Incentive and Service Differentiation in P2P Networks: A Game Theoretic Approach*

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Abstract—**Conventional Peer-to-Peer (P2P) networks do not provide service differentiation and incentive for users. Therefore, users can easily access information without contributing any information or service to a P2P community. This leads to the well known "free-riding" problem and consequently, most of the information requests are directed toward a small number of P2P nodes which are willing to share information or to provide service, hence, the "tragedy of the commons" occurs. The aim of this paper is to provide service differentiation based on the amount of services each node has provided to a P2P community. Since the differentiation is based on the amount of contribution, this encourages all nodes to share information/services in a P2P network. We first introduce a resource distribution mechanism for all information sharing nodes. This mechanism is a distributed in nature and has a linear time complexity and guarantees the "Pareto-optimal" resource allocation. In addition, the mechanism not only distributes resources in a way to increase the aggregated utility of the whole network, but also provides incentive for nodes in the P2P network to share information. Secondly, we model the whole resource request/distribution process as a competition game between all competing nodes. We show that this game has a Nash equilibrium. To realize this game, we propose a protocol such that all competing nodes can interact with the information providing node such that the Nash equilibria can be reached efficiently and dynamically. We also present a generalized mechanism which provides incentive for nodes having heterogeneous utility functions. Convergence analysis of the competition game is carried out. Examples are used to illustrate that the incentive protocol provides service differentiation and induces incentive for nodes to share information or to provide service, Lastly, the incentive protocol is adaptive to nodes arrival and departure events, and to different forms of network congestion.**

I. Introduction

There has been a lot of recent interest in P2P networks. As evidenced by traffic measurements of ISPs, a large percentage of existing network traffic is due to P2P applications [10], [4]. These applications aim to exploit the cooperative paradigm of information exchange to greatly increase the accessibility of information to a large population of network users. Unlike the client-server computing paradigm, P2P paradigm allows individual user (or node) to play the role of a server and a client at the same time. Therefore, nodes in a P2P network can assist each other in file searching, file lookup[14], [15], [18], [20] as well as file transfer in an anonymous manner[3]. For the file searching process, the P2P networks evolve from a centralized file/directory lookup approach (e.g., Napster) to a distributed objects query approach (e.g., Gnutella). Whereas distributed object queries can be effected by some form of controlled flooding, the new generation of P2P networks (e.g., Chord and CAN) use the method of *consistent hashing* to improve the efficiency of file lookup.

Current research effort focuses on improving the performance of file searching/lookup in P2P networks, but there exist some fundamental and challenging issues that remain unanswered. The *free-riding* and the *tragedy of the commons* are two of such problems. It was reported in [1] that nearly 70% of P2P users do not share any file in a P2P community and these users simply free-ride on other users who share information. Since there are few users who are willing to share information or to provide file transfer services, nearly 50% of all file search responses come from the top 1% of information sharing nodes. Therefore, nodes that share information and resources are prone to congestion, which leads to the tragedy of the commons problem [9]. In short, the current P2P network does not provide *service differentiation*, so there is no *incentive* for users to share information or to provide file transfer services.

Note that there are some oversimplified mechanisms that have been implemented in P2P softwares in order to encourage people to share information. For example, Kazaa [10], considers the "participation level" of each peer. The "participation level" is calculated by the ratio between a peer's recent uploads and downloads. But this ratio is not accumulated over time, and only provides differentiation for query requests. Another P2P system, eMule [4], established a credit system where credits are exchanged between any two specific nodes. During the competition of downloads, the information providing node reduces the queueing delay for the node which previously provide more upload service to that information providing node. Note that both of these mechanisms do not provide any analytical or concrete solution as to why such kind of mechanisms can work and why fairness can be maintained.

In this paper, we propose a protocol to provide service differentiation based on the contribution level of individual node. Our protocol targets on the file transfer process because the amount of data transfer per unit time is much higher than that of the object lookup/query. In this context, a node which offers popular files for sharing and provides more service (via file upload) to the P2P community will achieve a higher contribution level. As a result, when such a node later asks for a file transfer, it will be granted a higher utility than competing nodes having lower contribution levels. We address the challenges of incorporating such *incentive-compatible* resource distribution mechanism in the file transfer process such that we can: (i) encourage nodes to share information or provide services with their peers, (ii) achieve fair service differentiation between network users, and (iii) maximize the social welfare[16] or the aggregated perceived utility of the users. It is important to point out that our incentive protocol can be adopted by various P2P systems which use either the distributed query (e.g., Gnutella) or the consistent hashing approach (e.g., Chord or CAN).

The proposed incentive-compatible resource distribution mechanism has the following properties:

* A preliminary version of this paper appeared in ACM SIGMETRICS 2004.
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1. **Fairness:** nodes which have contributed more to the P2P network should gain more resources or achieve higher utility in the resource sharing.
2. **Avoidance of resource wastage:** the mechanism will not assign more resource to a node than it can consume. In case there is congestion on the communication path, the mechanism can adapt to the congestion level and re-distribute the resources accordingly.
3. **Adaptability and Scalability:** The mechanism can adapt to dynamic events such as node join/leave. Since the mechanism runs at each participating node, its performance is scalable as the size of the P2P network increases.
4. **Maximization of individual and social utility**: Each node has an incentive to follow the incentive protocol to maximize its utility in a competition game. On the other hand, the resource distribution maximizes the aggregate perceived utility from an information sharing node's perspective of view.

As we will show, the proposed mechanism makes different requesting users to bid for resource and thereby creating a *dynamic competitive game*. In order to assure every node in the P2P network will follow the mechanism honestly, the dynamic game created should be *strategic-proof* and *κ-collusion-proof*. The first property implies that following the proposed mechanism is the *best* strategy for each user in a P2P network. The second property implies that users cannot gain extra resource by cooperatively deceiving the system.

