MATH 2230 Complex Variables with Applications (2014-2015, Term 1)
Homework 5

1. (SEC.33,No.4)
   Show that $\log(i^2) \neq 2\log i$ when the branch
   $$\log z = \ln r + i\theta \qquad (r > 0, \frac{3\pi}{4} < \theta < \frac{11\pi}{4})$$
   is used.(Compare this with the example in Sec.33.)

2. (SEC.33,No.5)
   (a)Show that the two square roots of i are
   $$e^{\frac{i\pi}{4}} \;\; and \;\; e^{\frac{i5\pi}{4}}$$
   Then show that
   $$\log(e^{\frac{i\pi}{4}}) = (2n + \frac{1}{4})\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots)$$
   and
   $$\log(e^{\frac{i5\pi}{4}}) = [(2n+1) + \frac{1}{4}]\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots).$$
   Conclude that
   $$\log(i^{\frac{1}{2}}) = (n + \frac{1}{4})\pi i \qquad (n = 0, \pm 1, \pm 2, \ldots).$$
   (b)Show that
   $$\log(i^{\frac{1}{2}}) = \frac{1}{2}\log i,$$
   as stated in Example 5,Sec.32,by finding the values on the right-hand side of this equation and then comparing them with the final result in part (a).

3. (SEC.33,No.9)
   Suppose that the point $z = x+iy$ lies in the horizontal strip $\alpha < y < \alpha + 2\pi$. Show that when the branch $\log z = \ln r + i\theta (r > 0, \alpha < \theta < \alpha + 2\pi)$ of the logarithmic function is used, $\log(e^z) = z$.[Compare with equation(5),Sec.31.]