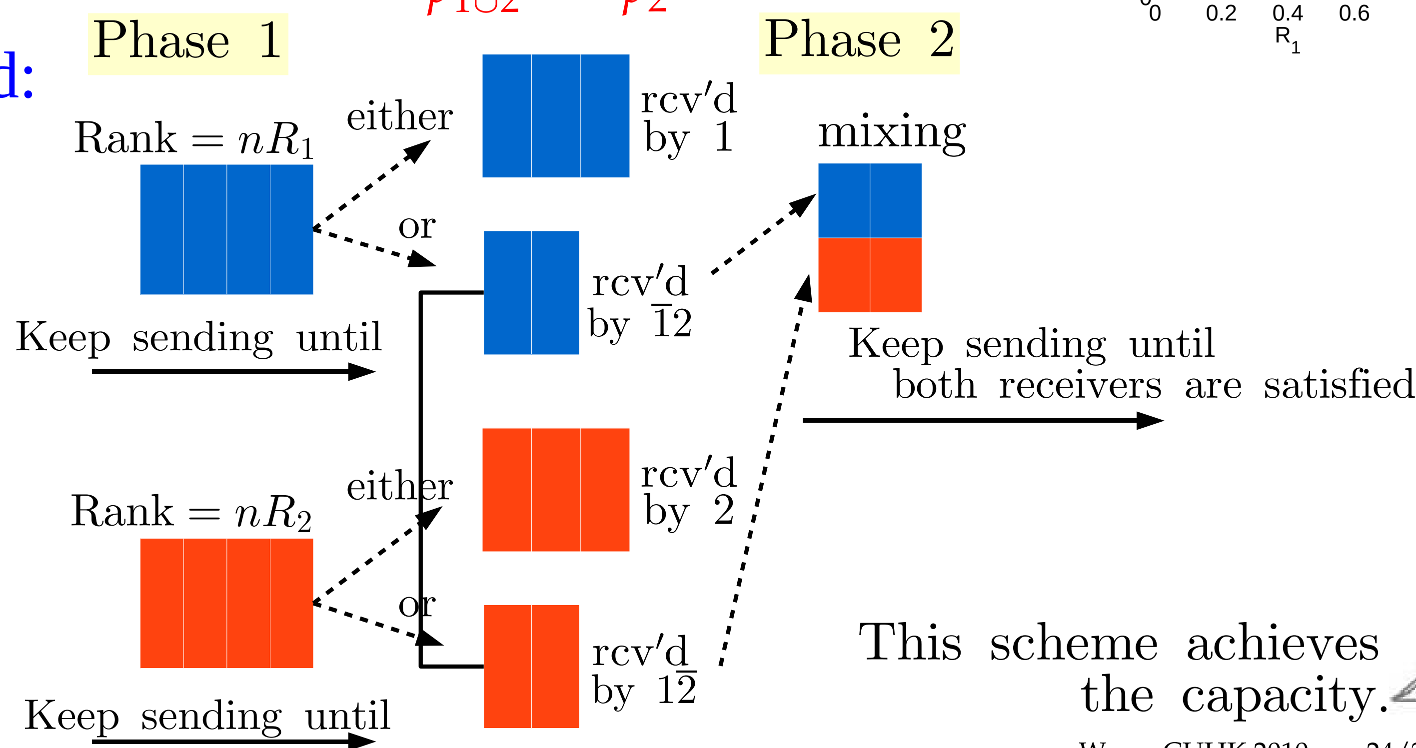
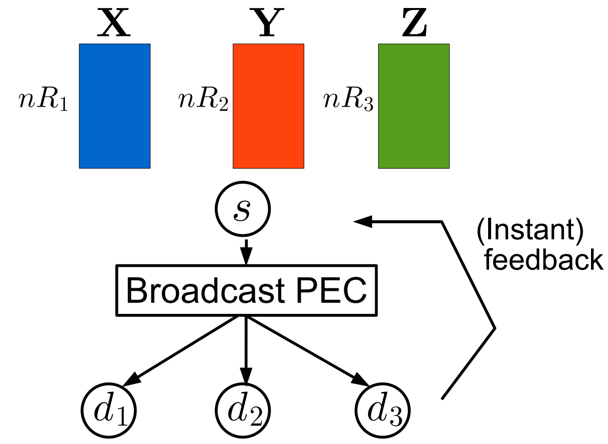


This scheme achieves the capacity.

What if $M \geq 3$?



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- The CH. parameters become more involved.
  - $M = 2$: p_{12}, p_{12^c}, p_{1^c2}, $p_{1^c2^c}$.
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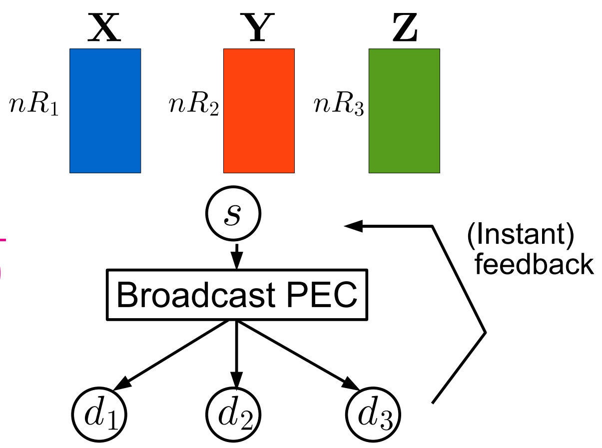

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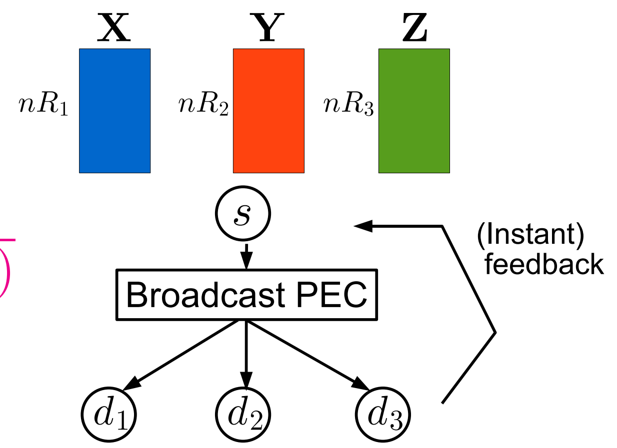

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- Can we also quantify the Shannon capacity for $M \geq 3$?

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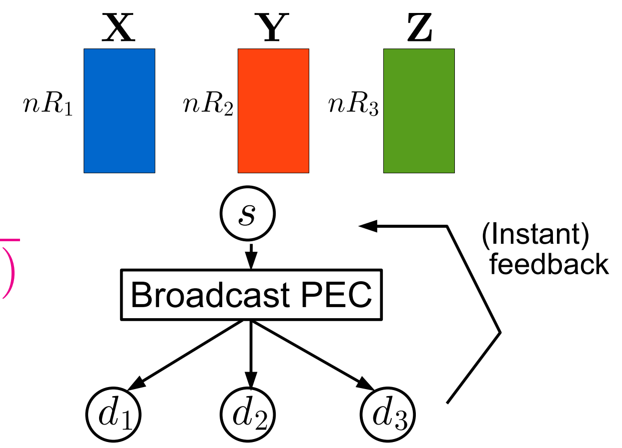
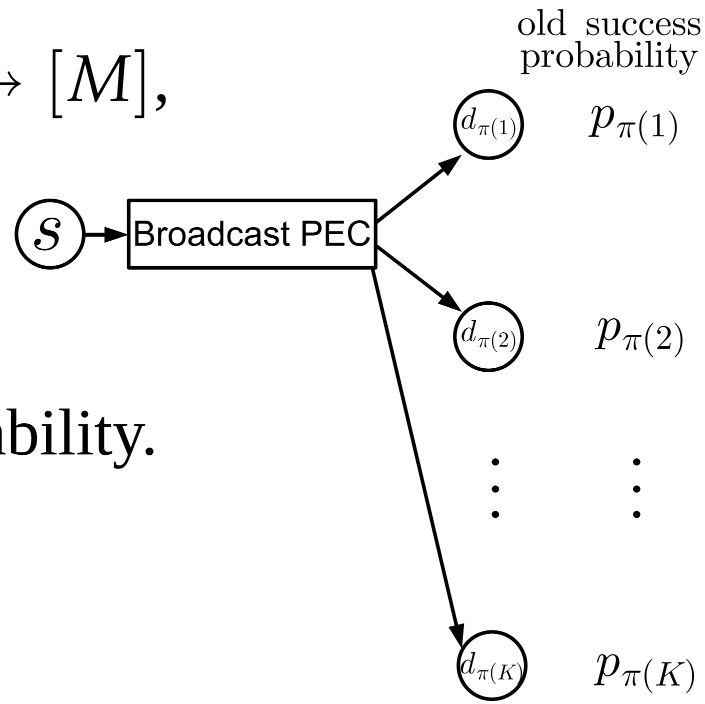
- The CH. parameters become more involved.
  - $M = 2$: p_{12}, p_{12^c}, p_{1^c2}, $p_{1^c2^c}$.
  - $M \geq 3$: the success probability $p_{S\overline{([M]\backslash S)}}$ that a packet is received *by and only by* $d_i \in S$. We have 2^M such parameters.



- Can we also quantify the Shannon capacity for $M \geq 3$?
  - Generalization of the outer bound is straightforward.
  - Generalization of the inner bound is more difficult.

Simple Cap. Outer Bound

- For any permutation $\pi : [M] \mapsto [M]$,
- p_k: The marginal success probability.



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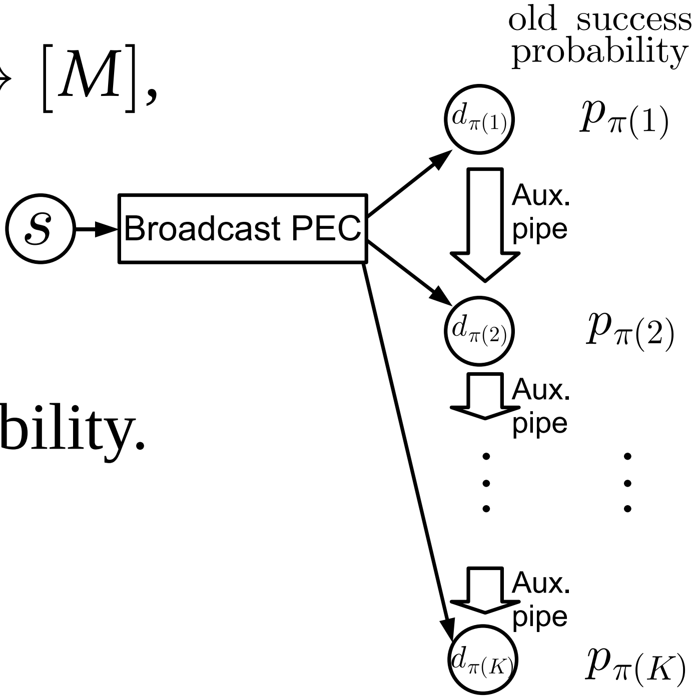

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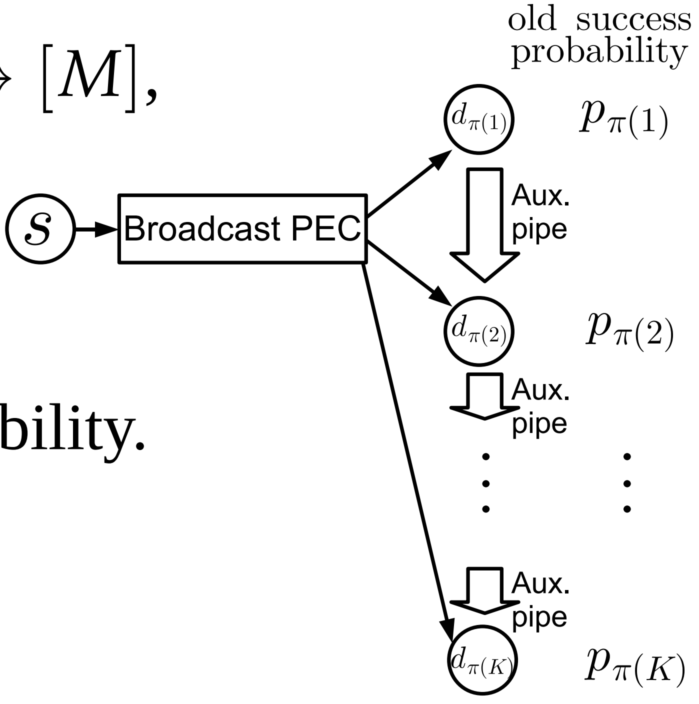

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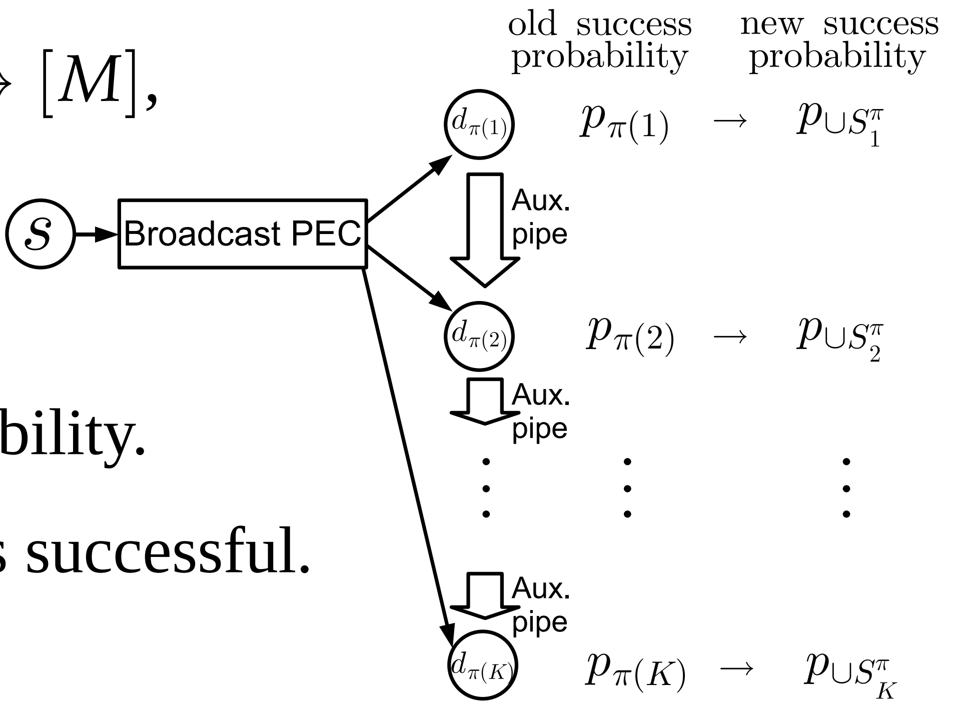

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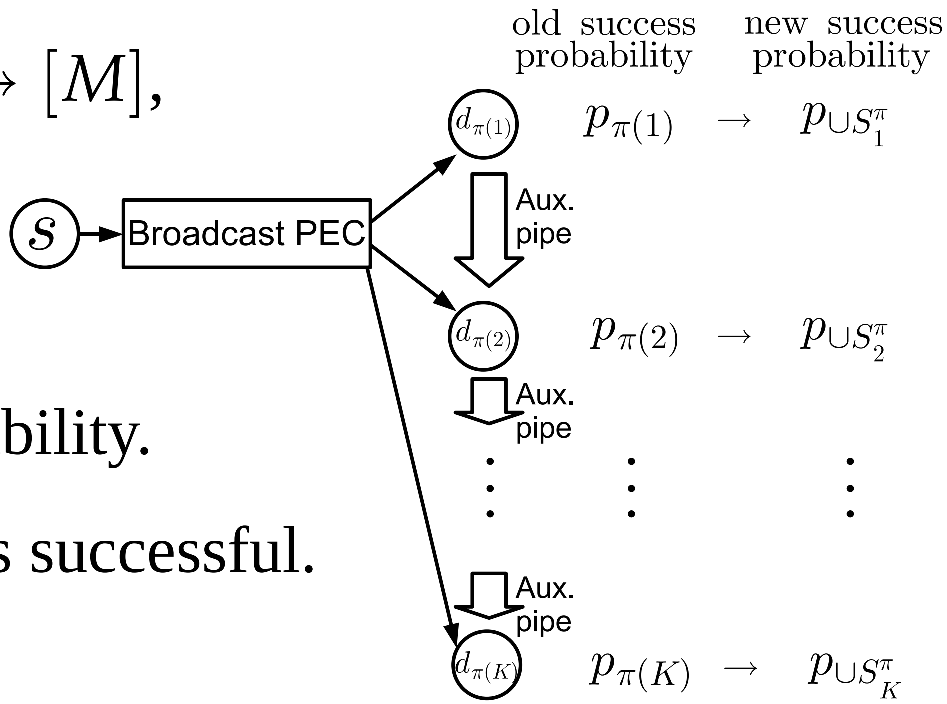

