THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4210 Financial Mathematics 2024-2025 Term 1
Homework Assignment 1
Due Date: 11:59PM, 29 September, 2024

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website `https://www.cuhk.edu.hk/policy/academichonesty/`

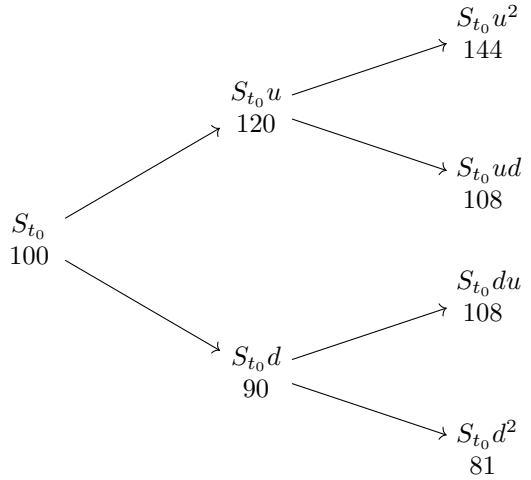
It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

_____ _____
Signature Date

General Regulations

- All assignments will be submitted and graded on CUHK Blackboard. You can view your grades and submit regrade requests here as well.
- Late assignments will receive a grade of 0.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Consider a two-step binomial tree model with a discrete compounding rate and the following parameters: $S_{t_0} = 100$, $u = 1.2$, $d = 0.9$, and $1 + r\Delta t = 1.1$. Find the price and the replication strategy of the American put with a strike price of $K = 110$. For the American put, the option holder can exercise the option at any of the times t_0, t_1, t_2. The option's payoff at time t_k is given by $(K - S_{t_k})_+$,



2. In a two-step binomial tree model with a one-step interest rate of $r = 0.05$, $S_0 = 100$, $u = 1.2$, $d = 0.8$, and $\Delta t = 1$, consider a contingent claim that expires after two years. The payoff is $S_T^2 \cdot \mathbb{1}_{\{S_T > 90\}}$.

(a) Find the initial price and the replication strategy of the European version of the option.

(b) Find the initial price and the replication strategy of the American version of the option.

3. Consider a two step binomial tree with the following parameters: $S_{t_0} = 100$, $u = 1.2$, $d = 0.8$, $r = 0.05$, $\Delta t = 1$. Find the European knock-out call option with a strike price of 95 and a barrier of 90 and the corresponding replication strategy. (The payout of the knockout call option at time t_2 is given by

$$(S_{t_2} - K)_+ \cdot \mathbb{1}_{\left\{\min_{k\in\{0,1,2\}} S_{t_k} > H\right\}},$$

where K is the strike price and H is the barrier price.)

4. Given $X \sim N(\mu, \sigma^2)$, compute $\mathbb{E}(e^{tX})$ for a constant t.