A. *Related work*

Let us briefly present some related work. In [7], the authors address one possible mechanism for *centralized* P2P systems like Napster. Our work, on the other hand, can be applied to both centralized or distributed P2P networks. Zhong et al. [21] discuss shortcomings of *micro-payment* and reputation system. They propose a cheat-proof, credit-based mechanism for mobile ad-hoc networks. However, they did not address how to provide incentive and service differentiation in the P2P setting. In [6], the authors discuss the economic behavior of P2P storage networks. Our work, on the other hand, focuses on the file-transfer and bandwidth allocation of a P2P system and we use the mechanism design approach in designing a competitive game in a P2P system. In [11], the authors proposed a budget-balance virtual money exchange mechanism to bring incentive into P2P networks. Our work, in the same context of the problem, uses a game-theoretic approach, and provides a stronger solution concept which is incentive-compatible. Lastly, algorithmic mechanism design [13], [17] provides a theoretical framework for designing incentive mechanisms.

B. *Paper organization*

The balance of our paper is as follows. In Section II, we provide a general overview of the interaction between the information providing node and information seeking nodes. In Section III, we present the resource distribution mechanism and its properties. In Section IV, we present the dynamic game model and how it can be applied to a P2P network. In Section V, we present a generalized mechanism that deals with incentive and heterogeneous perceived utilities. Convergence analysis of the dynamic game is presented in Section VI. In Section VII, we present the performance evaluation of the proposed mechanism and competition game. Section VIII concludes.

II. **Incentive P2P System Overview**

Let us provide an overview of our incentive P2P system. In particular, we illustrate the interaction between different nodes during the file transfer process. In later sections, we will formally present the development of the resource distribution mechanism and its properties.

Each node in our incentive P2P network can play the role of a server and a client at the same time. During a file transfer process, the node which performs the file sharing service (e.g., uploading files to other nodes) is called the *"source node"*, which is denoted by $\mathcal{N}_s$. Nodes which request for file download from $\mathcal{N}_s$ are called the *"competing nodes"*, which are denoted as $\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_N$, where N is the number of competing nodes. Each node in our incentive P2P network has a contribution value, which indicates how much service that node has provided to the whole P2P community. Due to the lack of space, we will not discuss the architecture issues on how to provide a scalable and secure contribution value reporting system. Please refer to [8].

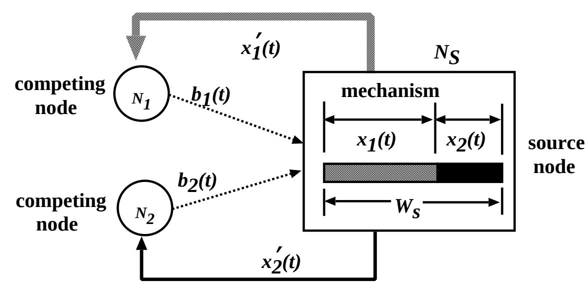


Fig. 1. Illustrating two competing nodes and a source node.

A scenario where there are two competing nodes $\mathcal{N}_1$ and $\mathcal{N}_2$ which request file download service from the source node $\mathcal{N}_s$. is illustrated in Figure 1. The source node has an upload bandwidth resource of $\mathcal{W}_s$ (in unit of bps). From time to time, these competing nodes send messages $b_1(t)$ and $b_2(t)$ (in unit of bps) to $\mathcal{N}_s$, telling $\mathcal{N}_s$ how much transfer bandwidth they want. Upon receiving these messages, $\mathcal{N}_s$ will use a resource distribution mechanism (to be presented in Section III) to distribute its bandwidth resource $\mathcal{W}_s$ based on the values of $b_1(t)$, $b_2(t)$, as well as their contribution values which are denoted by $C_1(t)$ and $C_2(t)$ respectively. As a result, $\mathcal{N}_s$ sends information to $\mathcal{N}_1$ and $\mathcal{N}_2$ with bandwidth $x_1(t)$ and $x_2(t)$ respectively. However, it is possible that there is network congestion along the communication path between $\mathcal{N}_s$ to $\mathcal{N}_1$ (or $\mathcal{N}_2$), therefore, packets may be lost and the actual received bandwidth at node $\mathcal{N}_1$ and $\mathcal{N}_2$ are $x'_1(t) \leq x_1(t)$ and $x'_2(t) \leq x_2(t)$ respectively.

The message $b_i(t)$ plays two important roles. First, it can be regarded as the *"bandwidth bidding"* message from the perspective of the competing node $\mathcal{N}_i$. Another usage of $b_i(t)$ is that it is a *confirmation* to the source node $\mathcal{N}_s$ that $\mathcal{N}_i$ has received certain amount of service (measured in unit of bps). Therefore, $\mathcal{N}_s$ can use this message as evidence to update its contribution. In general, the message $b(t)$ helps the source node to determine the proper bandwidth assignment. If a competing node is inactive or failed, the source node will assume that the competing node cannot receive any data and therefore, it will not send any more packet to the competing node. The source node, on the other hand, can *adjust* the bandwidth resource assignment whenever it receives a bidding message. The justifications for this adjustment are: (1) a new arriving competing node may request $\mathcal{N}_s$ for a new file download; (2) an existing competing node finishes its file transfer service; (3) due to the network congestion situation, a competing node replies different values of bidding messages throughout the file download session. To efficiently utilize the bandwidth resource $\mathcal{W}_s$ and to improve the rate of contribution increase for $\mathcal{N}_s$, the source node needs to adjust bandwidth distribution among competing nodes. Lastly, Figure

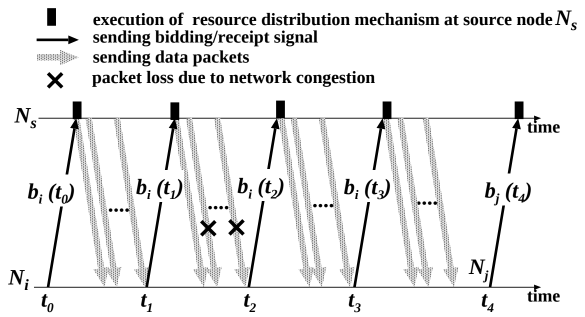


Fig. 2. Interaction between competing nodes and a source node.

