

TOPICS IN GRAPH ALGORITHMS (CSCI5320-22S)

Homework 2

Due: 5pm March 1, 2022

1. **(Due 5pm Feb 21)** Design polynomial-time algorithms as fast as you can to solve CLIQUE, VERTEX COVER, and DOMINATING SET for $k = 3$.
2. A *permutation matrix* P is a 0,1-matrix that has exactly one 1 in each row and column. Prove that any square matrix of nonnegative integers can be expressed as the sum of k permutation matrices if and only if all row sums and column sums equal k .
3. A warehouse worker need to load n bags to the truck at the door. Bags are scattered in different locations of the warehouse and the worker can carry two bags at a time. Design an efficient algorithm to find a strategy for the worker to minimize his total walking distance.
4. There are n activities to be held simultaneously with activity i having capacity of accommodating c_i persons, and m persons indicate their preferences of activities. Design an efficient algorithm to assign activities to persons that maximizes the total number of participants.
5. Prove NP-hardness of the following problems for graph G :
 - (a) Does G contain k vertices where every pairwise distance is at most l ?
 - (b) Does G contain a bridgeless connected spanning subgraph with at most k edges?
 - (c) Does G contain k vertices V' such that $G[V']$ is a tree?
6. Prove that it is NP-complete to determine whether we can remove $\leq k$ vertices from a graph to make it a bipartite graph.
7. Use a reduction from 3SAT to prove the NP-completeness of the following problem: Does a digraph G contain two vertex-disjoint (s, t) -paths P_1 and P_2 of length l_1 and l_2 respectively?