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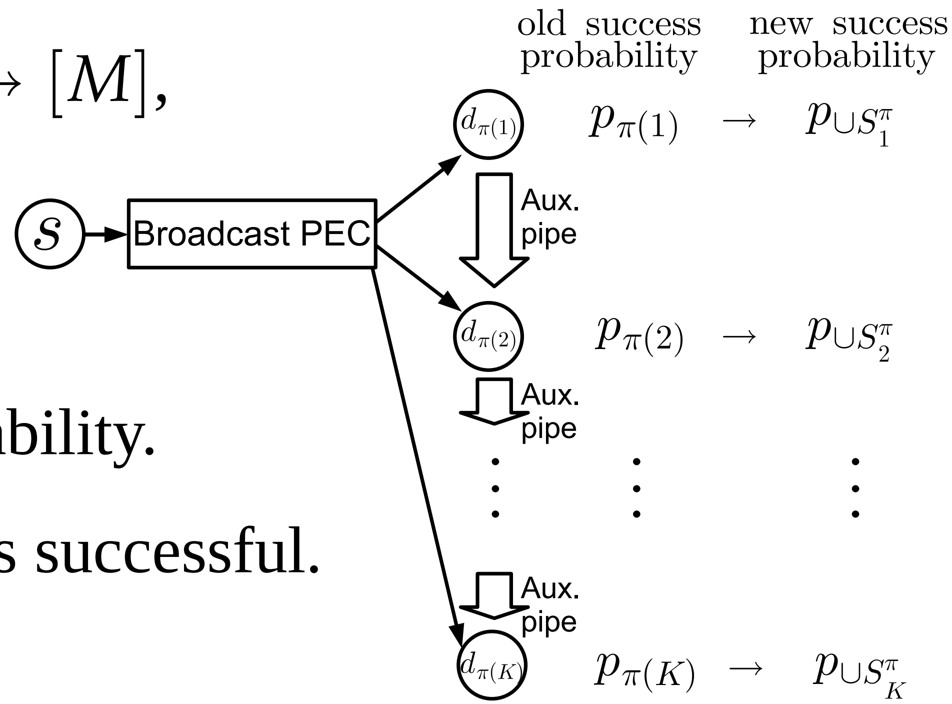

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- A capacity outer bound is thus $\forall \pi, \quad \sum_{k=1}^{M} \frac{R_{\pi(k)}}{p_{\cup S_k^\pi}} \leq 1.$

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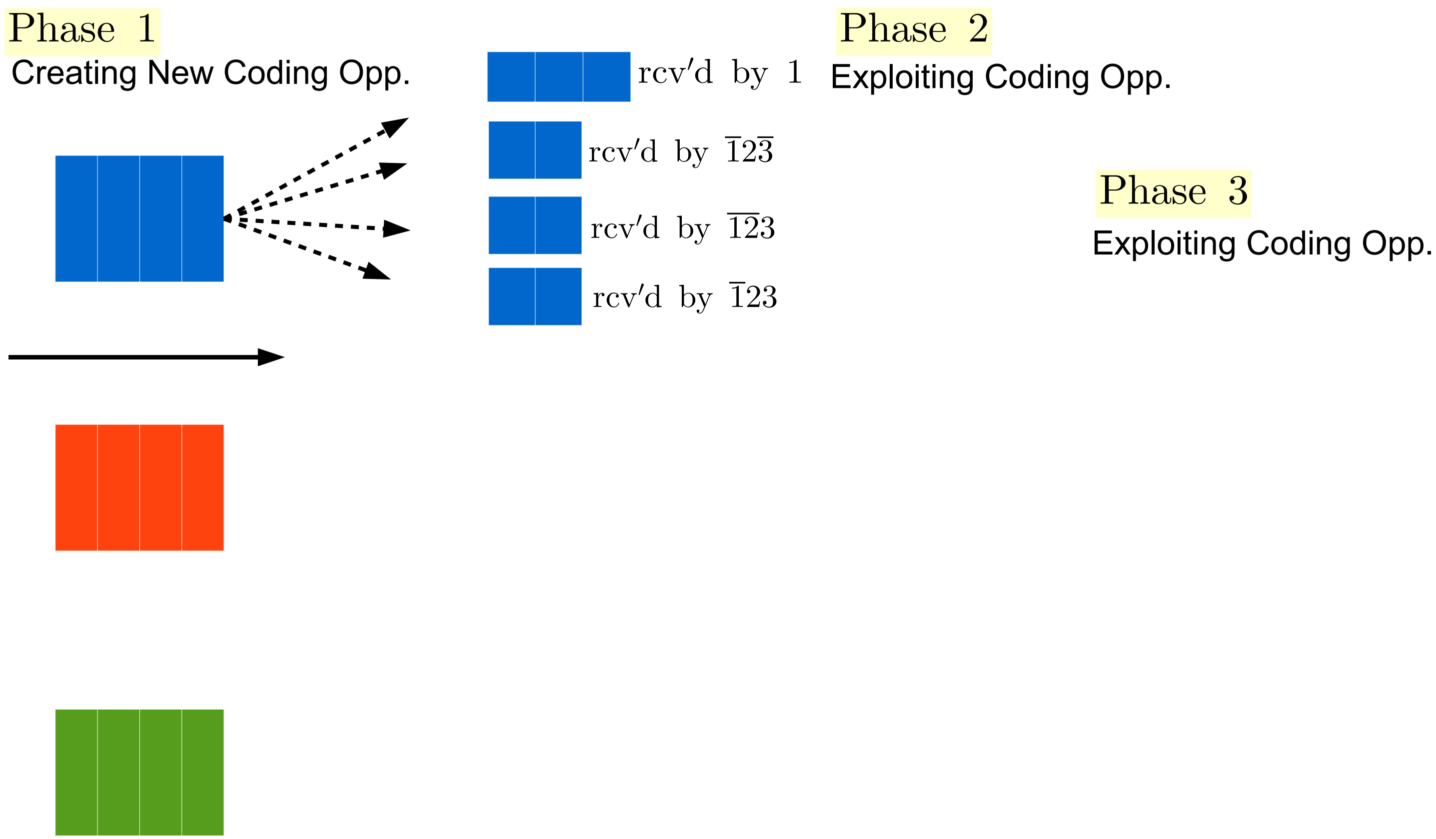

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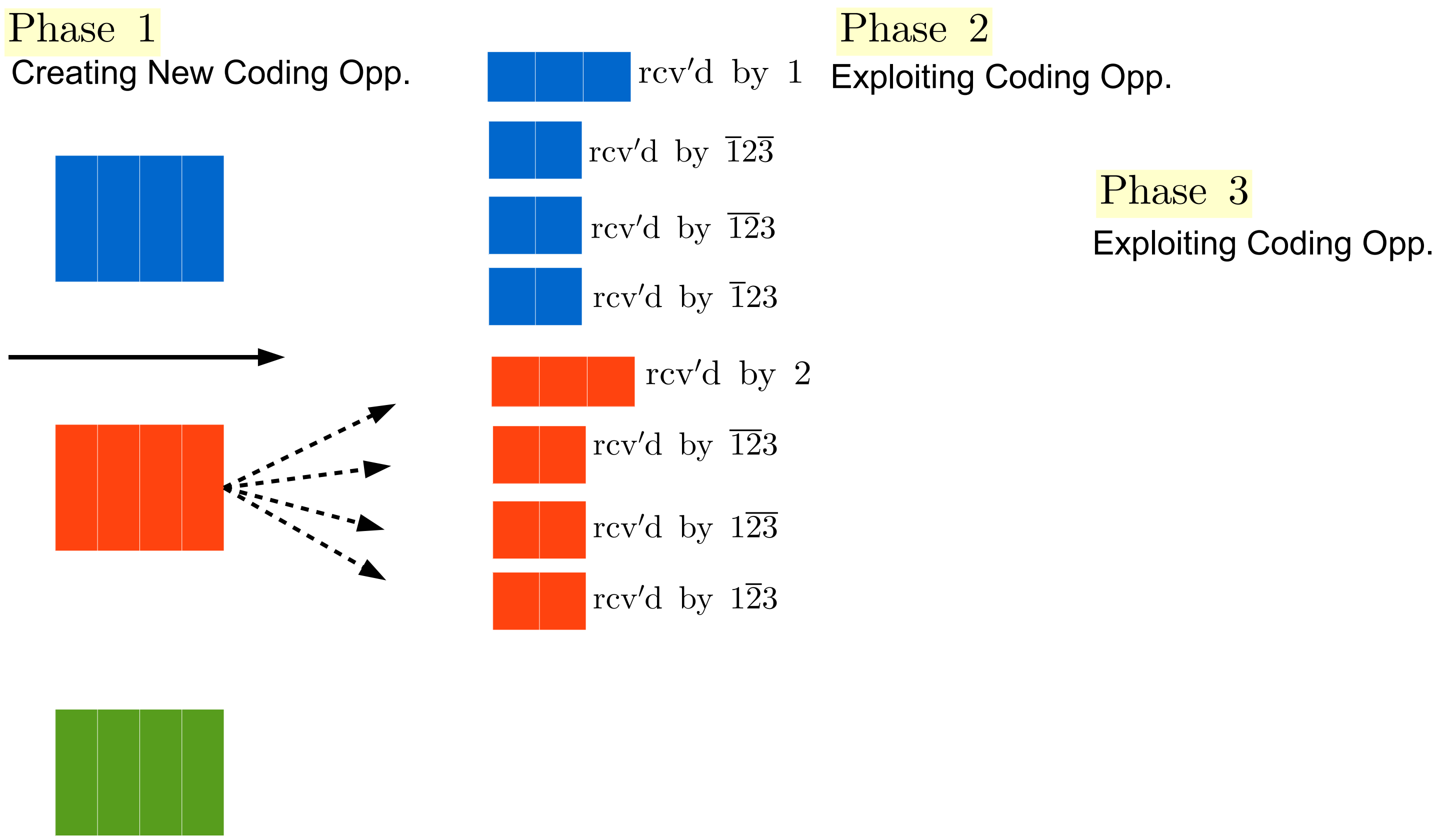

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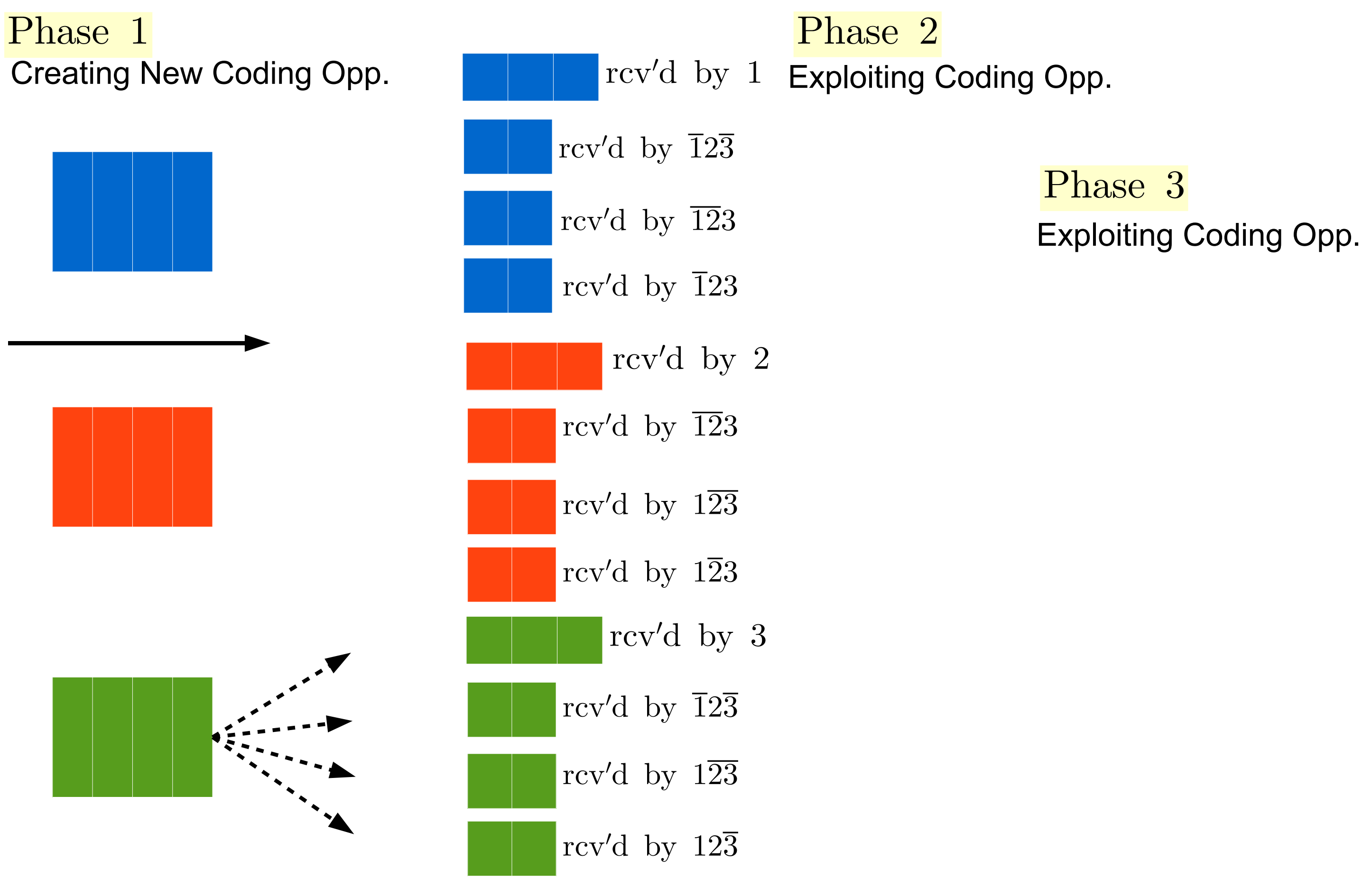

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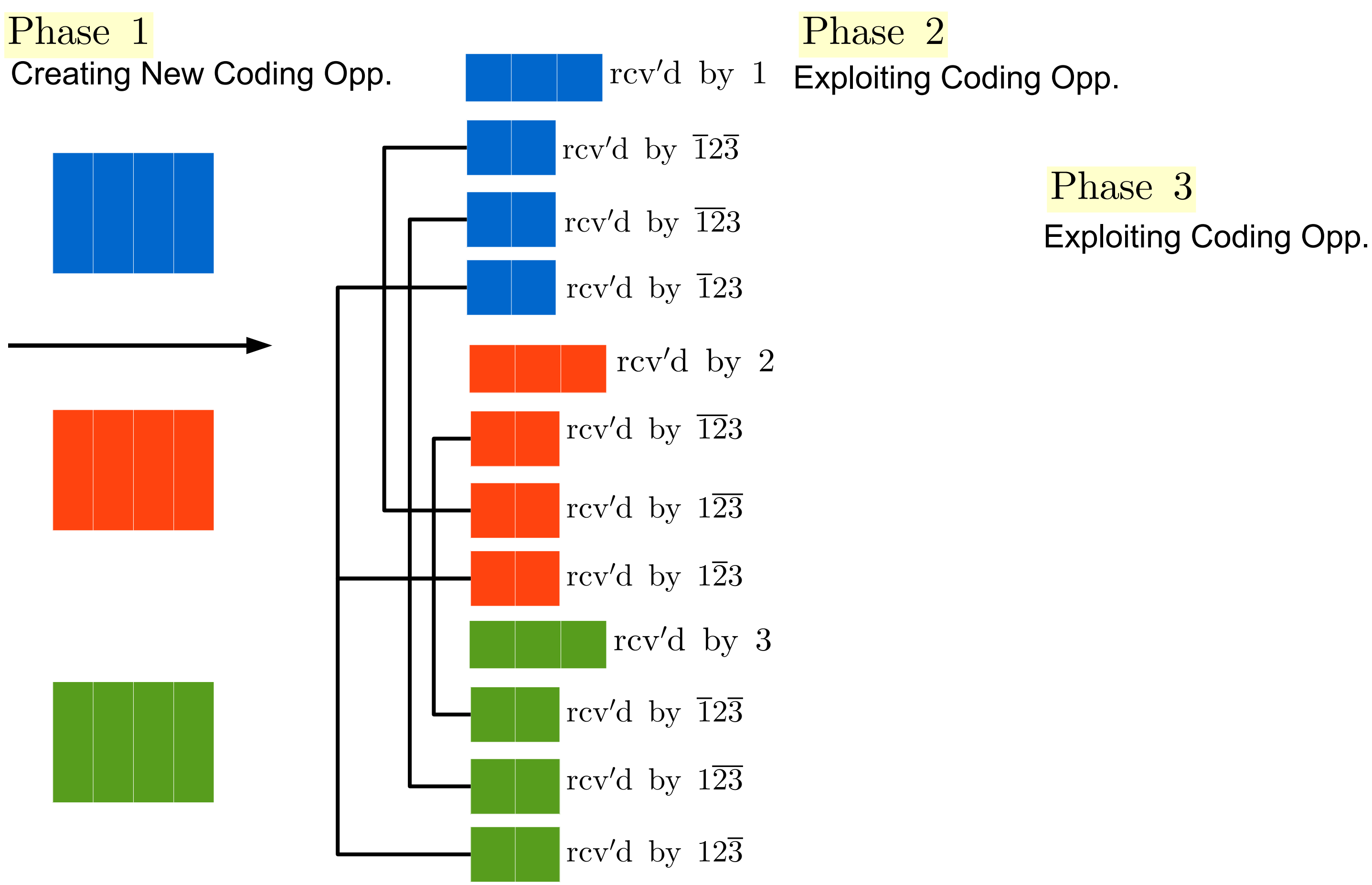