2 illustrates the interaction between the competing nodes and the source node $\mathcal{N}_s$. At time t_0, the competing node $\mathcal{N}_i$ requests for a file transfer of a large file F_i and sends a bidding message $b_i(t_0)$ to $\mathcal{N}_s$. After verifying the identity and contribution level of $\mathcal{N}_i$, $\mathcal{N}_s$ uses the resource distribution mechanism to determine the sending bandwidth $x_i(t_0)$, and delivers some data packets of F_i to $\mathcal{N}_i$ based on this rate allocation. After receiving these data packets, $\mathcal{N}_i$ sends another bidding/receipt $b_i(t_1)$ at time t_1, then $\mathcal{N}_s$ determines the the new resource allocation and sends some additional data packets of file F_i based on $x_i(t_1)$. Note that at this round of the data delivery, some data packets are lost due to network congestion, therefore, $\mathcal{N}_i$ sends a bidding/receipt $b_i(t_2)$ to $\mathcal{N}_s$ at time t_2, with $b_i(t_2) < b_i(t_3)$. The source node $\mathcal{N}_s$ adjusts the resource allocation and delivers additional data packets of file F_i to $\mathcal{N}_i$ at a lower rate. At time t_4, a new competing node $\mathcal{N}_j$ requests for a file transfer of the file F_j from $\mathcal{N}_s$ and it sends its bidding message $b_j(t_4)$, $\mathcal{N}_s$ adjusts the resource allocation based on the latest biddings of these two competing nodes $\mathcal{N}_i$ and $\mathcal{N}_j$.

III. Resource Distribution Mechanism

In this section, we discuss how the source node, say $\mathcal{N}_s$, implements a mechanism to distribute its bandwidth resource $\mathcal{W}_s$ (in Mb/s) among all its competing nodes $\mathcal{N}_1, \cdots, \mathcal{N}_N$. For ease of presentation, we start with some simple mechanisms and will discuss their shortcomings. Then we introduce more complex features so as to provide service differentiation and incentive.

Even Sharing Mechanism (ESM): one naive mechanism is to *evenly* divide the resource $\mathcal{W}_s$ among all competing nodes. When there are N competing nodes requesting for file downloads, $\mathcal{N}_s$ transmits a file to a competing node $\mathcal{N}_i$ with an assigned bandwidth x_i:

$$x_i \quad = \quad \frac{\mathcal{W}_s}{N} \text{ for } i = 1, \ldots, N. \qquad (1)$$

Although this mechanism seems fair in distributing the resource, there are some inherent problems. First, the bandwidth resource wastage may be significant. The wastage can occur in at least two forms: (1) if the connection between $\mathcal{N}_s$ and a competing node is congested, then the assigned bandwidth is not fully utilized, (2) the physical download bandwidth of a competing node may be less than the assigned bandwidth of $\mathcal{W}_s/N$, so the source node $\mathcal{N}_s$ cannot deliver information at that rate. Note that resource wastage also implies that $\mathcal{N}_s$ contributes some service to the community, but the amount of work may not be counted toward its contribution. Another problem of this type of mechanism is that it provides no service differentiation among competing nodes. Therefore, rational users have no incentive to share information or service, consequently we have the tragedy of the commons problem.

Resource Bidding Mechanisms (RBM): the objective of this mechanism is to overcome the resource wastage problem mentioned above. Under this mechanism, every competing node is required to send a bidding message periodically to $\mathcal{N}_s$. Let $b_i(t)$ be the bidding message from the competing node $\mathcal{N}_i$ at time t and it indicates the *"maximum"* bandwidth (in unit of bps) that $\mathcal{N}_i$ can receive at time t. Given all the bidding messages from competing nodes, $\mathcal{N}_s$ has the knowledge of the *upper bounds* bandwidth assignment and will not assign any bandwidth higher than $b_i(t)$ to $\mathcal{N}_i$ at time t. Notice that it seems possible for some competing nodes to request for more bandwidth than they really need; we will discuss the rational bidding values of competing nodes in Section IV.

One important property of the RBM mechanism is that it provides the *max-min fairness*[2]. Suppose $\vec{x} = [x_1, \ldots, x_N]$ is the bandwidth allocation for all N competing nodes with the feasible domain $x_i \in [0, b_i]$ for $i = 1, .., N$. Then a feasible allocation is max-min fair if and only if an increase of x_i within its domain of feasible allocation must be at the cost of a decrease of some x_j, where $x_j \leq x_i$. In other words, the max-min allocation gives the competing node with the smallest bidding value the largest feasible bandwidth while not wasting any resource for the source node $\mathcal{N}_s$. One can show that there exists a unique max-min fair allocation vector $\vec{x}$, and it can be obtained by the *"progressive filling algorithm"* (or the water filling algorithm). The algorithm initializes all $x_i = 0$, then will increase all competing nodes' bandwidth resource at the *same rate*, until one or several competing nodes hit their limits (i.e., $x_i = b_i$), then the amount of allocated resources for these competing nodes will not be increased any more. The algorithm will continue to increase the resource of other competing nodes at the same rate. The algorithm terminates when all competing nodes hit their limits, or the total resource $\mathcal{W}_s$ is fully utilized. Mathematically, we express the max-min resource distribution as follows. Let $\mathcal{N}_{\hat{1}}, .., \mathcal{N}_{\hat{N}}$ as N competing nodes sorted based on the non-decreasing value of b_i. The resource distribution of the RBM mechanism is

$$x_{\hat{k}} \quad = \quad \min \left\{ b_{\hat{k}}, \frac{\mathcal{W}_s - \sum_{\hat{i}=1}^{\hat{k}-1} x_{\hat{i}}}{N - \hat{k} + 1} \right\} \qquad \hat{k} = 1, \ldots, N. \quad (2)$$

