THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3310 2024-2025
Homework Assignment 1
Due Date: September 24, 2024

Please show your answer in detail.

1. Solve the following ODE using method of integrating factor

$$y' + \frac{1}{x}y - sin(x) = 0, \quad x > 0$$

with condition $y(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$.

2. Solve the following second order ODE using method of integrating factor

$$y'' - 4y - x^2 - 2x - 1 = 0$$

with conditions $y(0) = e^4 + e^{-4} - \frac{3}{8}$ and $y(2) = -\frac{3}{8}$.

3. Please show that

$$\int_0^{2\pi} \cos kx \cos mx \, dx = \begin{cases} 2\pi, \text{ if } k = m = 0 \\ \pi, \text{ if } k = m \neq 0 \\ 0, \text{ if } k \neq m \end{cases}$$

and that

$$\int_0^{2\pi} \sin kx \sin mx \, dx = \begin{cases} 0, \text{ if } k = m = 0 \\ \pi, \text{ if } k = m \neq 0 \\ 0, \text{ if } k \neq m \end{cases}$$

where m, k are non-negative integer.

4. Find a possible Fourier series solution to the following differential equation

$$-2y''(x) + y(x) = f(x)$$

where $x \in (-L, L)$ and

$$f(x) = \begin{cases} 0, & if \quad -L < x < 0, \\ \frac{1}{L}, & if \ 0 \leq x < L \end{cases}$$

5. Solve the following PDE using Fourier series

$$\begin{cases} u_t - 2 = u_{xx}, & 0 < x < 2\pi, t > 0 \\ u_{xx}(t, 0) = -2 = u_{xx}(t, 2\pi), & t > 0 \\ u(0, x) = x - x^2, & 0 < x < 2\pi \end{cases}$$

Hint: in the class, the strategy to solve homogeneous type PDE is to assume $u(t, x) = \sum f_n(t)g_n(x)$, here a good try is to add one more term $f(t)$ or $g(x)$ to eliminate the effect of the constant term 2 in the PDE.

6. Let f be a 2π-periodic function and $f(t) = sinh(t) = \frac{e^t - e^{-t}}{2}$ for $t \in (-\pi, \pi)$, then find the real Fourier series and prove that

$$\frac{sinh(1)}{sinh(\pi)} = \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^{k-1} k}{k^2 + 1} \sin(k)$$