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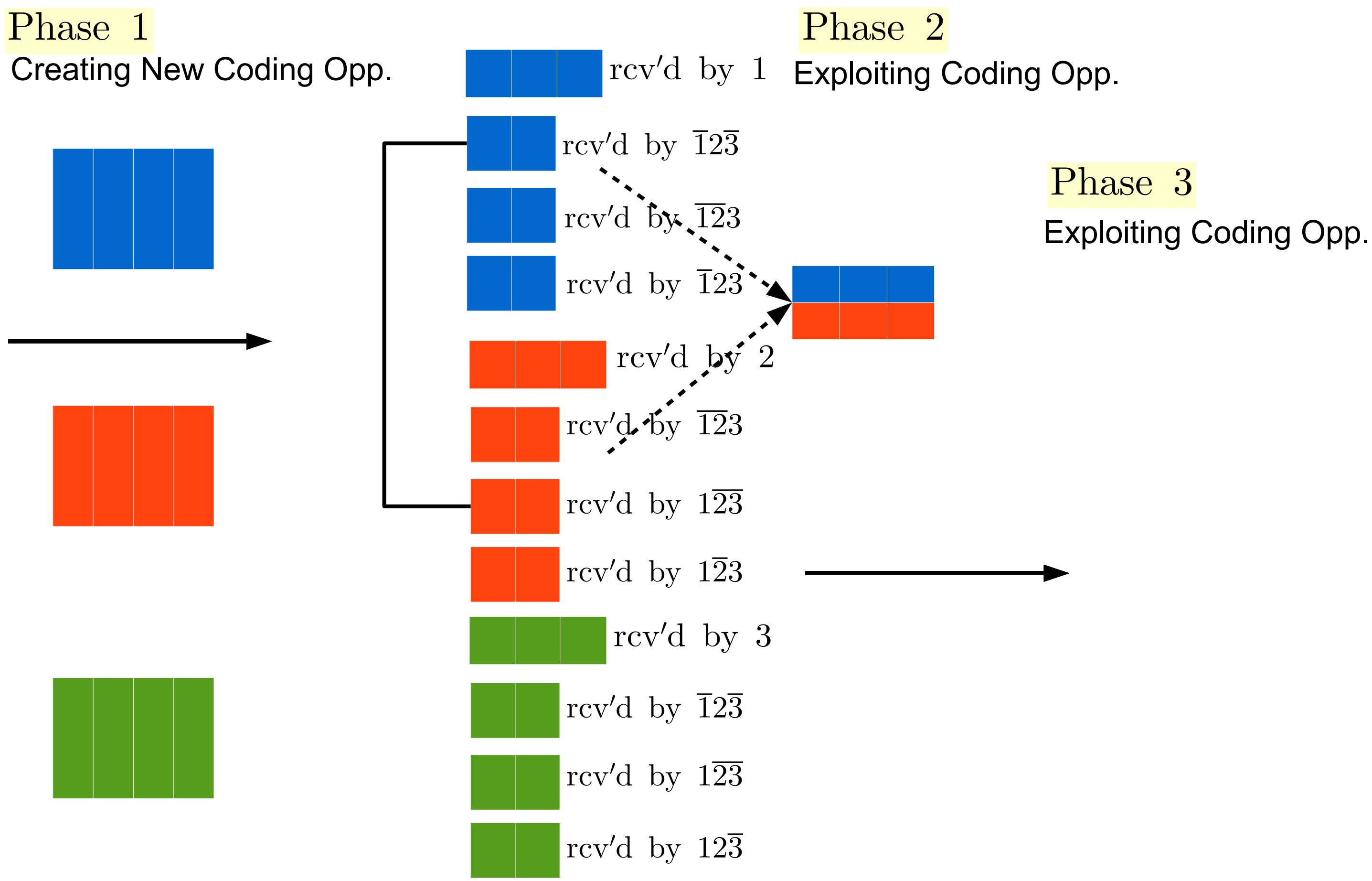

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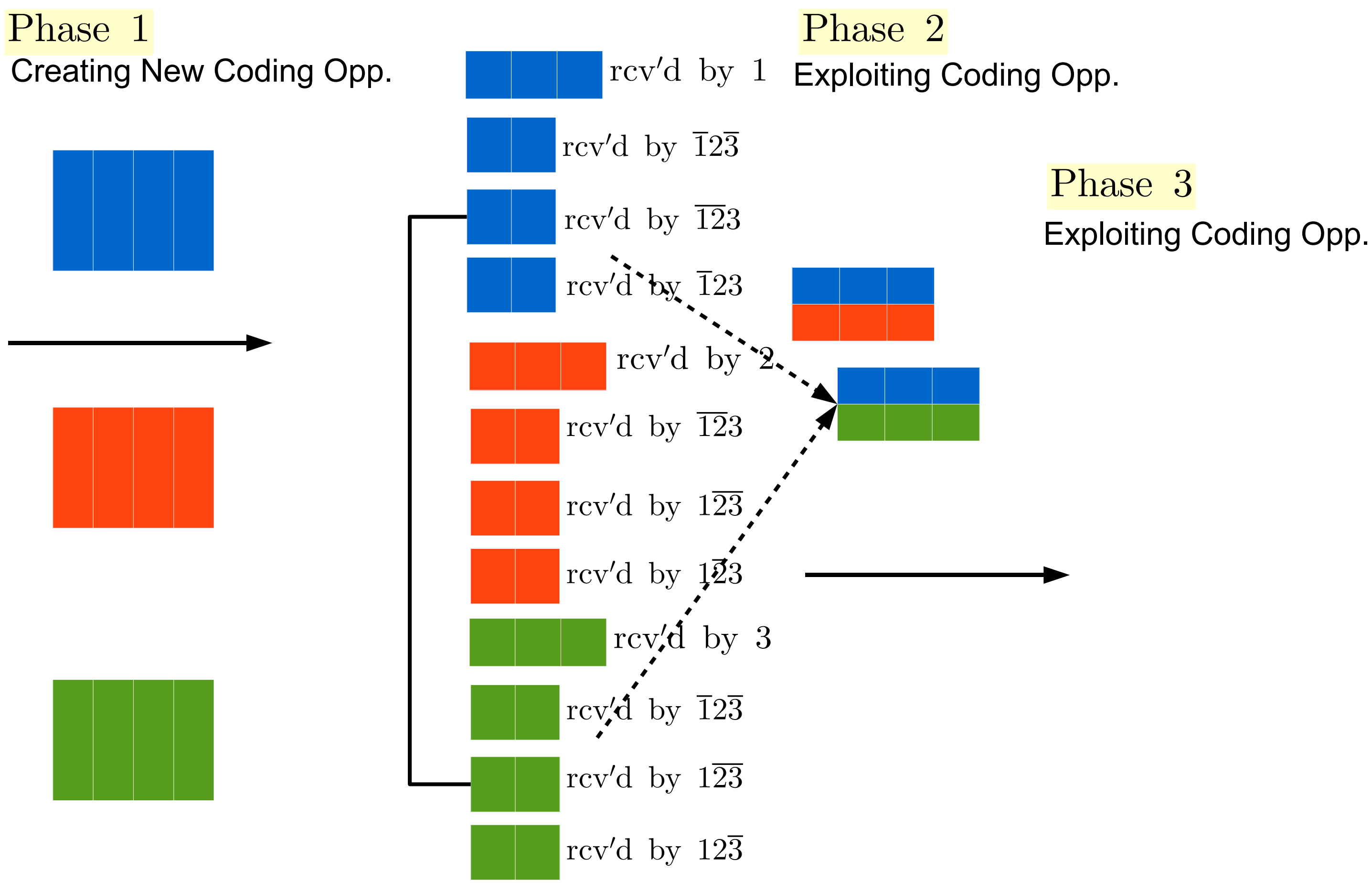

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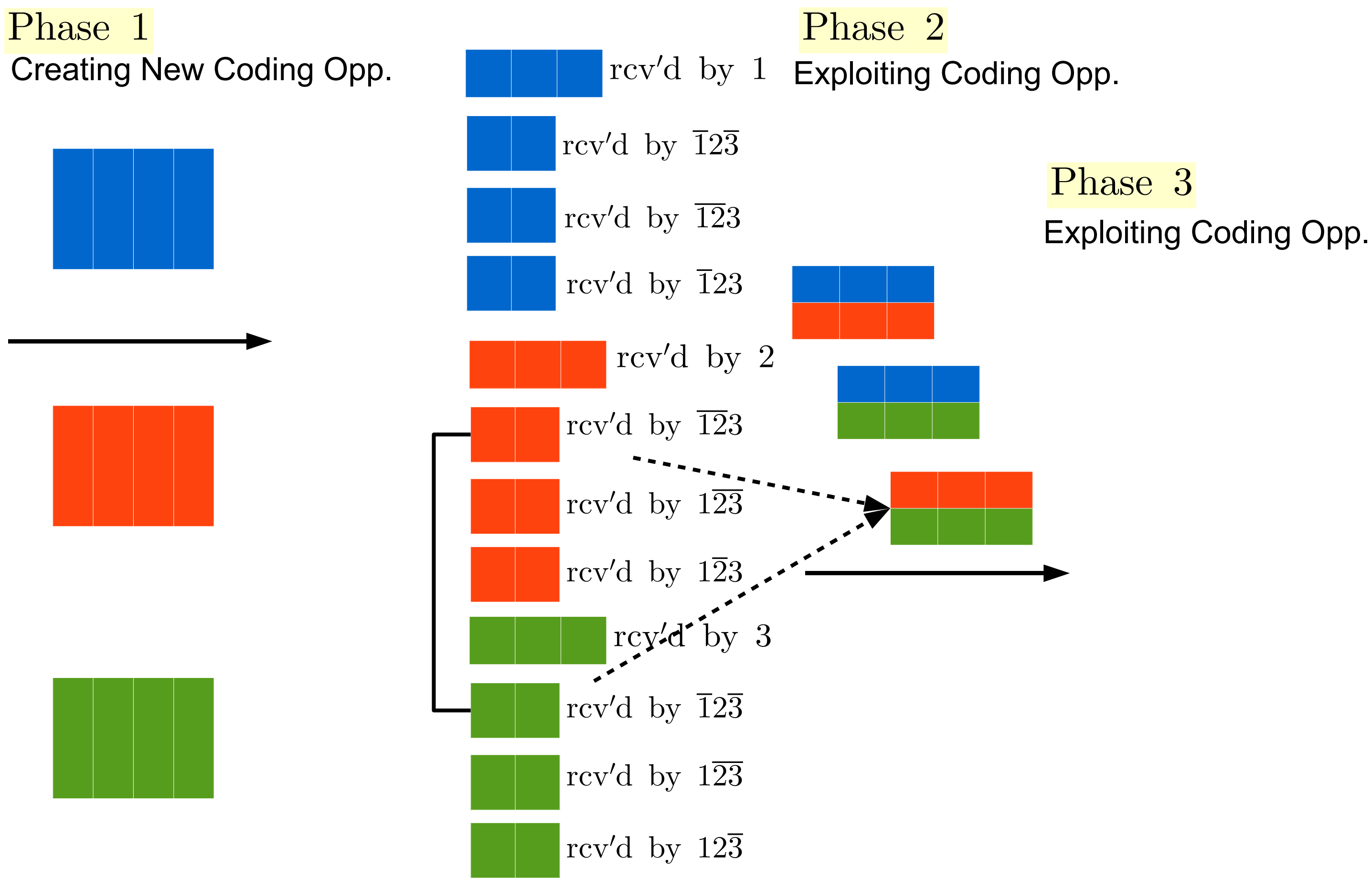

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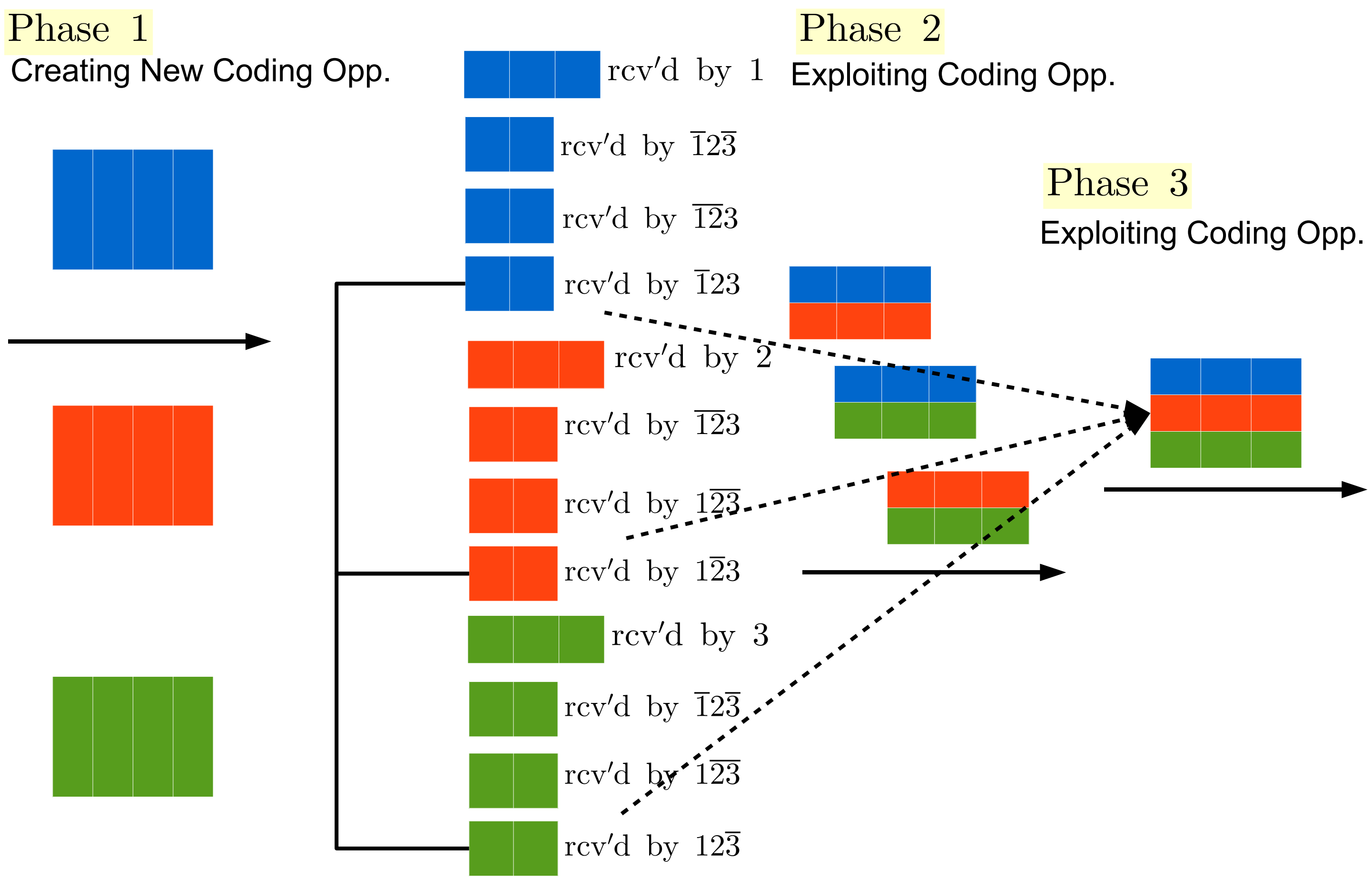

