香港中文大學
The Chinese University of Hong Kong

Course Examinations 2021-22 2nd Term

Course : CSCI5320 – Topics in Graph Algorithms

Time : 2-4pm, April 26, 2022

This exam has 4 pages consisting of
3 parts with 11 problems of 45 points.

Student ID : ....

Name : ....

Part	Points
I	
II	
III	
Total	

Part I (5 problems with 15 points) Circle correct statements. Each correct answer gives you 1 point, but each extra one costs you 1 point if you circle more than 15 statements.

1. Consider the MST problem and let G be a weighted connected graph.
   (a) No MST can contain a heaviest edge.
   (b) No MST can contain the heaviest edge even if G has a unique heaviest edge.
   (c) For any lightest edge e in G, there is an MST containing e.
   (d) The red rule alone suffices for finding an MST in G.
   (e) G has a unique MST if all edge weights are distinct.
2. If a parameterized problem Π admits an FPT algorithm, then
   (a) Π can be solved in $f(k) + n^c$ time for some constant c.
   (b) Π can be solved in polynomial time for $k \leq 99$.
   (c) Π has a polynomial-size kernel.
   (d) The unparameterized version of Π is NP-complete.
   (e) There is an FPT reduction from CLIQUE to Π.
3. Determine problems solvable in FPT time:
   (a) Find k vertices in a graph to cover the minimum number of edges.
   (b) Find k vertices in a graph to cover the maximum number of edges.
   (c) Find k vertices in a planar graph to cover the maximum number of edges.
   (d) Find a path of length k in a graph.
   (e) Find k vertices in a cubic graph to dominate the maximum number of vertices.
4. Determine methods that enable us to solve VERTEX COVER in FPT time.
   (a) Bounded search tree.
   (b) Kernelization.
   (c) Iterative compression.
   (d) Color coding.
   (e) Indirect certificating.
5. If a parameterized problem Π is W[1]-complete, then
   (a) Π cannot be solved in FPT time.
   (b) There are FPT reductions from Π to all problems in W[1].
   (c) There is a polynomial-time reduction from CLIQUE to Π.
   (d) If Π can be solved in FPT time, so can CLIQUE.
   (e) If some problem in W[1] cannot be solved in FPT time, neither can Π .

Part II (5 points each for 2 problems)

1. Determine the number of distinct minimum spanning trees in the graph of Figure 1. Briefly explain the key ideas in obtaining your answer.

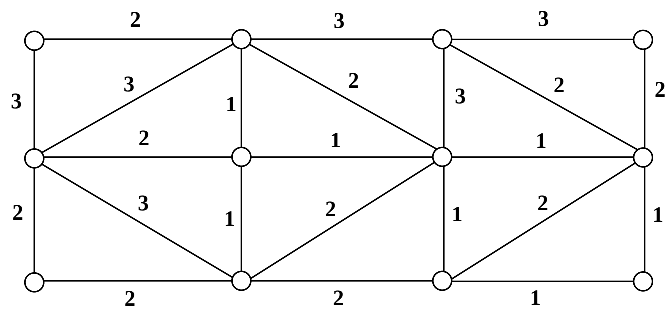

Figure 1: Graph G for MST.

2. For the maze shown in Figure 2, use a graph to determine whether it is possible to enter at ENTRANCE, walk through each doorway exactly once, and get out from EXIT. If yes, give such a walk. Otherwise close fewest doorways to make it possible and give a required walk for the new configuration. Give justifications to your answer.

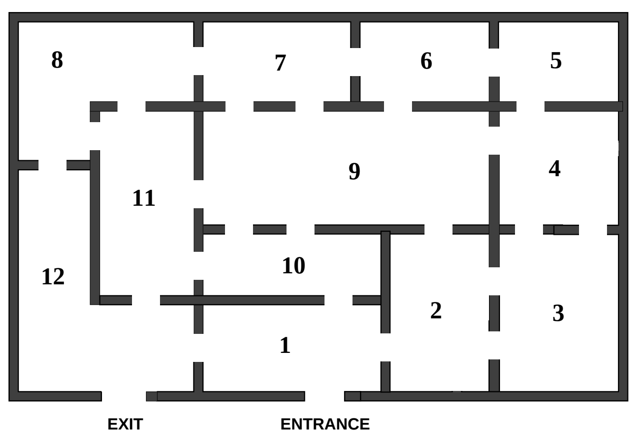

Figure 2: Floor plan of a maze

Part III (5 points each for 4 problems) For each (sub)problem, you get 20% of its points if you write "I don't know" only.

1. For the following values of k in the INDEPENDENT SET problem, give either an NP-completeness proof or a polynomial-time algorithm for the problem.

   (a) $k = n/3$.

   (b) $k = n - 10$.

   (c) $k = n - \log_2 n$.

2. Design an FPT algorithm to solve the following problem: determine whether an edge-bicolored graph $G = (V, E_b \cup E_r)$ contains at most k vertices V' such that all vertices in $G - V'$ are monochromatic, where a *monochromatic vertex* has the same color for all its incident edges.

3. Find in $O(n^2)$ time a kernel of size $O(k^2)$ for the following problem on an n-by-n 0-1 matrix M: Can we remove a total of at most k rows/columns from M to obtain an all-zero matrix?

4. Let G be a graph where all but k vertices have degree at most 9. Design an FPT algorithm to solve the following problem on G: Find an induced subgraph H with k vertices that contains as many edges as possible.

— **End of exam paper**