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TOPICS IN GRAPH ALGORITHMS (CSCI5320-22S)

Homework 1
Due: 5pm Feb 14, 2022

1. For the graph G in Figure 1,

   (a) determine the number of different minimum spanning trees,

   (b) determine whether $\text{mst}(G - xy) = \text{mst}(G)$, and

   (c) determine the maximum weight for edge uv to make $\text{mst}(G + uv) < \text{mst}(G)$,

   where $\text{mst}(G)$ denotes the cost of a minimum spanning tree in G.

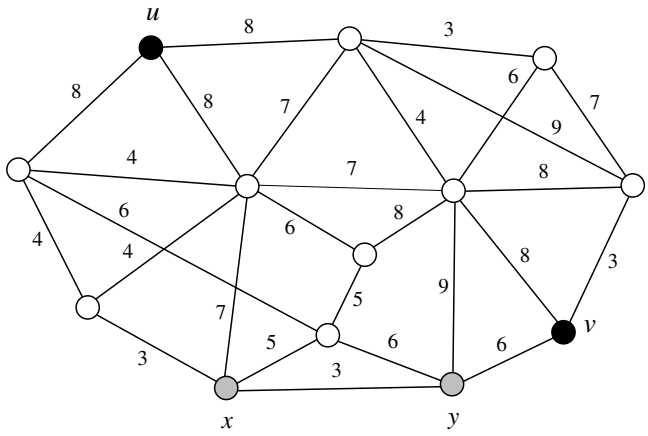

Figure 1: Graph G

2. Prove the correctness of algorithm **Greedy-MST** for the red rule.

3. For the following rules, determine ones that lead to correct algorithms for finding a minimum spanning tree T in a weighted connected graph G. Give proofs for correct ones and counterexamples for incorrect ones.

   (a) Find a cut X, add to T a lightest edge $e \in X$, and delete e from G.

   (b) Find a cut X, add to T a lightest edge $e \in X$, and contract e.

   (c) Find a cut X, and delete a heaviest edge of X from G.

   (d) Find a vertex v incident with no blue edge, and color a lightest edge incident with v blue.

4. Let T be an MST of graph G. Design an efficient algorithm to update T for the following changes in G:

   (a) addition of edges,

   (b) deletion of edges, and

   (c) weight change of edges.

5. For the purpose of edge reweighting in connection with shortest paths, give an easy way to compute the function $h : V \to Z$ for a dag $G = (V, E; w)$ with $w : E \to Z$.

6. Let G be a digraph where all negative edges have the source vertex s as tail vertices. Determine whether Dijkstra's algorithm works correctly for G when G has no negative cycles.