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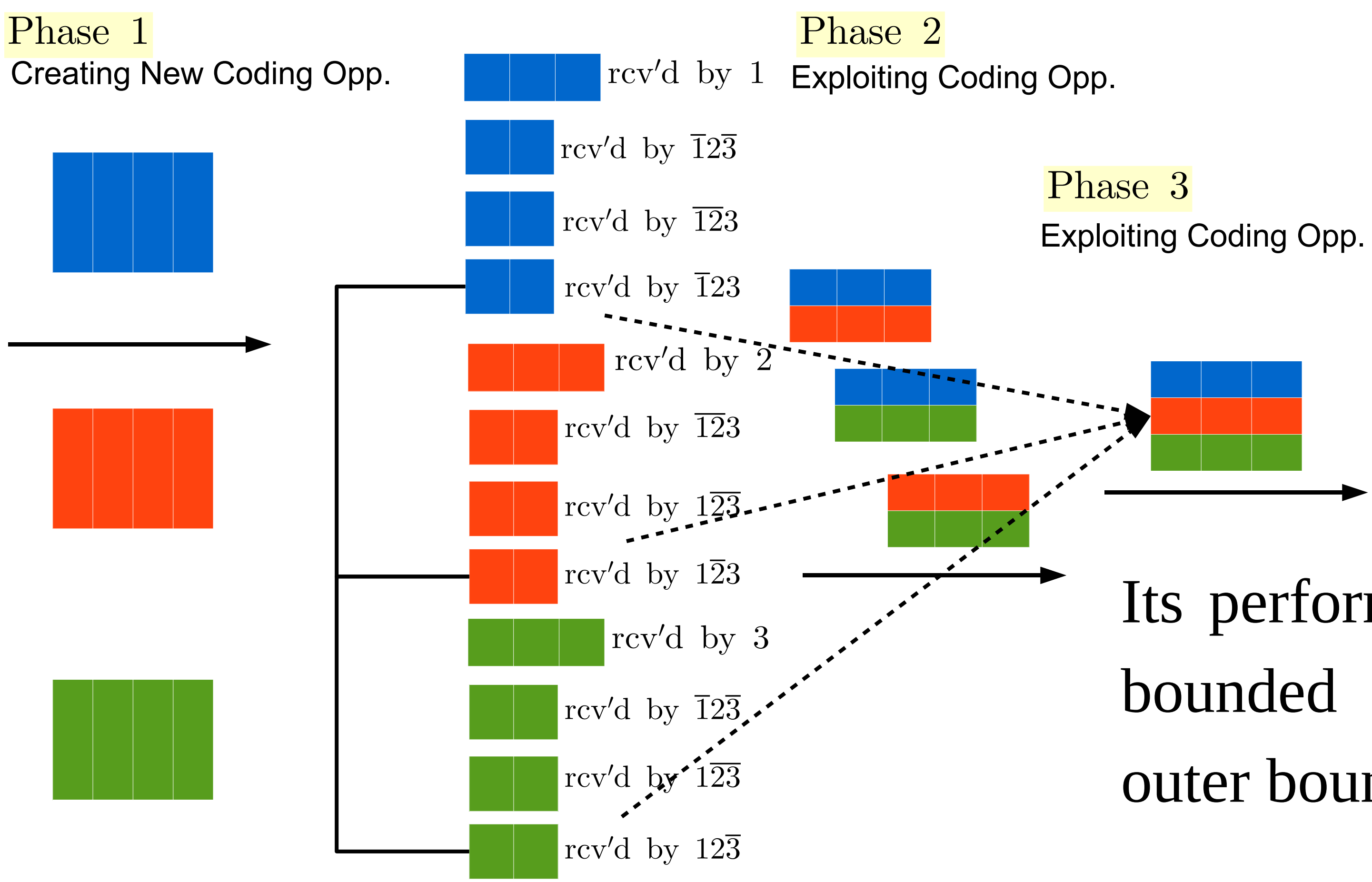
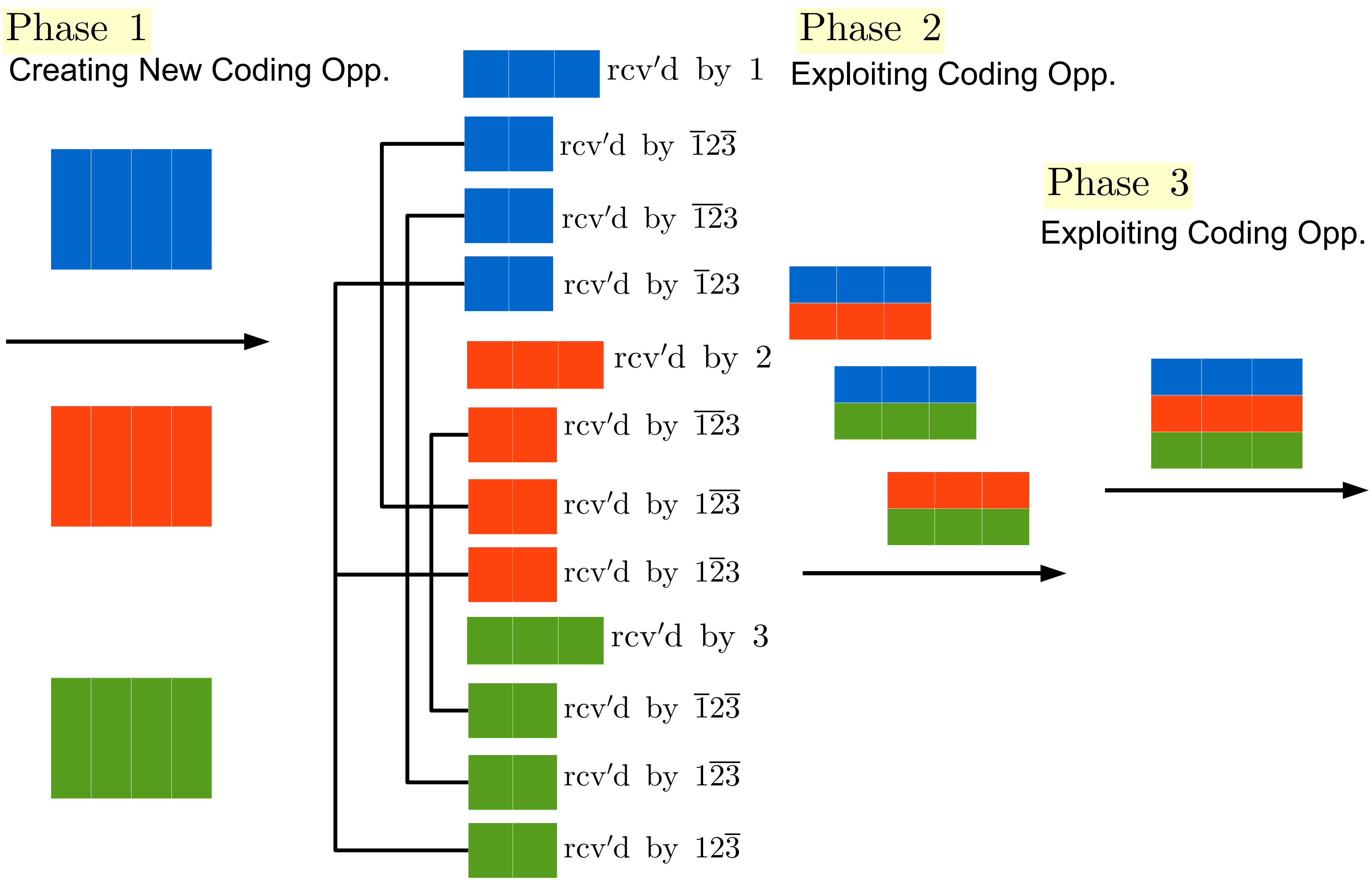


Its performance is strictly bounded away from the outer bound.

What Went Wrong?



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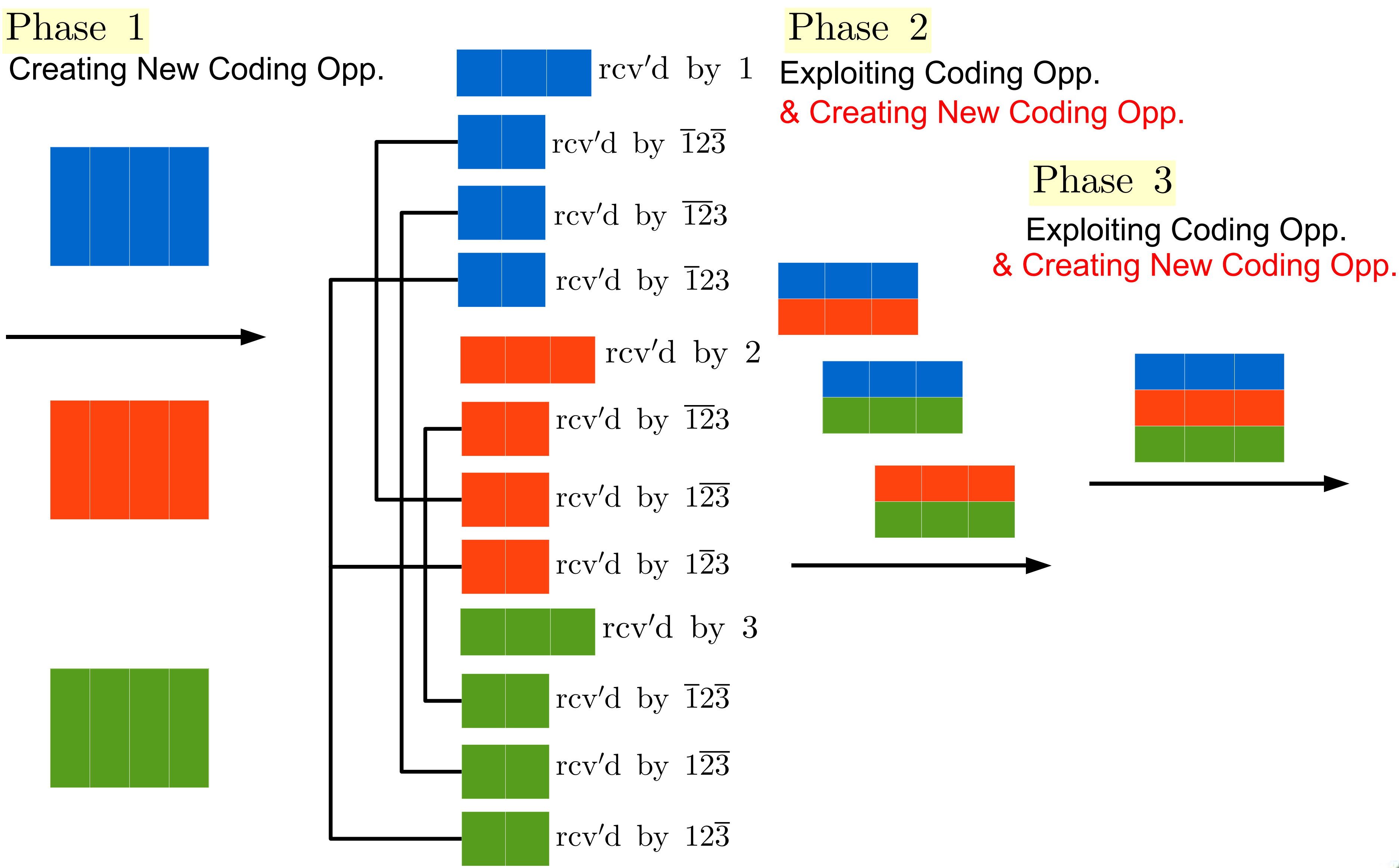

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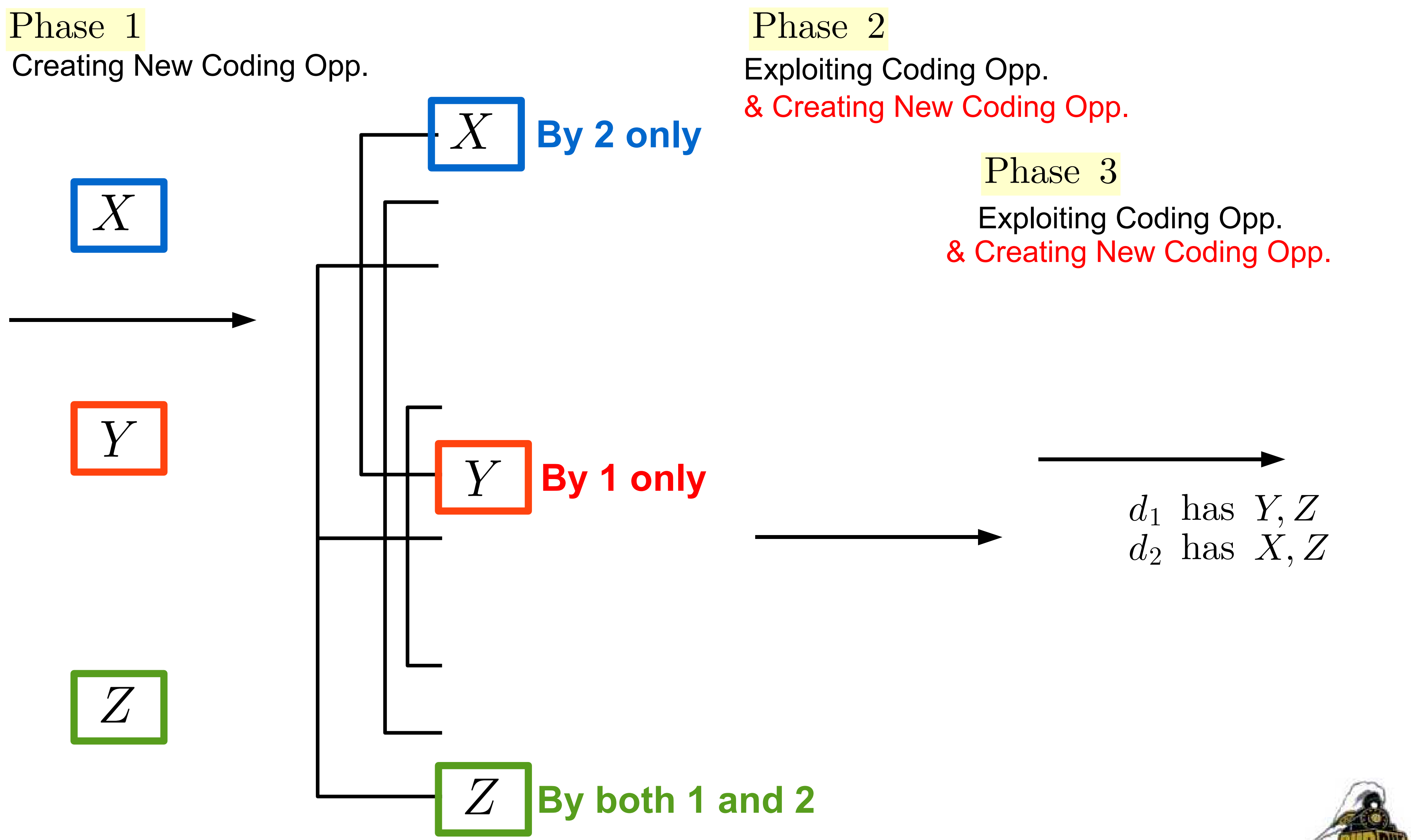
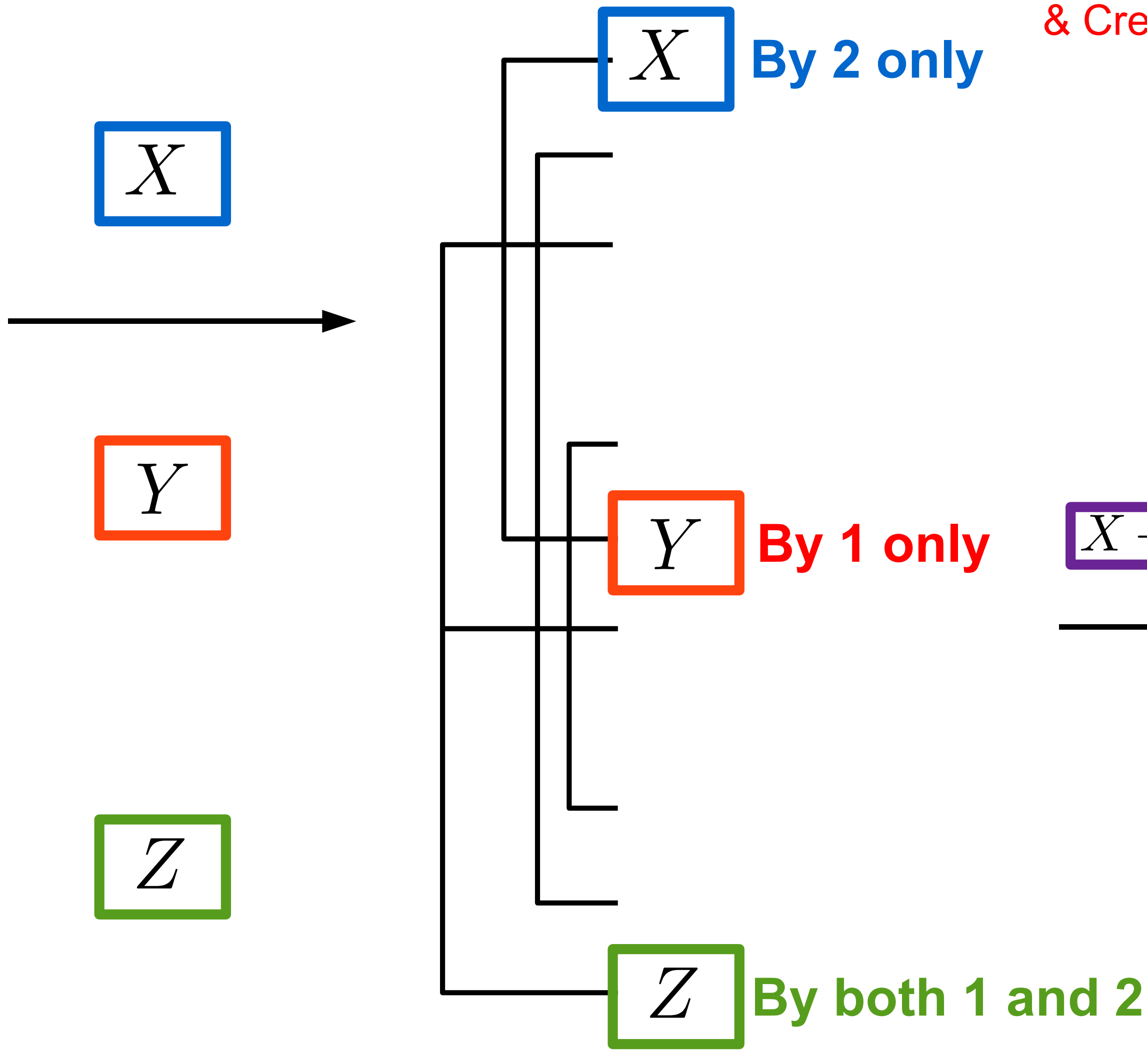


What Went Wrong?

