Homework 1

Please submit, through Blackboard, solutions to *all* of the following problems. The deadline for submissions is 18:00 on Monday 20th May 2024. Please let me know if any of the problems are unclear or have typos.

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(1.1) Let $u = (1, -2, 1)$, $v = (0, 2, 3)$ and $w = (2, 0, -1)$ be vectors in $\mathbb{R}^3$. Find $\alpha, \beta, \gamma \in \mathbb{R}$ such that
$$(1, 1, 1) = \alpha u + \beta v + \gamma w.$$

(1.2) Let $x, y \in \mathbb{R}^n$.

    a) Show that either $(x + y) \cdot x \geq 0$, or $(x + y) \cdot y \geq 0$.

    b) Assume now that x and y are non-zero vectors of the same length. Show that, if the vector $x + y$ is non-zero, then it bisects the angle between x and y.

(1.3) For some $m \in \mathbb{N}$, let $x_1, \ldots, x_m \in \mathbb{R}^n$ be unit vectors. The centre of mass of these vectors is defined to be

$$C := \frac{1}{m} \sum_{i=1}^{m} x_i \in \mathbb{R}^n.$$

    a) Show that the length of C is less than or equal to 1.

    b) When does the length of C equal 1? Justify your answer.