Figure 3(a) illustrates the RBM with four competing nodes of $\vec{b} = [1, 2.5, 2.5, 4]$ (in unit of Mbps) and the resource bandwidth $\mathcal{W}_s = 7$ Mbps. The resource allocation is $\vec{x} = [1, 2, 2, 2]$ (in unit of Mbps), which is depicted by the *"shaded region"* in the figure. Although the RBM avoids resource wastage, but it doesn't

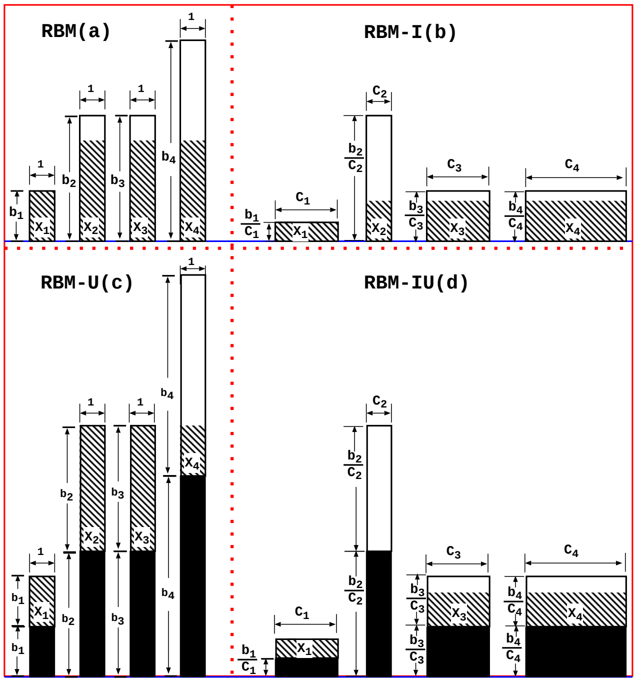


Fig. 3. Resource distribution mechanisms: (a) RBM; (b) RBM-I; (c) RBM-U; (d) RBM-IU. The shaded region represents the amount of resource allocation for individual node.

provide any *incentive* for nodes to share information. Two competing nodes with the same value of biddings will obtain the same amount of resource regardless of their actual contribution to the P2P community.

Resource Bidding Mechanism with Incentive (RBM-I): To provide incentive, this mechanism takes the contribution level of competing nodes into account. Let C_i be the contribution value of the competing nodes $\mathcal{N}_i$ and this value reflects the amount of work that $\mathcal{N}_i$ has performed, for example, sharing and uploading files for other nodes. The contribution value C_i can be retrieved from some distributed database agents at the beginning of the file transfer process, or every time when the source node receives the bidding message $b_i(t)$ from the competing node $\mathcal{N}_i$.

One can implement the resource bidding mechanism with incentive (RBM-I) by enhancing the progressive filling algorithm as follows. We distribute bandwidth resource to all competing nodes at the same time but with different rates. In particular, the competing node $\mathcal{N}_i$ will have a resource assignment rate of C_i. Also, once the assigned resource to $\mathcal{N}_i$ reaches its limit of b_i, $\mathcal{N}_i$ will be taken out from the resource distribution. Therefore, one can view the mechanism as a weighted max-min resource distribution. Mathematically, we express the RBM-I as follows. Let $\mathcal{N}_{\hat{1}}, \ldots, \mathcal{N}_{\hat{N}}$ as N competing nodes sorted based on the non-decreasing value of b_i/C_i. The resource distribution is

$$x_{\hat{k}} \;=\; \min\left\{ b_{\hat{k}}, \frac{C_{\hat{k}}\left(\mathcal{W}_s - \sum_{i=1}^{\hat{k}-1} x_{\hat{i}}\right)}{\sum_{j=\hat{k}}^{N} C_j} \right\} \quad \hat{k} = 1, \ldots, N. \quad (3)$$

Using the previous example in RBM but now with contributions $\vec{C} = [2.5, 1, 2.5, 4]$, the resource allocation is $\vec{x} = [1, 0.8, 2, 3.2]$ (in unit of Mbps), which is showed in Figure3(b). One important property of this mechanism is that if two competing nodes have the *same* bandwidth bidding values and without hitting their limits, then the assigned bandwidth will be *proportional* to their contribution values (i.e., $\mathcal{N}_2$ and $\mathcal{N}_3$).

Resource Bidding Mechanism with Utility Feature (RBM-U): The focus of this mechanism is the efficiency of the resource allocation from the perspective of the competing nodes' satisfaction. Consider a case of two competing nodes $\mathcal{N}_i$ and $\mathcal{N}_j$ which have the same contribution values. If the bandwidth resource at the source node is $\mathcal{W}_s = 1$ Mbps and the two bidding messages are $b_i(t) = 10$ Mbps and $b_j(t) = 1$ Mbps. Based on the RBM mechanism, they will receive a bandwidth resource of 0.5 Mbps each. Although the resource at $\mathcal{N}_s$ is efficiently utilized, but the degree of satisfactions of these two competing nodes are obviously different. To overcome this problem, we use the concept of *utility*[16] to represent the degree of satisfaction of a competing node given certain allocated bandwidth resource.