Phase 1

Creating New Coding Opp.



Phase 2

Exploiting Coding Opp.

& Creating New Coding Opp.

Phase 3

Exploiting Coding Opp.

& Creating New Coding Opp.

$X + Y$

rcv'd by $\overline{1}23$

d_1 has Y, Z

d_2 has X, Z

d_3 has $X + Y$

What Went Wrong?

Phase 1
Creating New Coding Opp.

Phase 2
Exploiting Coding Opp.
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Phase 3
Exploiting Coding Opp.
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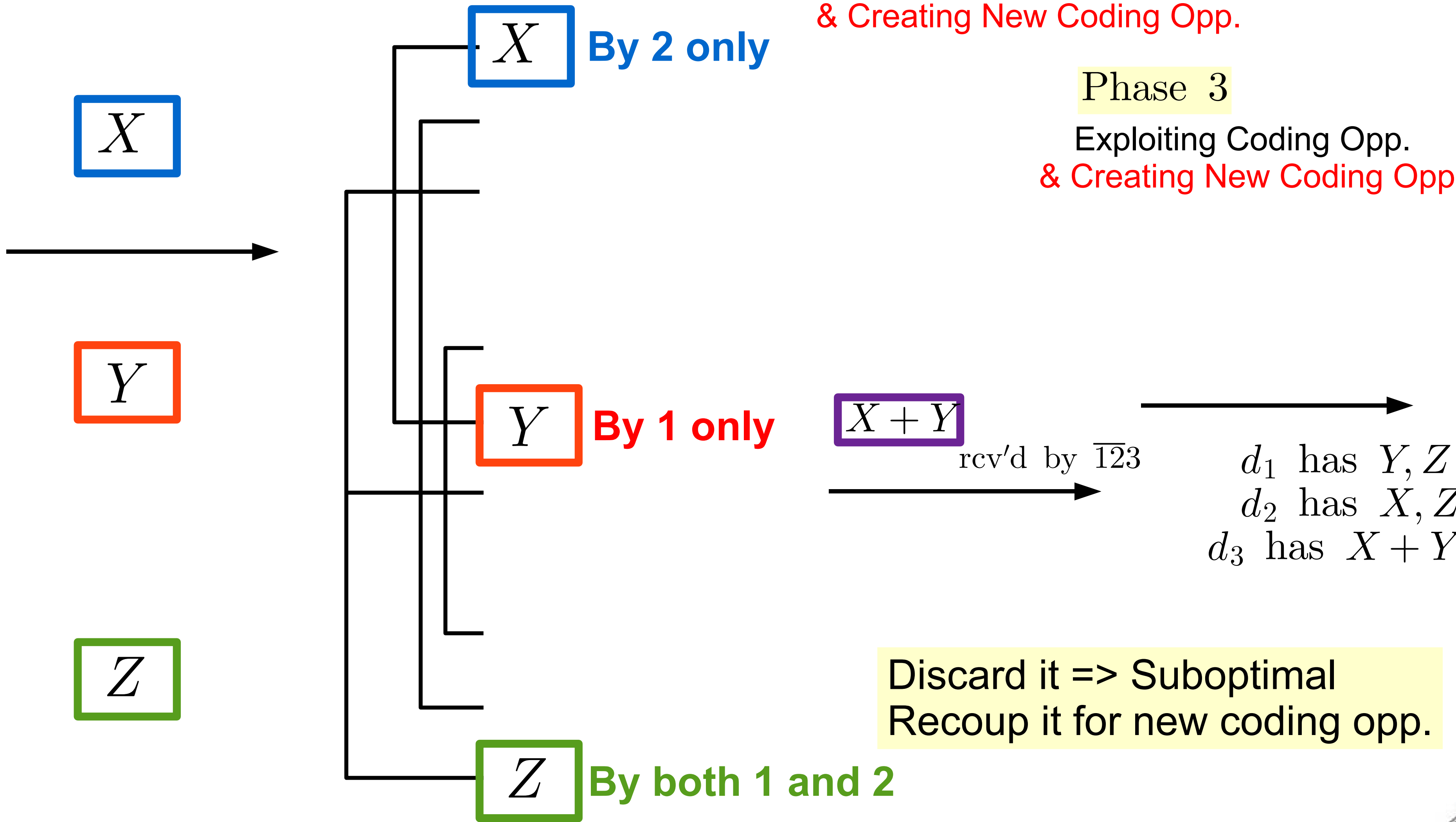
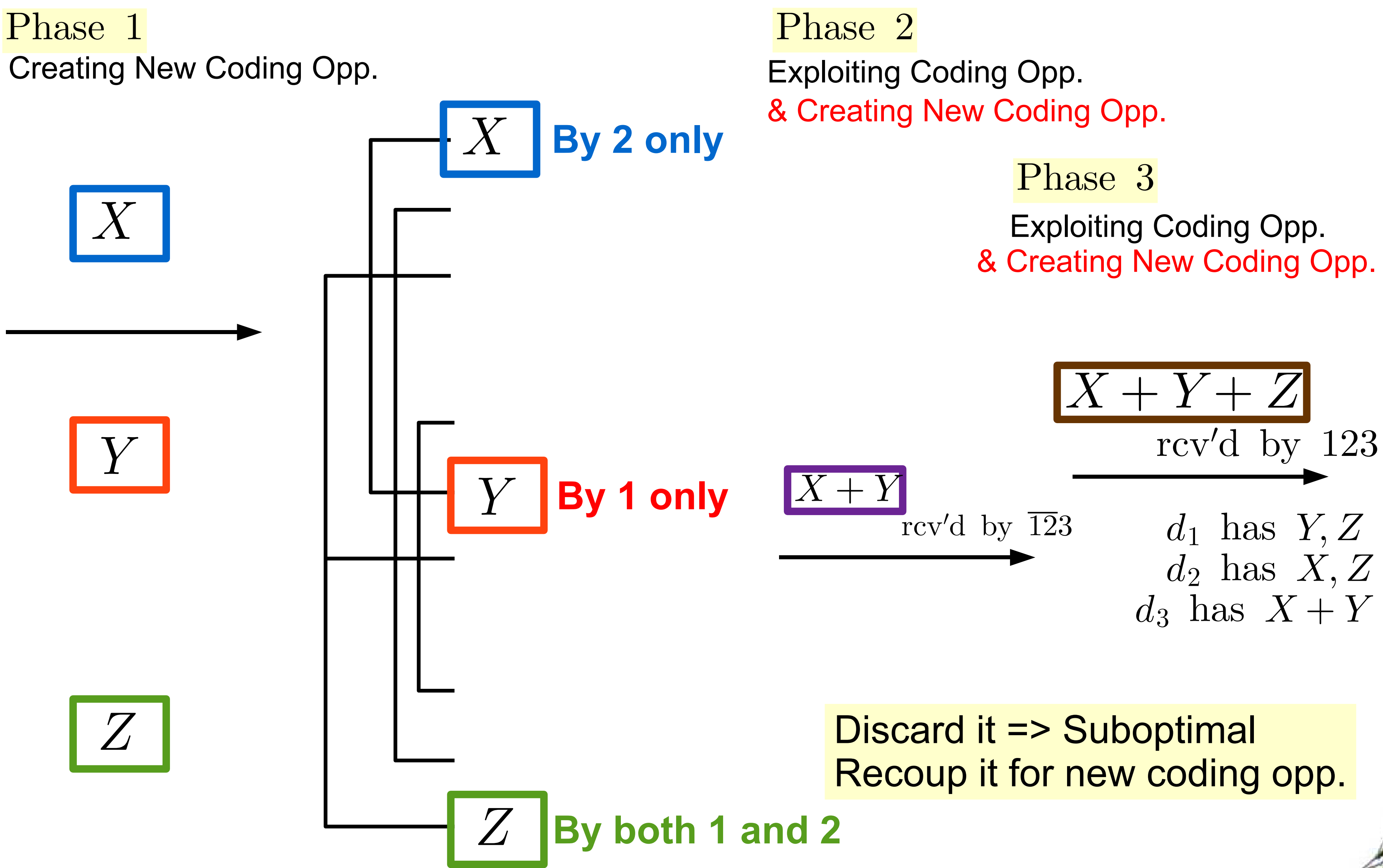


Discard it => Suboptimal
Recoup it for new coding opp.

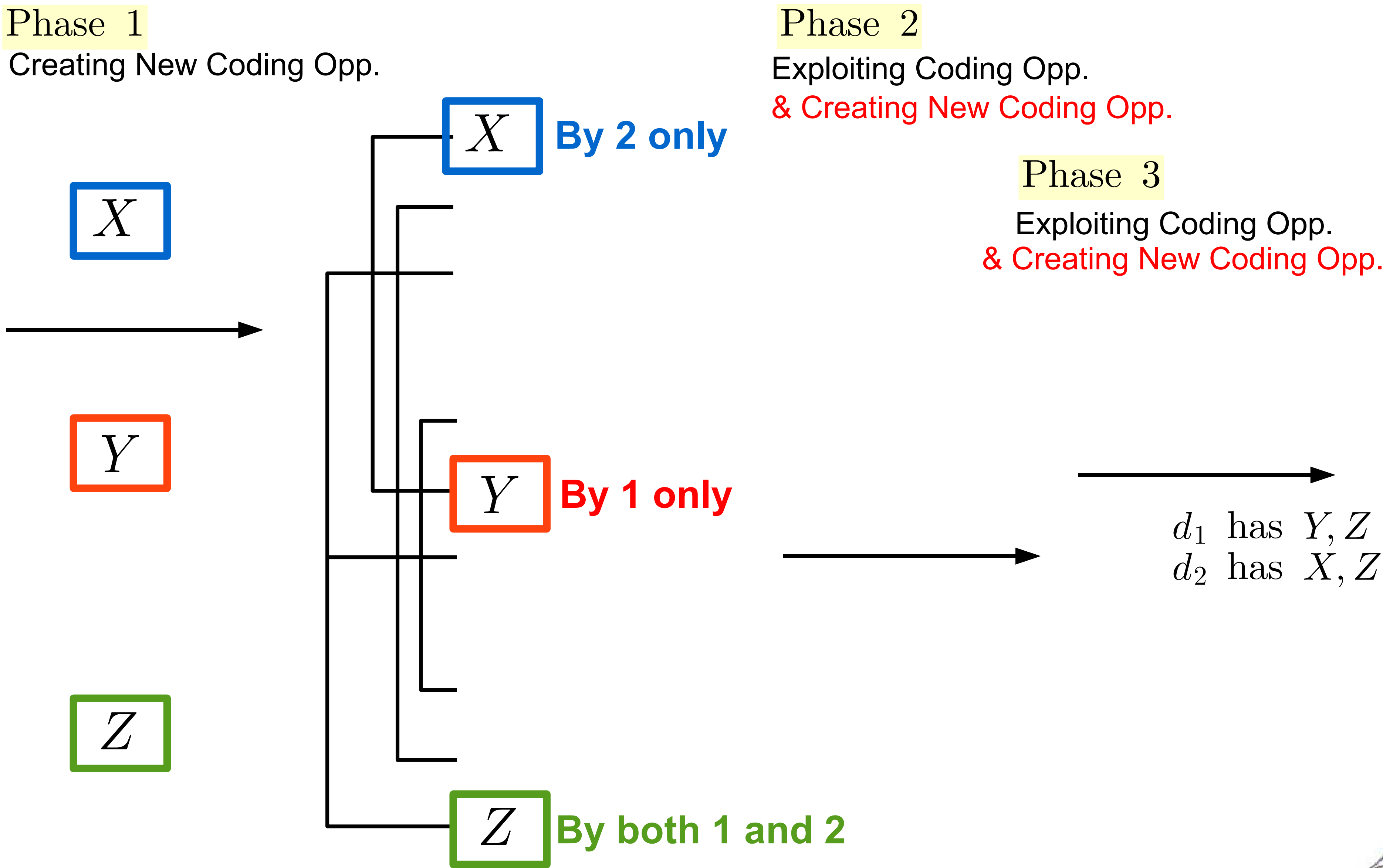
What Went Wrong?



New Cap. Inner Bound

- Again, we need code alignment in order to recoup the overheard coding opportunities during Phases 2 to M.
- That is, the overheard coding vectors $[X+Y]$ has to remain aligned in the subsequent mixing stages.
- We propose a new Packet Evolution scheme.
- For each packet,
  - The overhearing status keeps evolving to create more coding opportunities.
  - The representative coding vector keeps evolving to ensure code alignment.

The Packet Evolution Scheme



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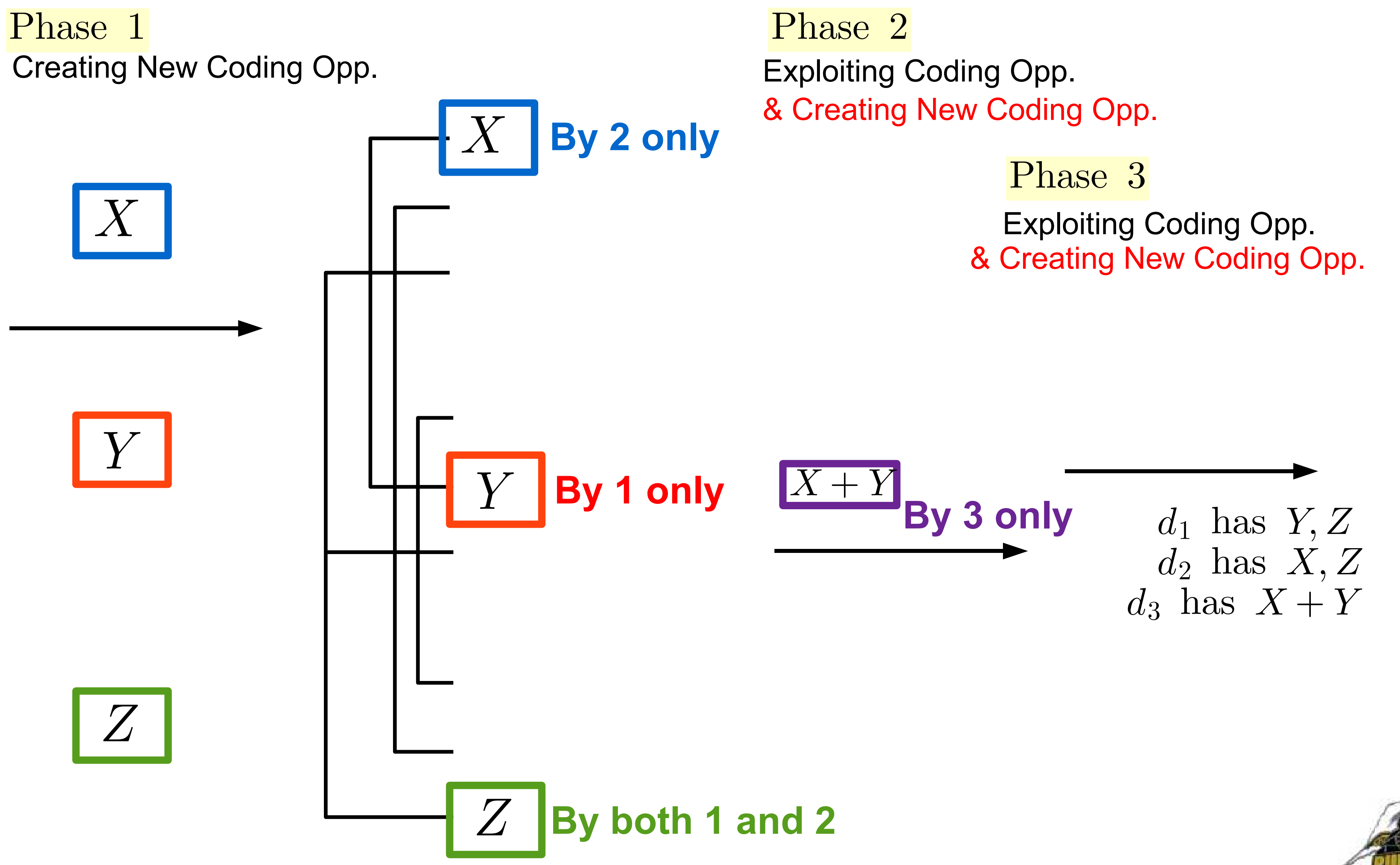

