

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530A: Games & Strategic Thinking 2024-2025 Term 1
Homework Assignment 1
Due Date: 19 October, 2024 (Saturday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the “Submission Details” is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website <https://www.cuhk.edu.hk/policy/academichonesty/>

It is also understood that the course teacher will not grade assignments without a properly signed declaration by the student concerned.

Signature

Date

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, [here are a few tips](#).

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Do the following:

1. Go to [2024R1 Games and Strategic Thinking \(UGEB2530A\)](#)
 2. Choose Tools in the left-hand column
 3. Scroll down to the bottom of the page
 4. The green Gradescope icon will be there
- Late assignments will receive a grade of 0.
 - Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

For the declaration sheet:

Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via [Gradescope](#).

Or

Write your name on the first page of your submitted homework, and simply write out the sentence “I have read the university regulations.”

- Write your solutions on A4 white paper or use an iPad or other similar device to present your answers and submit a digital form via Gradescope. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Please be aware that you can only use a ball-point pen to write your answers for any exams.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Evaluate the following matrix products.

$$\begin{array}{ll} \text{(a)} \begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} & \text{(c)} \begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 4 & 3 \\ 6 & 5 \end{pmatrix} \\ \text{(b)} \begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} & \text{(d)} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \end{pmatrix} \end{array}$$

2. Answer each question separately:

- (a) The D18 auto parts store chain has two outlets, one in Shatin and one in Kowloon Tong. Among other things, it sells wiper blades, windshield cleaning fluid, and floor mats. The monthly sales of these items at the two stores for two months are given in the following tables:

		August Sales	
		Shatin	Kowloon Tong
Product	Wiper Blades	20	15
	Cleaning Fluid (bottles)	10	12
	Floor Mats	8	4

		September Sales	
		Shatin	Kowloon Tong
Product	Wiper Blades	23	12
	Cleaning Fluid (bottles)	8	12
	Floor Mats	4	5

Use matrix arithmetic to calculate the change in sales of each product in each store from August to September.

- (b) The revenue generated by sales in the Shatin and Kowloon Tong branches of the D18 auto parts store was as follows:

		August Sales in Hong Kong (SAR) Dollars	
		Shatin	Kowloon Tong
Product	Wiper Blades	140.00	105.00
	Cleaning Fluid (bottles)	30.00	36.00
	Floor Mats	96.00	48.00

If the Hong Kong (SAR) dollar was worth \$0.90 Chinese yuan (renminbi) at the time, compute the revenue in Chinese yuan (renminbi).

- (c) The D18 auto parts store had the following sales in its Shatin store:

		Shatin
Product	Wiper Blades	20
	Cleaning Fluid (bottles)	10
	Floor Mats	8

The store sells wiper blades for \$7.00 each, cleaning fluid for \$3.00 per bottle, and floor mats for \$12.00 each. Use matrix multiplication to find the total revenue generated by sales of these items (products).

- (d) August sales at the D18 auto parts stores in Shatin and Kowloon Tong are given in the following table.

		Shatin	Kowloon Tong
Product	Wiper Blades	20	15
	Cleaning Fluid (bottles)	10	12
	Floor Mats	8	4

The **usual** selling prices for these items are \$7.00 each for wiper blades, \$3.00 per bottle for cleaning fluid, and \$12.00 each for floor mats. The **discount** prices for D18 members are \$6.00 each for wiper blades, \$2.00 per bottle for cleaning fluid, and \$10.00 each for floor mats. Use matrix multiplication to compute the total revenue at each store, assuming first that all items (products) were sold at the usual prices, and then that they were all sold at the discount prices.

3. Suppose a die and 2 coins are tossed together. Let x be the number obtained from the dice and y be the number of heads shown among the coins.

(a) Fill in the blanks in the following tables:

x	1	2	3	4	5	6
$P(\text{getting } x)$						

y	0	1	2
$P(\text{getting } y)$			

(b) Using part (a), fill in the blanks in the following table:

z	1	2	3	4	5	6	7	8
$P(x + y = z)$								

(c) Now, evaluate the expected value of $x + y$.