We first define the family of utility functions we consider in this paper. Given an allocated bandwidth x, the utility of the node $\mathcal{N}_i$ is denoted by $U_i(x)$. The utility function we consider in this work satisfies the following three assumptions: (a) $U_i(x)$ is concave (or the marginal utility $\frac{dU_i(x)}{dx}$ is non-increasing $\forall x \geq 0$), (b) $U_i(0) = 0$ and, (c) the utility depends on the ratio of $\frac{x}{b}$. In other words, $U_i(x_i) = U_j(x_j)$ whenever $\frac{x_i}{b_i} = \frac{x_j}{b_j}$ for any two competing nodes $\mathcal{N}_i$ and $\mathcal{N}_j$. The justifications for the above assumptions are as follows. First, the utility function is concave, which is often used to represent *elastic traffic* such as file transfer[16]. Concavity implies that the marginal utility is non-increasing as one increases the allocated bandwidth resource x. This captures the physical characteristics of elastic traffic: the utility increases significantly when a competing node starts receiving service. The increase of utility becomes less significant when the receiving bandwidth is nearly saturated. Second, the utility is zero when a competing node is not allocated any bandwidth. Third, because utility measures the satisfaction of a competing node, naturally, it is a function of the *fraction* of allocated resource over the bidding resource. Furthermore, this assumption normalizes the utility of all nodes so we can *compare* the degree of satisfaction of different nodes.

The objective of the RBM-U mechanism is to maximize the social (or aggregated) utility. Formally, we have:

$$\max \sum_{i=1}^{N} U_i(x_i) \qquad \text{s.t.} \sum_{i=1}^{N} x_i \leq \mathcal{W}_s \;\; \text{and} \;\; x_i \in [0, b_i] \; \forall \; i.$$

It is important to point out that the implication of this maximization problem is to allocate resource to the competing node which currently has the *largest* marginal utility (i.e., largest $dU_i(x)/dx$). The allocation process starts with $x_i = 0$ for

$i = 1,..N$, then assigns resource to the node which has the largest marginal utility and ends when the resource $\mathcal{W}_s$ is used up, or all the competing nodes are fully satisfied with $x_i = b_i \forall i$.

In reality, each competing node's utility function may not be known to other nodes. But after allocating certain amount of bandwidth to a competing node, a source node can infer the utility of the competing node by a "perceived utility". From now on, we use "utility" to indicate the "perceived utility" by the source node. We will discuss how competing nodes maximize their underlying true utilities in the next section.

Definition *1:* We define the *perceived utility* of node i by any source node to be $U_i(x_i)$, where x_i is the assigned bandwidth to node i. This is an estimation on the true utility of node i by the source node.

Let us consider the following form of perceived utility function which satisfies the above three assumptions:

$$U_i(x_i) = \log\left(\frac{x_i}{b_i} + 1\right) \qquad \text{where } x_i \in [0, b_i].$$

Let us state the connection between the underlying true utility vs. the perceived utility. Although b_i indicates the "maximum" receiving bandwidth of node i, node i may lie about this information in order to obtain more bandwidth during the resource competition (will be discussed in a later section). Whenever b_i is the true value of the node's "maximum" receiving bandwidth, the perceived utility represents the true utility.

The marginal utility is $U_i' = (x_i + b_i)^{-1}$. Therefore, the RBM-U mechanism tries to increase the resource to the competing node which has the smallest value of $x_i + b_i$ at any time. Using the previous example of RBM of 4 competing nodes with $\vec{b} = [1, 2.5, 2.5, 4]$ and $\mathcal{W}_s = 7$ Mbps, we use the above perceived utility function and the resource allocation which maximizes the aggregated utility is $\vec{x} = [1, 2.5, 2.5, 1]$ (in unit of Mbps). This result is depicted in Figure 3(c). The figure shows graphically how the mechanism works. Each competing node, say $\mathcal{N}_i$, has a lower limit height which is equal to b_i (e.g., the darken region). The enhanced progressive filling algorithm distributes resource first to the competing node that has the *lowest* depth since that node has the *largest* marginal utility at that point. When the assigned resource to node $\mathcal{N}_i$ equals to its maximum bidding b_i, node $\mathcal{N}_i$ is taken out from the resource distribution. The algorithm terminates when all nodes reach their maximum allocation, or when the resource $\mathcal{W}_s$ is fully utilized.

Presenting the above mechanisms is important because it gives the understanding on how we "derive" the final resource distribution mechanism as follows.

Resource Bidding Mechanism with Incentive and Utility Feature (RBM-IU): One can view the RBM-IU mechanism as a *generalization* of the previous discussed mechanisms. This mechanism considers both the utilities of competing nodes and their contribution values. Each competing node, say $\mathcal{N}_i$, has its contribution value C_i and bidding message b_i. Mathematically, the RBM-IU performs the following constraint optimization:

$$\max \sum_{i=1}^{N} C_i \log\left(\frac{x_i}{b_i} + 1\right) \quad \text{s.t.} \sum_{i=1}^{N} x_i \leq \mathcal{W}_s, x_i \in [0, b_i] \ \forall \ i.$$