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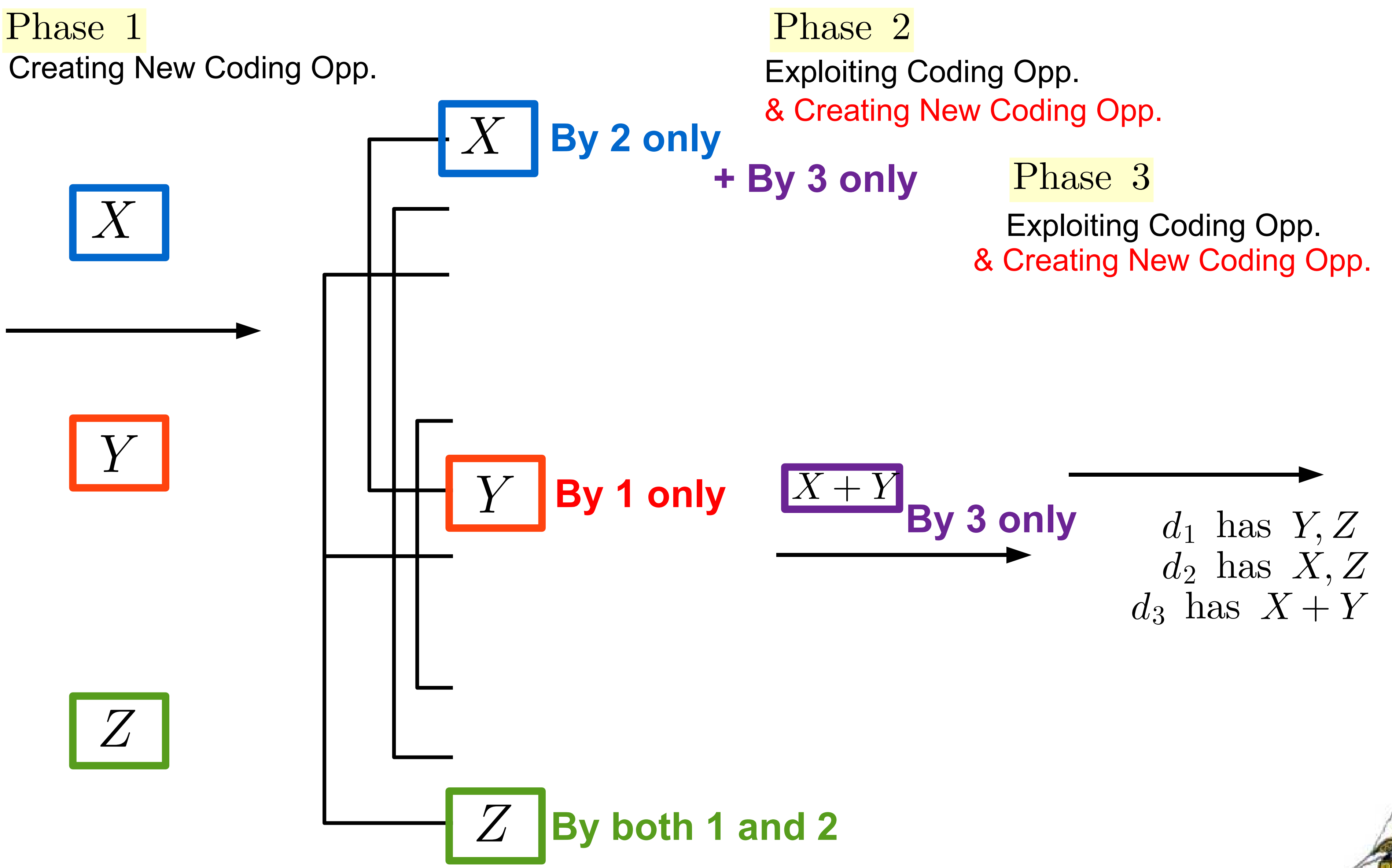

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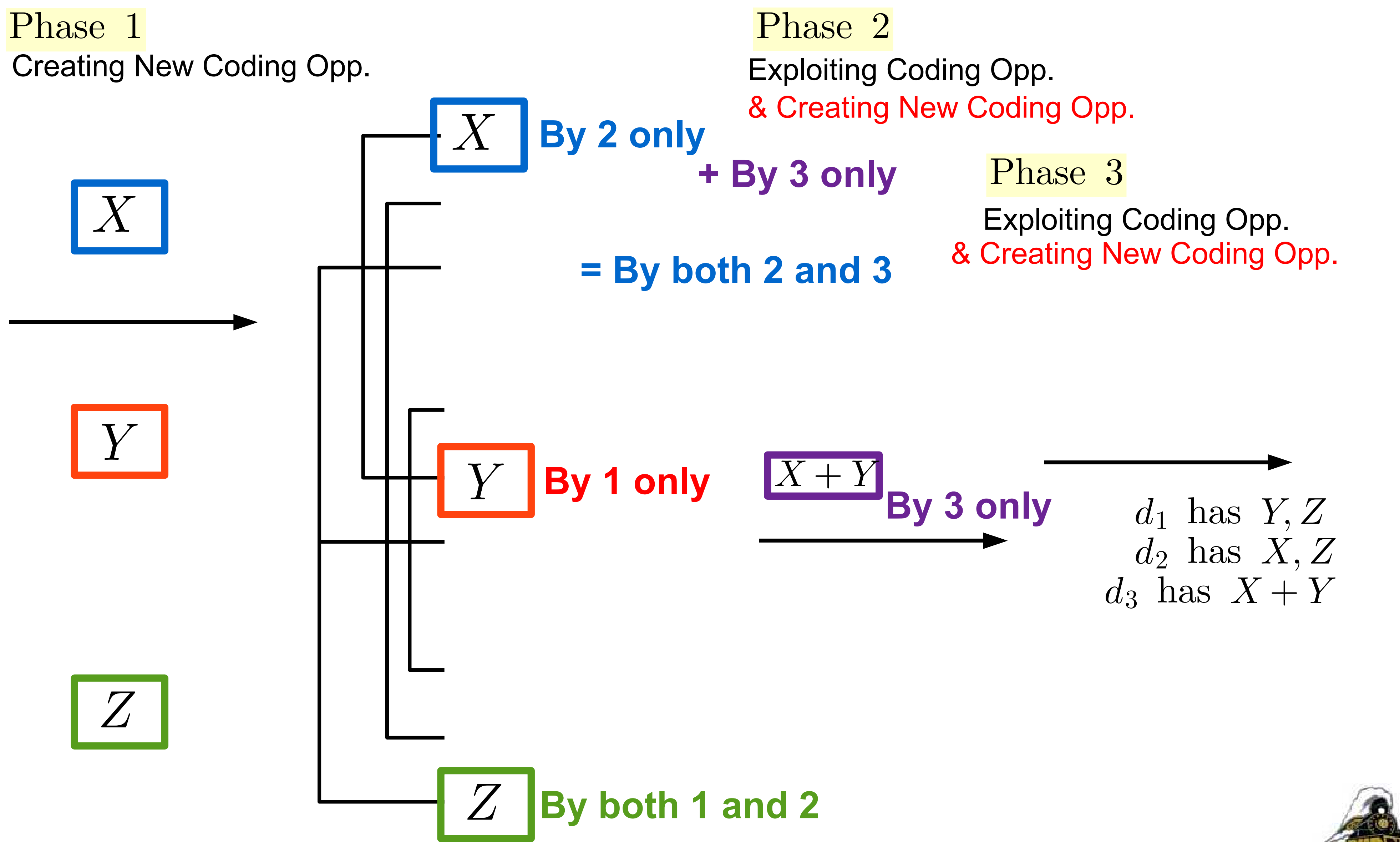

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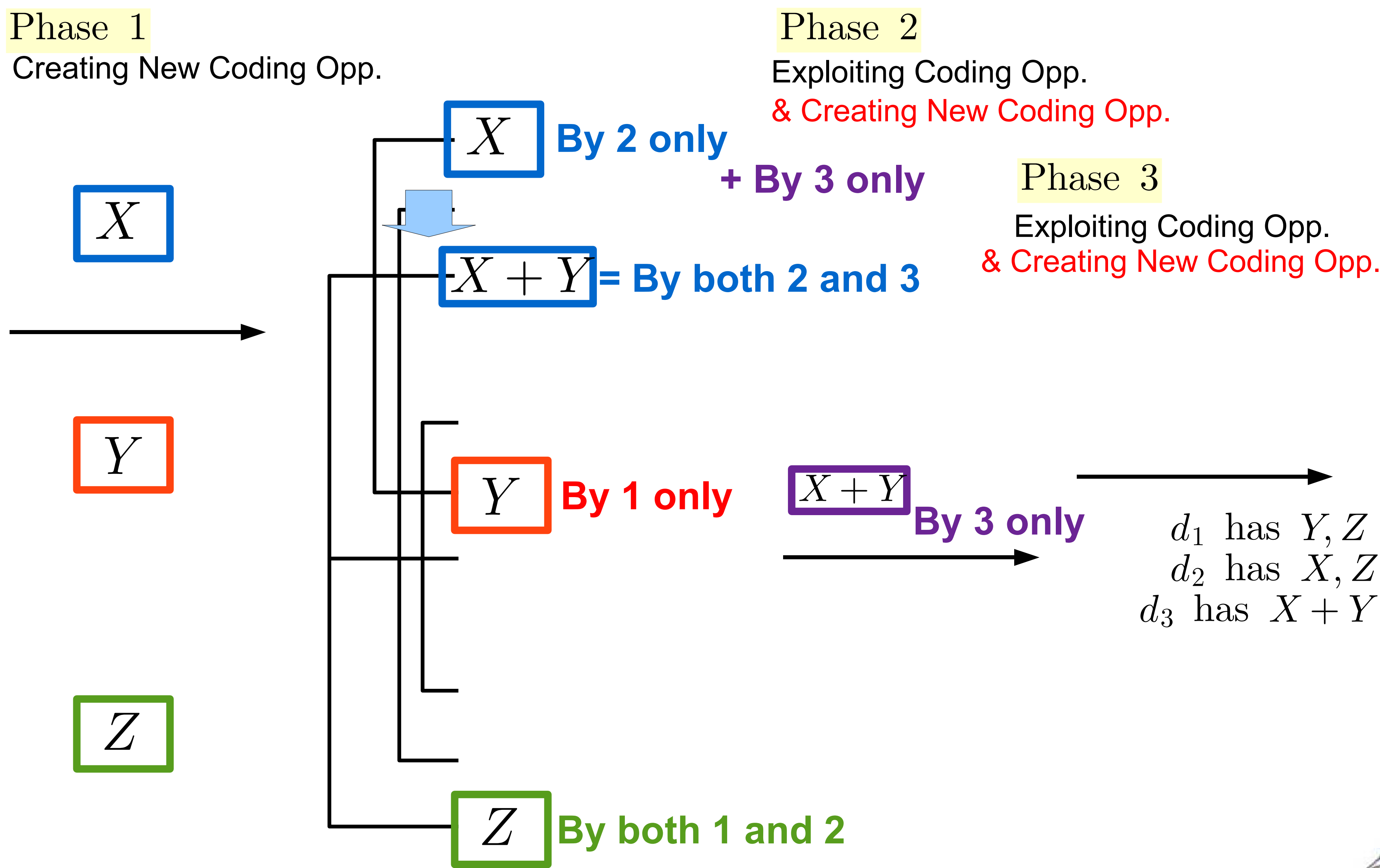

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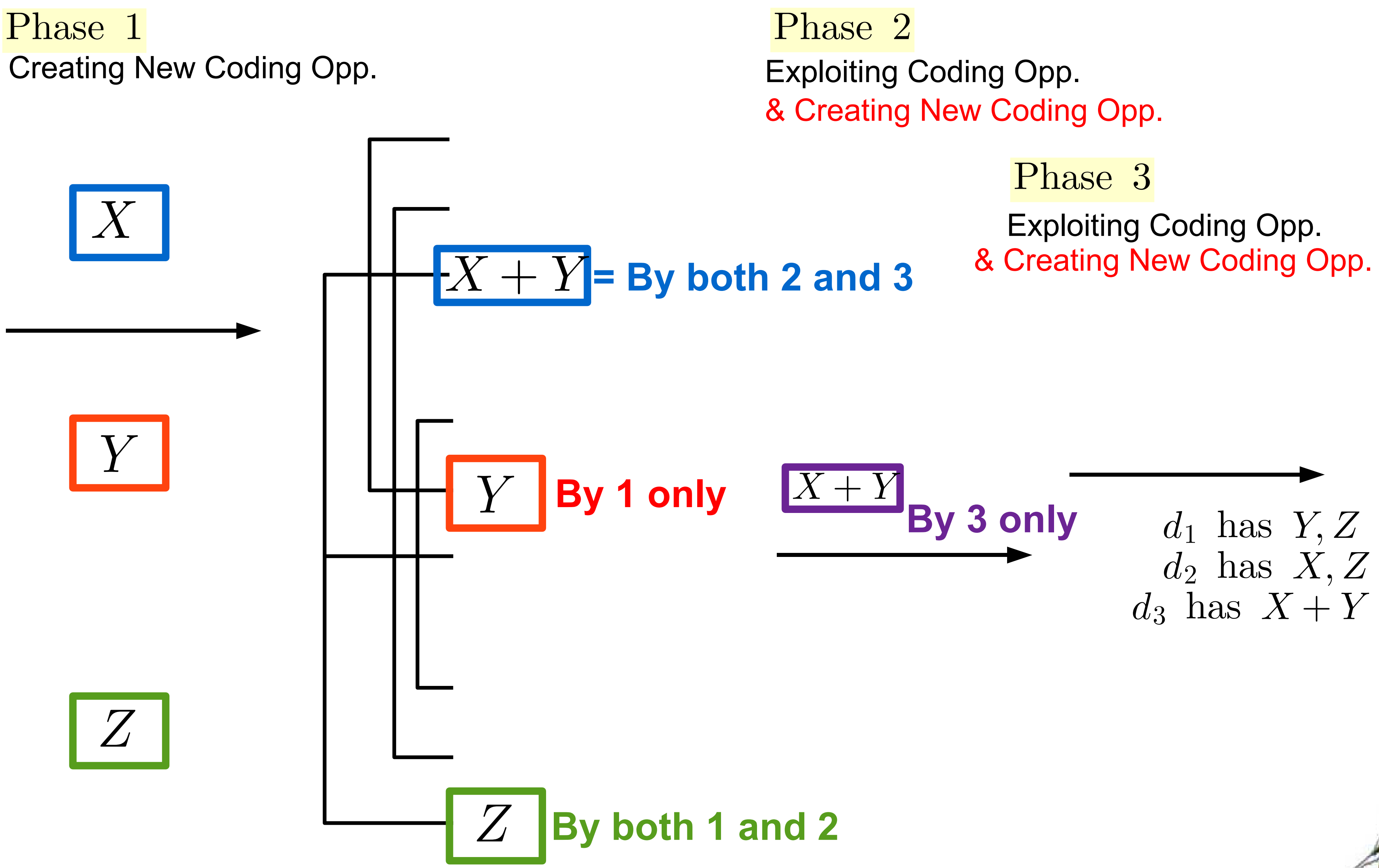