4. Consider the bimatrix of the game:

$$(A, B) = \begin{pmatrix} (4, -1) & (2, 3) \\ (0, 5) & (3, 1) \end{pmatrix}.$$

- (a) Suppose Player I uses $(0.3, 0.7)$ and Player II uses $(0.6, 0.4)$. Find the expected payoffs of the two players.
- (b) Suppose Player I uses $(0.3, 0.7)$. Find the best strategy for Player II.
- (c) Suppose Player II uses $(0.6, 0.4)$. Find the best strategy for Player I.
5. There are two boxes labeled 1 and 2. Box 1 has a \$1 coin in it and Box 2 has a \$2 coin in it. Mary chooses one of the boxes and secretly triples the amount in the box. Peter chooses one box, without knowing what Mary has chosen, and gets the money inside. Then Mary gets the money in the other box.
- (a) Write down the bimatrix of the game. (Use Mary as the row player and Peter as the column player.)
- (b) Suppose Mary chooses Box 1 with a probability of 0.2 and Peter chooses Box 1 with a probability of 0.3. Find the expected payoffs of the two players.

- (c) Suppose Mary chooses Box 1 with a probability of 0.2. Find the best strategy for Peter.
- (d) Suppose Peter chooses Box 1 with a probability of 0.3. Find the best strategy for Mary.
6. In a rock-paper-scissors game, the loser pays the total number of fingers in each two-gesture round to the winner. The payoff is 0 if there is a draw.
- (a) Write down the game matrix (payoff of player 1) of the game. (Use rock, paper, scissors, as the order of strategies.)
- (b) Suppose player 1 uses $(0.2, 0.4, 0.4)$ and player 2 uses $(0.3, 0.5, 0.2)$. Find the expected payoff of player 1.
- (c) If player 1 uses $(0.2, 0.4, 0.4)$, what is the best strategy for player 2.
- (d) If player 2 uses $(0.3, 0.5, 0.2)$, what is the best strategy for player 1.
- (e) By considering equalizing strategies, find a Nash equilibrium and the value of the game.
7. In a game, two players call out one of the numbers 1, 2, or 3 simultaneously. Let S be the sum of the two numbers. If S is even, then player 2 pays S dollars to player 1. If S is odd, then player 1 pays S dollars to player 2.
- (a) Write down the game matrix for the payoff of player 1.
- (b) Write down the game matrix for the payoff of player 2.
- (c) Find the expected payoff of player 1 if player 1 calls out the numbers 1, 2, 3 with probabilities 0.5, 0.3, 0.2 respectively, and player 2 calls out the numbers 1, 2, 3 with probabilities 0.1, 0.4, 0.5 respectively.
- (d) Suppose player 2 calls out the numbers 1,2,3 with probabilities 0.1, 0.4, 0.5 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?
8. For each of the following game matrices, determine whether there is a saddle point. Copy the game matrix and circle all saddle points of the matrix if there are any.

(a)
$$\begin{pmatrix} -1 & -4 & 4 & -2 \\ -4 & 4 & -1 & 0 \\ 2 & 3 & -1 & 1 \end{pmatrix}$$

(b)
$$\begin{pmatrix} -3 & 5 & -2 & 0 \\ 0 & -5 & -1 & -3 \\ 1 & 3 & 7 & 4 \\ -2 & 2 & 3 & 1 \end{pmatrix}$$

(c)
$$\begin{pmatrix} 4 & 3 & 5 & 3 \\ 2 & 1 & -1 & -20 \\ 3 & 3 & 4 & 3 \\ -16 & 0 & 16 & 1 \end{pmatrix}$$

9. Use the dominated strategy to obtain the optimal strategies for both players and determine the value of the game if any. The pay-off matrix is given below:

		Player B				
		I	II	III	IV	V
Player A	I	2	4	3	8	4
	II	5	6	3	7	8
	III	6	7	9	8	7
	IV	4	2	8	4	3

10. Solve the zero sum games, that is, find a maximin strategy for the row player, a minimax strategy for the column player and the value of the game, with the following game matrices.

(a) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}$

11. Solve the zero sum games with the following game matrices.

(a) $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 6 \\ 0 & 4 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$

12. Solve the zero sum game with game matrix

$$\begin{pmatrix} 5 & 3 & 8 & 1 \\ 2 & 3 & 5 & 10 \\ 7 & 5 & 6 & 2 \\ 6 & 4 & 3 & 1 \end{pmatrix}$$

13. Solve the following game matrices.

(a) $\begin{pmatrix} 1 & 0 & 4 \\ 0 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

$$(c) \begin{pmatrix} 2 & -4 & -4 \\ -4 & 6 & -4 \\ -4 & -4 & 16 \end{pmatrix}$$

$$(d) \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 3 \end{pmatrix}$$