The RBM-IU mechanism enhances the progressive filling algorithm as follows: (a) We treat the competing node $\mathcal{N}_i$ as a bucket with area b_i and width C_i. (b) The bucket of the competing node $\mathcal{N}_i$ is located at the height b_i/C_i, therefore the upper limit of the bucket is at the height of $2b_i/C_i$. (c) At any time, the RBM-IU mechanism increases the amount of resource into the competing node's bucket which currently has the lowest height. In other words, the bucket that has the largest weighted marginal utility (i.e., weighted by the contribution value). It is interesting to observe that when competing nodes have the same contribution value, the RBM-IU is equivalent to the RBM-U mechanism. The spirit of this mechanism is to increase the amount of resource of the competing node which has the largest weighted marginal utility of $C_i/(b_i + x_i)$ with the rate of C_i. Figure 3(d) illustrates the RBM-IU mechanism with $\vec{b} = [1, 2.5, 2.5, 4]$, $\vec{C} = [2.5, 1, 2.5, 4]$ and $\mathcal{W}_s = 7$ Mbps. The final resource allocation is $\vec{x} = [1, 0, 2.3, 3.7]$ (in unit of Mbps). From the figure, one can observe that the mechanism fills the bucket of $\mathcal{N}_i$ at most up to its area limit of b_i with the resource distribution rate of C_i. The bucket of $\mathcal{N}_i$ at the "resource level" $(x_i + b_i)/C_i$ is guaranteed to have the marginal utility $C_i/(x_i + b_i)$. The algorithm terminates when all competing nodes reach their resource limit, or when the resource $\mathcal{W}_s$ is fully utilized.

The RBM-IU mechanism can be expressed by the following pseudo-code. The source node $\mathcal{N}_s$ maintains a *sorted list* of competing nodes with b_i/C_i in ascending order.

RBM-IU Mechanism ()

1. **if** $(\sum_{i=1}^{N} b_i \leq \mathcal{W}_s)$ **return** $\vec{x} = \vec{b}$;`/*no congestion*/`
2. l=2; u=1; `/*upper and lower limits index*/`
3. v=C_1; w=$\mathcal{W}_s$; `/* filling rate and resource capacity*/`
4. level = $\frac{b_1}{C_1}$;`/*initialize resource level*/`
5. **while** (w> 0)
6.   **if** ((min$\{\frac{2b_u}{C_u}, \frac{b_l}{C_l}\}$ − level)*v≥w)
7.   level = level + w/v; w=0;
8.   **else if** $(\frac{2b_u}{C_u} < \frac{b_l}{C_l})$
9.   w −= ($\frac{2b_u}{C_u}$− level)*v; level = $\frac{2b_u}{C_u}$; v −= C_u; u++;
10.   **else**
11.   w −= ($\frac{b_l}{C_l}$− level)*v; level = $\frac{b_l}{C_l}$; v += C_l; l++;
12. **for** (each i)
13. $x_i = \min\{\max\{0, (\text{level} - \frac{b_i}{C_i}) * C_i\}, b_i\}$;
14. **return** $\vec{x}$;

Based on the above code, $\mathcal{N}_s$ performs the filling algorithm when the total bidding is greater than the total available resource. In determining the final "resource level", we have three cases in the *while* loop at line 5: (1) When the resource is used up, the loop ends with the final "resource level" (line 6-7). (2) If the next available resource level is at the upper limit (or bidding level) of some competing node, then we adjust the remaining amount of available resource and reduce the filling rate by that competing node's contribution value C_i since we won't give any more resource to that satisfied competing node (line 8-9). (3) If the next available resource level is a lower limit of some competing node, then we adjust the remaining amount of available resource and increase the filling rate by that competing node's contribution value C_i (line 11). The reason is this competing node will have the largest weighted marginal utility for its turn to gain the resource at a rate of C_i. Note that this is a *linear* algorithm with a complexity of $O(N)$ where N is the number of competing nodes at the source node $\mathcal{N}_s$. Therefore, resource distribution can be executed quickly.

Theorem *1: The RBM-IU mechanism solves the following constraint optimization problem:*

$$\max \sum_{i=1}^{N} C_i \log\left(\frac{x_i}{b_i}+1\right) \quad \text{s.t.} \sum_{i=1}^{N} x_i \leq \mathcal{W}_s, x_i \in [0, b_i] \ \forall\ i.$$

Proof: Let us consider an equivalent constraint optimization problem in a standard form as follows:

$$\max \sum_{i=1}^{N} C_i \log(x_i + b_i)$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} x_i \leq \mathcal{W}_s, x_i \leq b_i \ \forall\ i \text{ and } x_i \geq 0 \ \forall\ i.$$

We have the Lagrangian function:

$$L(x,\lambda) = \sum_{i=1}^{N} C_i \log(x_i+b_i) - \sum_{i=1}^{N} \lambda_i(x_i-b_i) - \lambda_0\left(\sum_{i=1}^{N} x_i - \mathcal{W}_s\right)$$

where each λ_i is the Lagrangian multiplier associated with the according "less than" constraint.

For any optimal solution x^*, the Karush-Kuhn-Tucker (KKT) condition [19] requires there exists a non-negative Lagrangian multiplier λ such that the following conditions are satisfied:

$$\frac{\partial L}{\partial x_i} = \frac{C_i}{x_i^* + b_i} - \lambda_i - \lambda_0 = 0 \ (\text{or } \ \leq 0 \text{ if } x_i^* = 0),$$

$$\frac{\partial L}{\partial \lambda_i} = x_i^* - b_i = 0 \ (\text{or } \ \leq 0 \text{ if } \lambda_i = 0),$$

$$\frac{\partial L}{\partial \lambda_0} = \sum_{i=1}^{N} x_i^* - \mathcal{W}_s = 0 \ (\text{or } \ \leq 0 \text{ if } \lambda_0 = 0).$$