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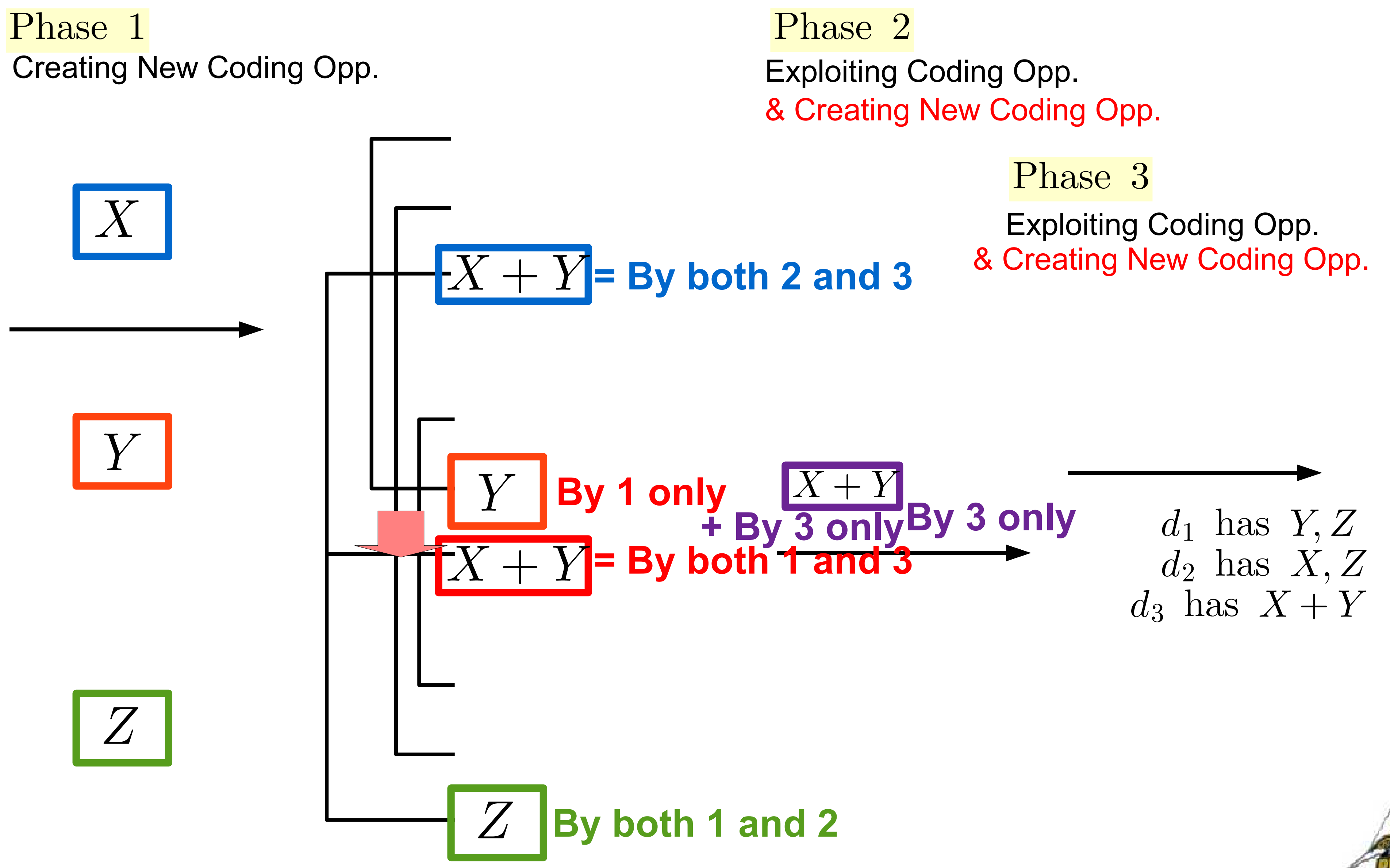

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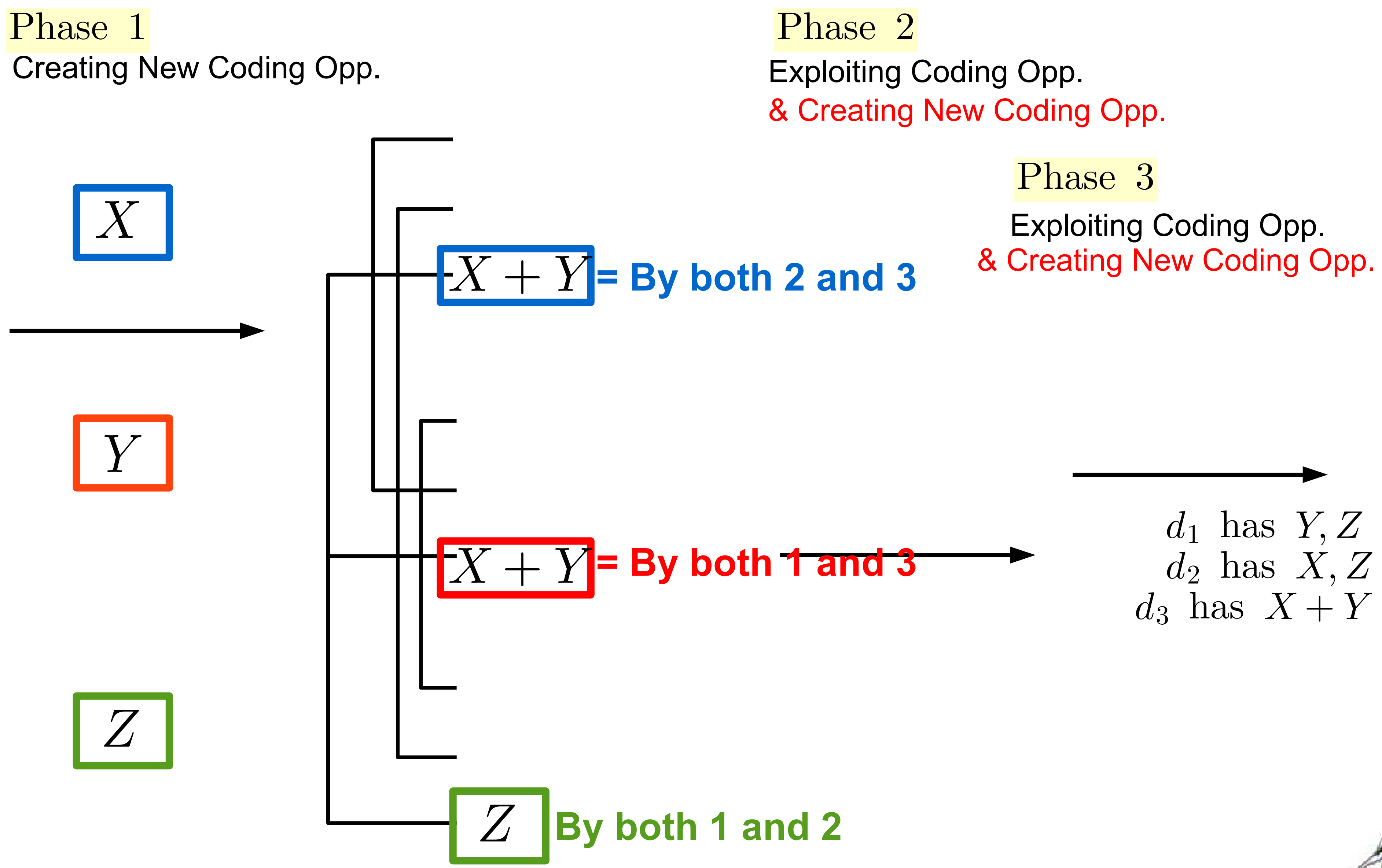

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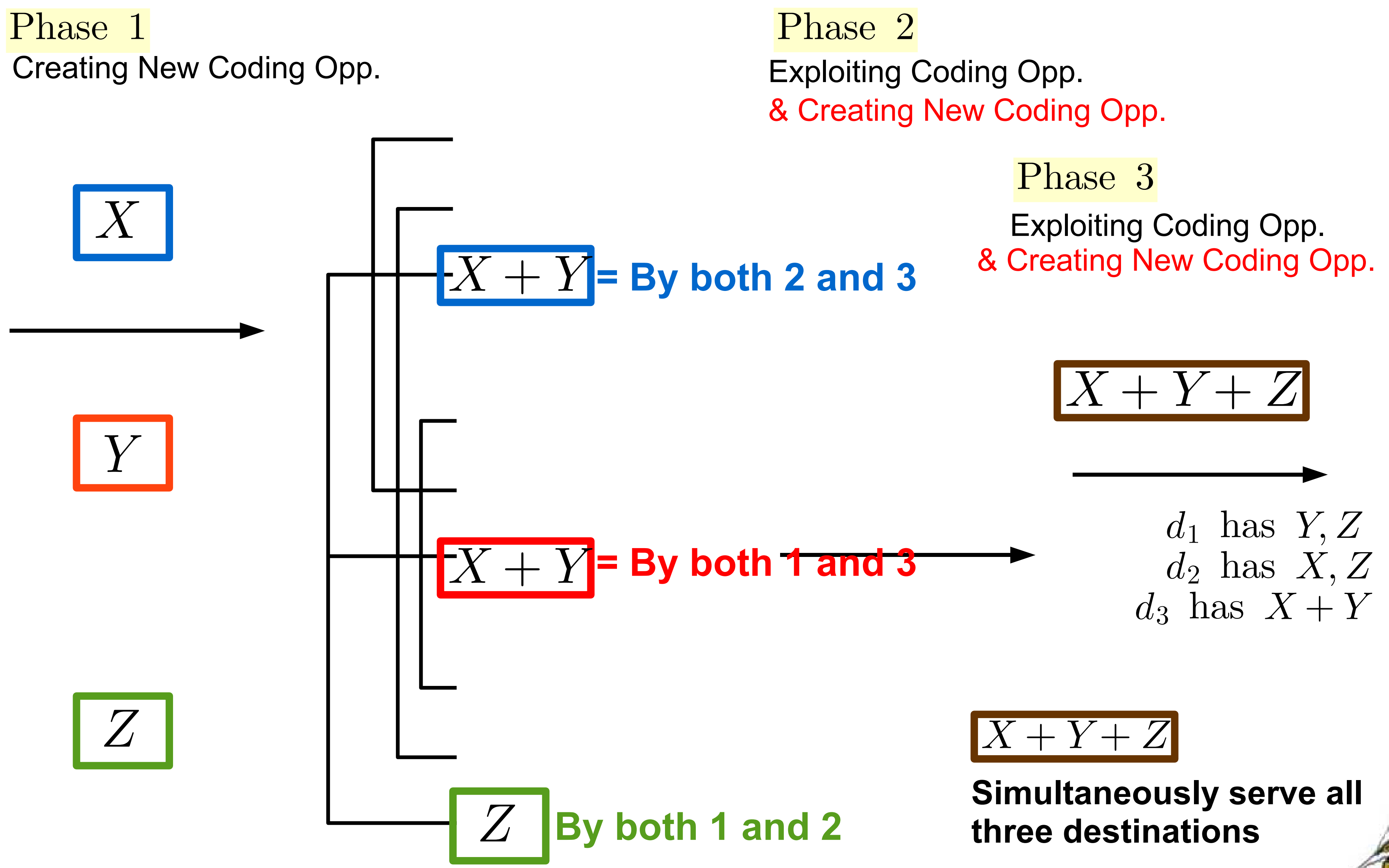

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- The overhearing status and the coding vector of each packet keep evolving.

Capacity Results

- Capacity outer bound: $\forall \pi, \sum_{k=1}^{M} \frac{R_{\pi(k)}}{p_{\cup S_k^{\pi}}} \leq 1.$
- By analyzing the throughput of the packet evolution scheme, we obtain new inner bounds for 1-to-M broadcast PECs with arbitrary $p_{S\overline{([M]\backslash S)}}$ parameters.

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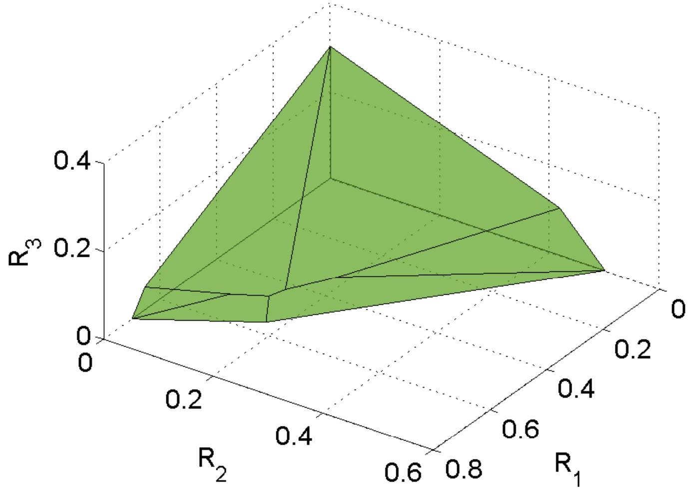

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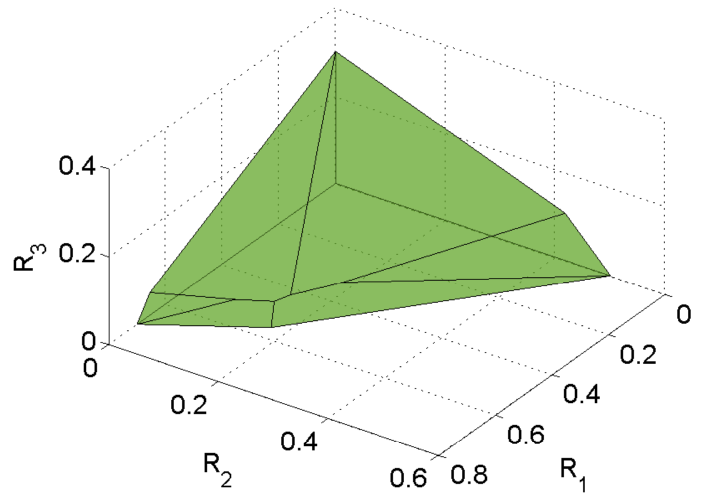

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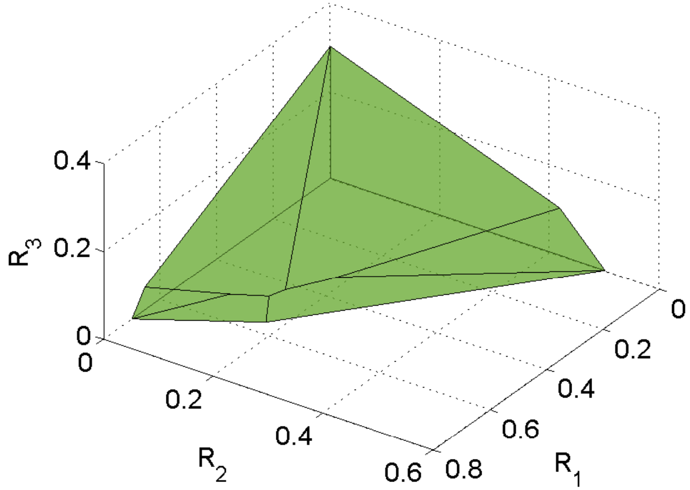

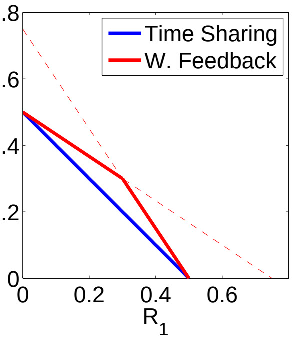

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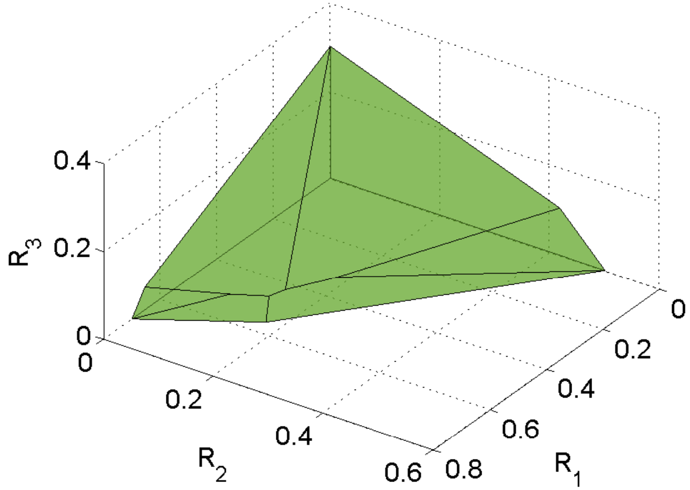

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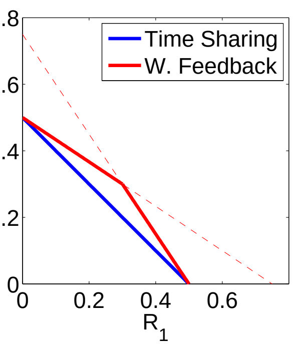


- For all our experiments, the outer/inner bounds always meet.