First, if $\sum_{i=1}^{N} b_i \leq \mathcal{W}_s$, the RBM-IU mechanism assigns $x_i = b_i$ for all i. By checking the KKT condition, λ_0 has to be 0, and $\lambda_i = C_i/(x_i + b_i)$ for all $i > 0$. Second, if $\sum_{i=1}^{N} b_i > \mathcal{W}_s$, the RBM-IU mechanism makes the resource filling at the level $h = (x_j + b_j)/C_j$ for some j, where $0 < x_j < b_j$. It uses up all resource so that $\sum_{i=1}^{N} x_i - \mathcal{W}_s = 0$, and λ_0 can be positive. Make $\lambda_0 = 1/h$, the KKT requires:

$$\frac{C_i}{x_i^* + b_i} - \lambda_i = \frac{1}{h} \ (\text{or } \ \leq \ \text{if } x_i^* = 0).$$

It is satisfied in all three cases: (a) When $0 < x_i < b_i$, $\lambda_i = 0$. This is the case when all nodes get the resource at the final resource level, so $\frac{C_i}{x_i^*+b_i} = \frac{1}{h}$. (b) When $x_i = 0$, node i is not filled because b_i/C_i is higher than h. Thus, $\frac{C_i}{x_i+b_i} - \frac{1}{h} \leq 0 \leq \lambda_i$. (c) When $x_i = b_i$, $\frac{C_i}{x_i+b_i} - \frac{1}{h} = \lambda_i \geq 0$. Because the resource level $\frac{x_i+b_i}{C_i}$ of node i must be less than or equal to the final resource level h.

By the strong concavity of the objective function and all linear constraints, the above KKT condition also guarantees that the solution from RBM-IU mechanism is the optimal solution for the constraint optimization problem. ∎

Moreover, the following two important theorems state some of the "*desirable*" properties of the RBM-IU mechanism.

Theorem *2: For any two competing nodes $\mathcal{N}_i, \mathcal{N}_j$, the mechanism RBM-IU assigns the bandwidth x_i and x_j such that:*

$$\text{if } \frac{C_i}{b_i} \geq \frac{C_j}{b_j} \Longrightarrow U_i(x_i) \geq U_j(x_j). \tag{4}$$

Proof: When $\frac{C_i}{b_i} \geq \frac{C_j}{b_j}$, the stated condition is equivalent to:

$$\frac{b_i}{C_i} \leq \frac{b_j}{C_j}. \tag{5}$$

So initially, node $\mathcal{N}_i$ has a lower resource level than node $\mathcal{N}_j$. Therefore, bucket i will hit its capacity faster than j. In the final bandwidth distribution, we have:

$$\frac{x_i + b_i}{C_i} \leq \frac{x_j + b_j}{C_j}. \tag{6}$$

When Eq. (6) meets the strictly less than condition, this implies that $x_i = b_i$. In this case, node $\mathcal{N}_i$ is fully satisfied and reaches its maximal utility value of $U_i(x_i) = \log 2$ and therefore $U_i(x_i) \geq U_j(x_j)$. When Eq. (6) meets the equality condition, we divide Eq. (5) by $\frac{x_i+b_i}{C_i} = \frac{x_j+b_j}{C_j}$, which gives:

$$\begin{aligned} \frac{b_i}{x_i+b_i} \leq \frac{b_j}{x_j+b_j} \quad &\Longrightarrow \quad \frac{x_i+b_i}{b_i} \geq \frac{x_j+b_j}{b_j} \\ &\Longrightarrow \quad \log\left(\frac{x_i+b_i}{b_i}\right) \geq \log\left(\frac{x_j+b_j}{b_j}\right) \\ &\Longrightarrow \quad U_i(x_i) \geq U_j(x_j) \end{aligned}$$ ∎

Remarks: The implication of this theorem is that a competing node which has the highest contribution per unit resource request than any other nodes will receive the highest utility. Therefore, the RBM-IU provides incentive to P2P system and makes the balanced utilities among all competing nodes.

Theorem *3: The resource allocation $\vec{x}$ is "Pareto optimal", which implies that the resource allocation vector cannot be improved further without reducing the utility of at least one competing node.*

Proof: There are two cases for terminating the RBM-IU mechanism. One is when the aggregated bidding $\sum_{i=1}^{N} b_i \leq \mathcal{W}_s$. In this case, and $\vec{x} = \vec{b}$ and all nodes are equally satisfied. The second case is when the total resource $\mathcal{W}_s$ is fully utilized. In this case, no matter how efficient is the other resource allocation vector, it has to decrease some amount of resource of some competing nodes so as to improve the utility of other nodes. ∎

IV. Resource Competition Game

In the proposed incentive P2P network, each competing node sends bidding messages which indicates its "maximum" receiving bandwidth to the source node, in return, the source node uses the mechanism RBM-IU for bandwidth resource distribution. In reality, each competing node can report bidding values strategically which may not necessarily be its "maximum" receiving bandwidth. Thus, by allowing each competing node to choose it bidding value freely, the mechanism RBM-IU forms a resource competition game among all competing nodes. The *interaction* between the competing nodes and the source node can be described by the game-theoretic framework[12]. We model the interaction of resource competition as a game and explore its solution and properties. Lastly, we discuss how this game can be incorporated into the P2P protocol such that it converges to the Nash equilibria.

A. Theoretical Competition Game

We model the resource bidding and distribution processes as a competition game among all the competing nodes. One basic postulate in the game theory is that the game structure is *common knowledge* to all players. In our competition game, we assume total amount of bandwidth resource $\mathcal{W}_s$ and all contribution values C_i's are common knowledge. This means that all nodes know the information, know that their rivals know the information, and know that their rivals know that they know the information, and so on. Also, we only consider the non-trivial situations when $\sum b_i > \mathcal{W}_s$. The competition game can be described as follows:

1. All the competing nodes are the players of the game.
2. The bidding message b_i is the strategy of the competing node $\mathcal{N}_i$. A bidding vector $\vec{b} = \{b_1, b_2, ..., b_N\}$ is a strategy profile where N is the number of competing nodes in the game.
3. The mechanism RBM-IU defines the rules and the structure of the game. We can regard mechanism RBM_IU as a mapping which has $\vec{C}$ and $\vec{b}$ as input parameters and returns $\vec{x}$ as output.
4. The outcome of the game is the vector $\vec{x}$ which represents the amount of bandwidth resource each competing node obtains.
5. The objective of each player is to maximize its assigned bandwidth x_i. We do not explicitly assume the utility function of each player. But as long as the utility function is a non-decreasing function in x_i, the objective to maximize x_i is equivalent to maximize its underlying true utility.