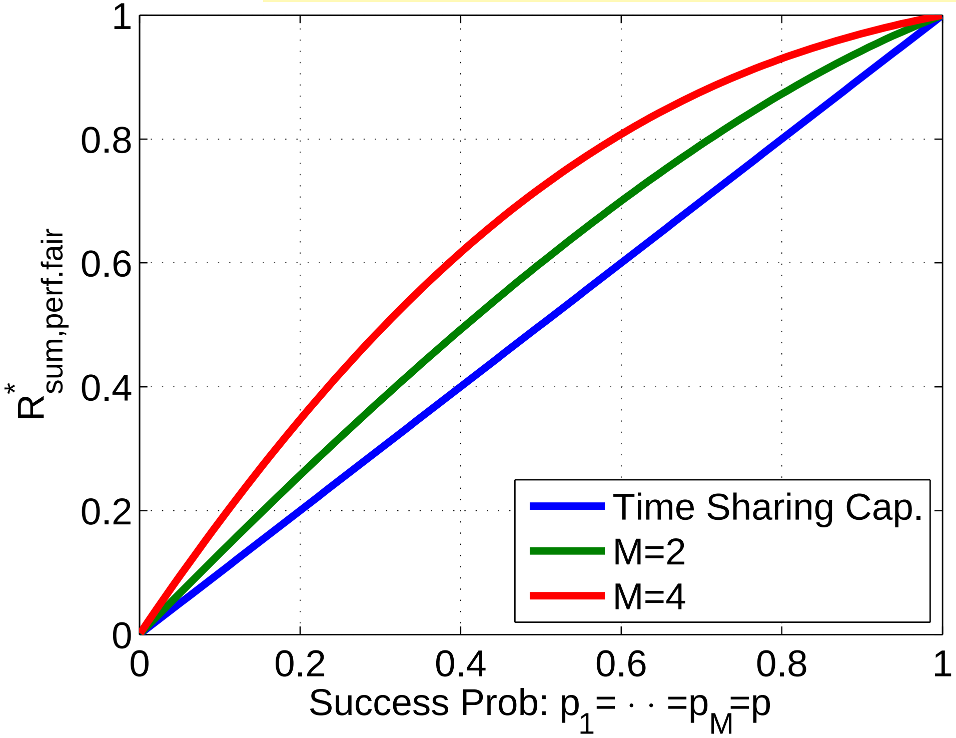
Numerical Evaluation

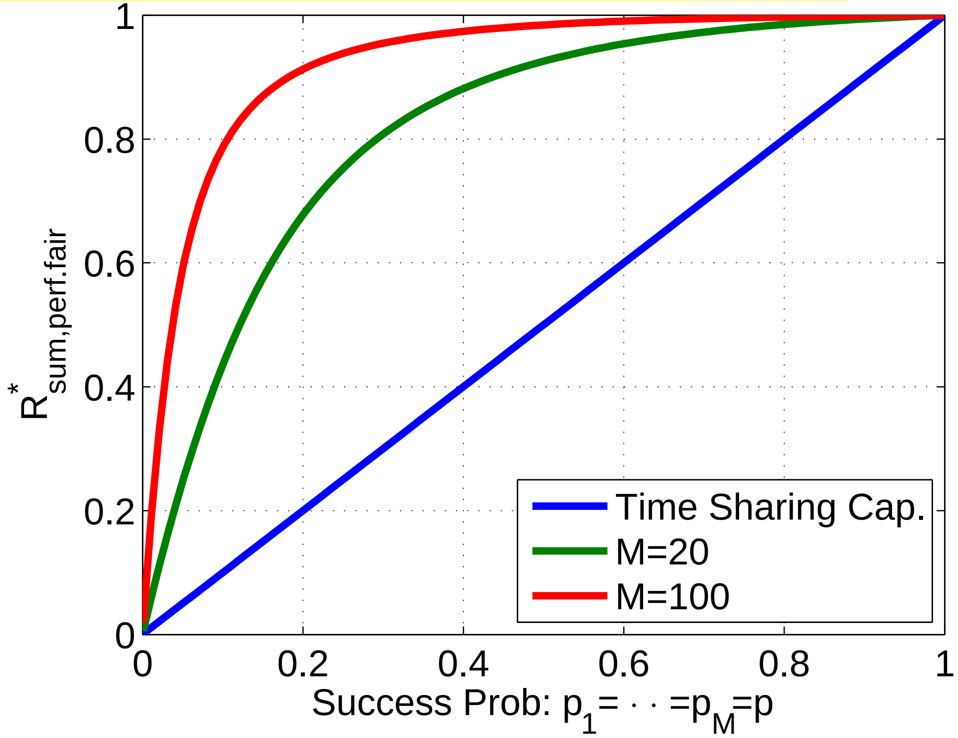
$$\forall \pi, \sum_{k=1}^{M} \frac{R_{\pi(k)}}{p_{\cup S_k^{\pi}}} \leq 1.$$

Symmetric spatially independent PECs: $p_1 = p_2 = \cdots = p_M = p$

Perfectly fair systems: $R_1 = R_2 = \cdots = R_M$

Sum rate capacity $\sum_{k=1}^{M} R_k$ vs. marginal success prob. p.





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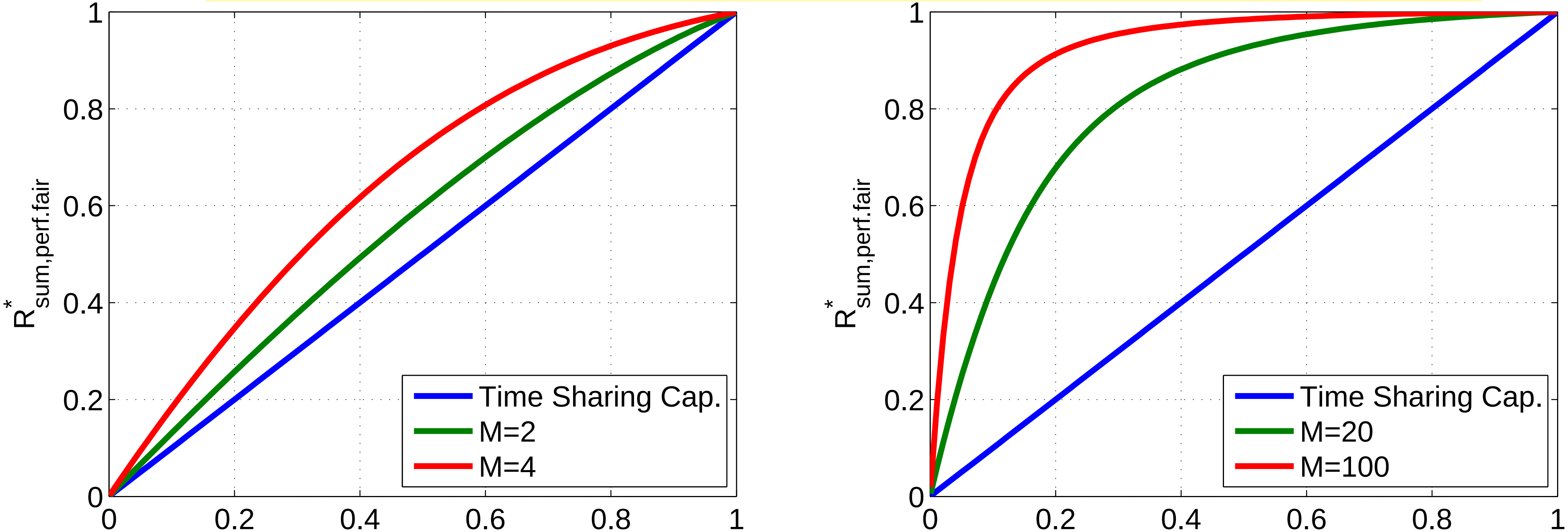

Corollary : When $M \to \infty$, the channel becomes effectively noiseless. [Larsson *et al.* 06]

The Coding Gain Is Real.

- In practice, per-packet feedback is costly.
- We modify the packet evolution scheme and develop a Mixing-reAlignment-Mixing (MAM) scheme that requires only infrequent periodic feedback.

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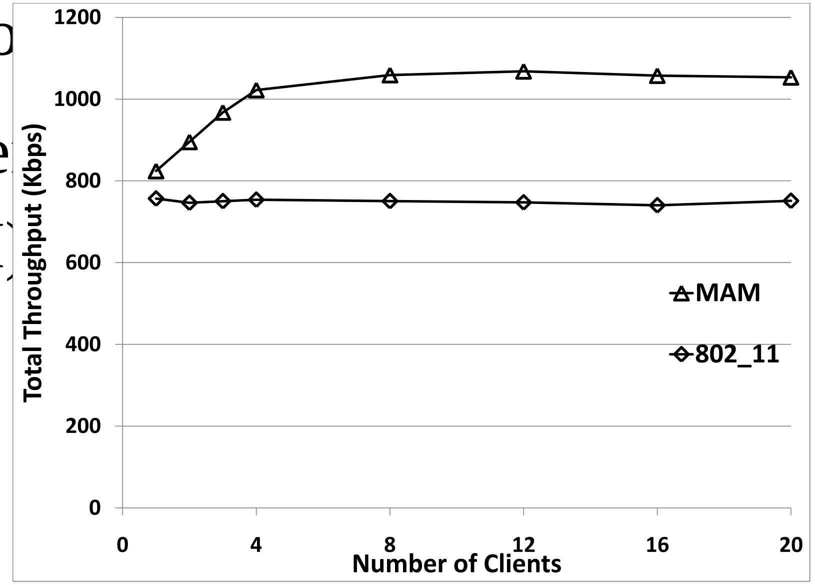
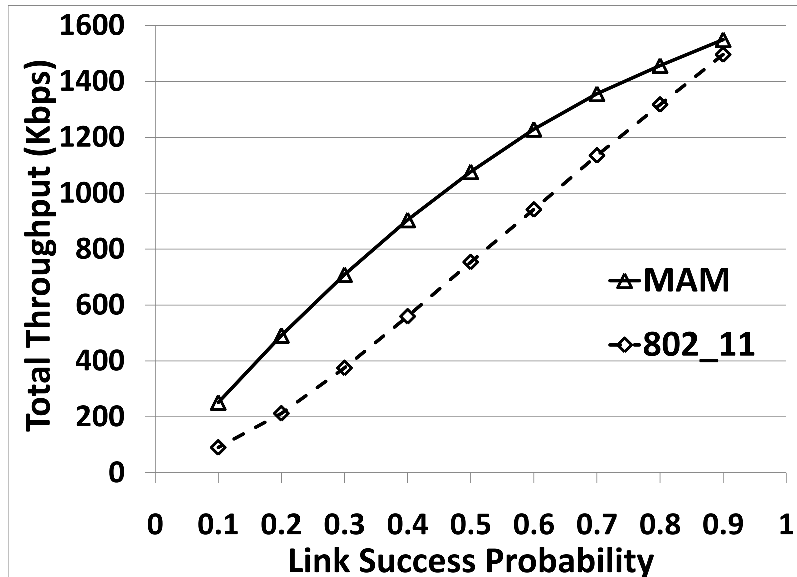
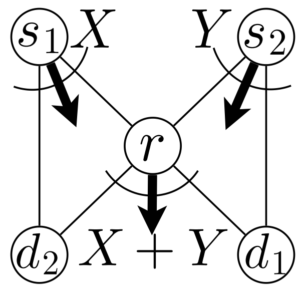
- We have implemented practical MAM in Glomosim simulator. Group sessions into groups of $M = 4$ sessions and perform MAM within each group. Rayleigh fading model with 802.11 CSMA-CD. Packet loss rate: 0.5.

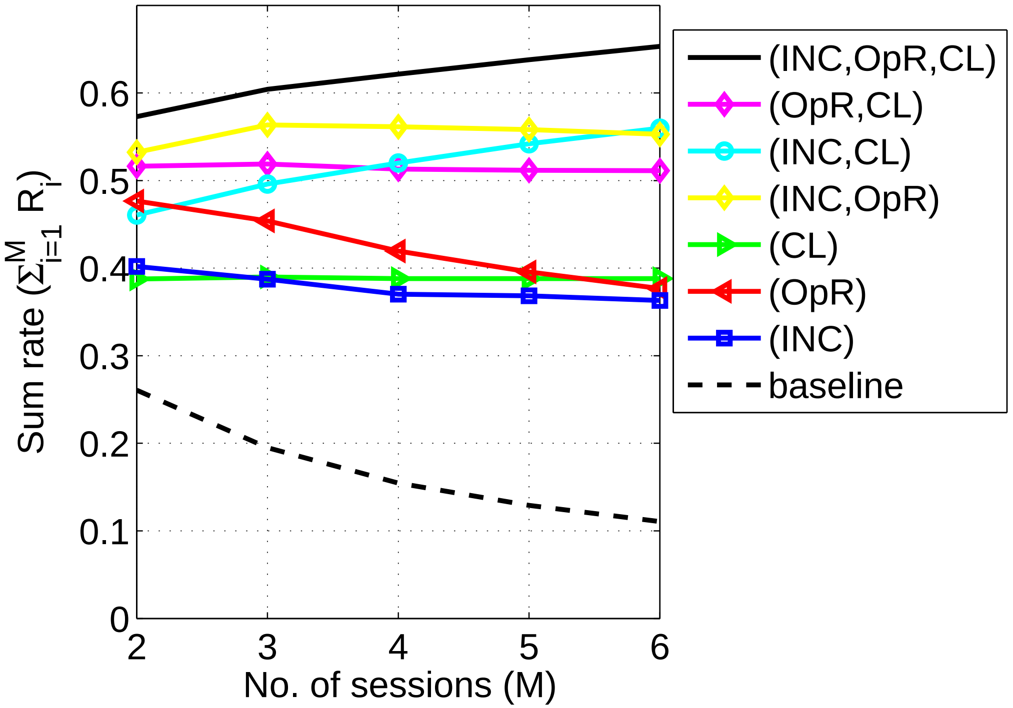




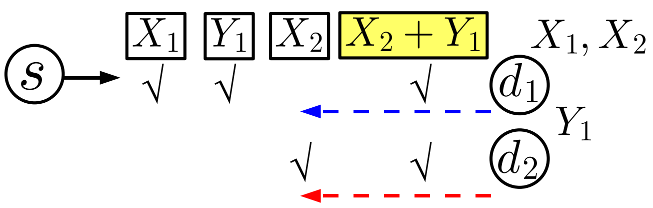
Summary

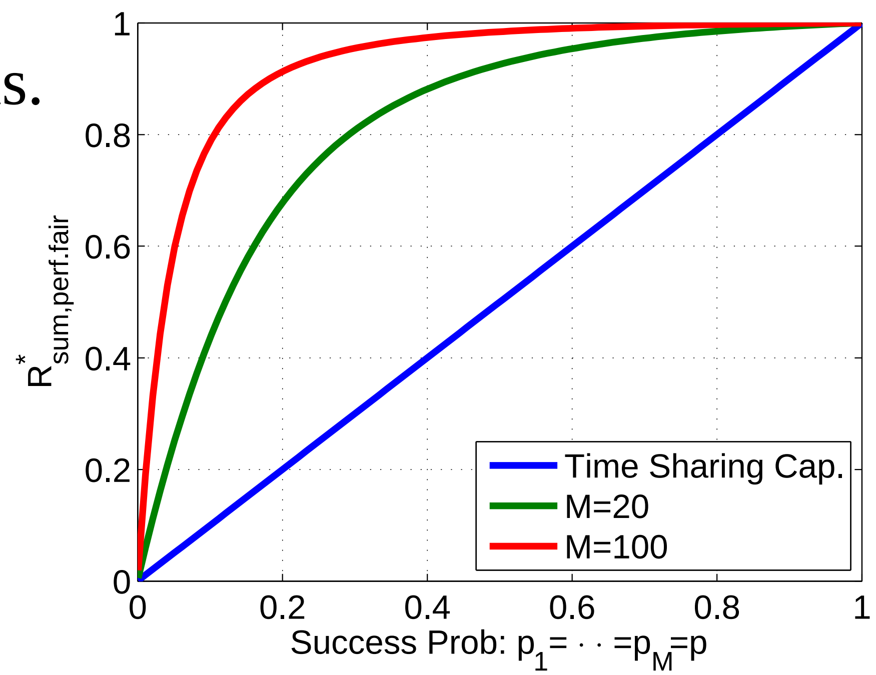
- The capacity of COPE-like protocols.





- The capacity of ER-like protocols.





- Provably tight for $M = 3$; Empirically tight for all M.

Conclusion

- Wireless network coding — From practice (ex: COPE, ER, and MORE protocols) back to theory.

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- Message side information vs. channel output feedback:
  - Side information brings larger gains but is harder to exploit.
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- From theory back to practice: Combining the information-theoretic and algorithmic studies.
  - Ex: How to guarantee termination in a noisy environment?
  - Ex: The linear independence guaranteed by $\mathsf{GF}(q)$, $q \to \infty$ does not hold with prob. 1 for the practical choice $\mathsf{GF}(2^8)$. How to guarantee decodability?

Questions?