Lemma *1: The mapping function* RBM-IU*:* $\vec{C} \times \vec{b} \rightarrow \vec{x}$ *is quasi-concave in each individual's strategy* b_i*.*

Proof: Consider any strategy profile $\vec{b} = \{b_i, \vec{b}_{-i}\}$, where $\vec{b}_{-i}$ is any fixed strategy profile (or bidding messages) of players other than $\mathcal{N}_i$. We regard the resource allocation of $\mathcal{N}_i$ as a function of its bidding value $x_i(b_i)$. When we increase b_i from zero gradually, $x_i(b_i)$ will also increase monotonically with $x_i(b_i) = b_i$. Because the weighted marginal utility $C_i(x_i + b_i)^{-1}$ must be among the largest for b_i close to zero. After that, when we continue to increase b_i, the marginal utility decreases. At some point when the marginal utility is not among the largest, the bandwidth allocation $x_i(b_i) < b_i$ and starts to decrease monotonically with b_i until $x_i(b_i) = 0$. From the one-peak pattern of $x_i(b_i)$, we know the upper-level contour set is convex and therefore, the function is quasi-concave in b_i. ∎

Theorem *4: There exists at least one Nash equilibrium in the competition game.*

Proof: Note that the bidding values do not have to be infinite because x_i becomes zero when b_i is larger than certain threshold. Accordingly, the strategy set is convex and compact. The mechanism represents a continuous function for resource distribution and from Lemma 1, it's quasi-concave in each b_i. Therefore, by Proposition 8.D.3 in [12], the game has at least one Nash equilibrium. ∎

Lemma *2: For any player, say* $\mathcal{N}_i$*, the strategy* $b_i^* = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$ *implies a resource allocation of* $x_i^* = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$ *for* $i = 1, \ldots, N$*.*

Proof: Let W' be the least amount of resource which gives $\mathcal{N}_i$ the resource $x_i = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$. This outcome will give $\mathcal{N}_i$ the "resource level" at height $h' = \frac{2b_i^*}{C_i} = \frac{2\mathcal{W}_s}{\sum_{j=1}^N C_j}$. Any other player, say $\mathcal{N}_k$, may report its strategy b'_k in two possible cases: (1) When $b'_k \le \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$, we have $2b'_k/C_k \le \frac{2\mathcal{W}_s}{\sum_{j=1}^N C_j} = h'$. Hence, $x'_k = b'_k \le \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$. (2) When $b'_k > \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$, we have $b'_k/C_k > \frac{\mathcal{W}_s}{\sum_{j=1}^N C_j}$. So $x'_k = (h' - b'_k/C_k)C_k < \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$. As a result, $W' = \sum_{k=1}^N x'_k \le \sum_{k=1}^N \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j} = \mathcal{W}_s$. So with $\mathcal{W}_s$ amount of resources, we can at least distribute $\frac{\mathcal{W}_s * C_i}{\sum_{j=1}^N C_j}$ amount of resources to player $\mathcal{N}_i$ and this is also the bidding value b_i. Therefore the RBM-IU mechanism will allocate exactly $x_i^* = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$ amount of resources to player $\mathcal{N}_i$. ∎

Remark: The implication of the above lemma is in guaranteeing that a player can gain its fair share of resource during the competition. For some players who have small contribution values, they will not suffer from resource starvation. But for free-riders, they will eventually gain zero resource in competition.

Theorem *5: The strategy profile* $b_i^* = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$ *for player* $\mathcal{N}_i$*, where* $i = 1, \ldots, N$*, is a Nash equilibrium.*

Proof: The aggregated bidding is $\sum_{i=1}^N b_i^* = \mathcal{W}_s$, so that $x_i^* = b_i^*$ for player $\mathcal{N}_i$, for $i = 1, \ldots, N$. From Lemma 2, any player $\mathcal{N}_k$ who insists on $b_k^* = \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$ gains $x_k = b_k^* = \frac{\mathcal{W}_s C_k}{\sum_{j=1}^N C_j}$. Therefore, regardless to the change of strategy from b_i^*, player $\mathcal{N}_i$ gains $x_i \le \mathcal{W}_s - \sum_{k \ne i} x_k^* = x_i^*$. ∎

Theorem *6: The strategy profile* $b_i^* = \frac{\mathcal{W}_s C_i}{\sum_{j=1}^N C_j}$ *for player* $\mathcal{N}_i$*, where* $i = 1, \ldots, N$*, is the unique Nash equilibrium.*

Proof: Suppose there exist another Nash Equilibrium $\{\hat{b_i}, \hat{x_i}\}$, where each player i uses strategy $\hat{b_i}$ and gains $\hat{x_i}$ amount of resource. At least, one of the players has $\hat{b_i} \ne b_i^*$. By Lemma 2, independent of what the strategies the other players use, strategy b_i^* induces $x_i^* = b_i^*$. Because the RBM-IU mechanism will not assign x_i larger than the bidding b_i, the first necessary condition for $\{\hat{b_i}, \hat{x_i}\}$ to be a Nash Equilibrium is $\hat{b_i} \ge b_i^*$ $\forall i$. Otherwise, b_i^* can always get more bandwidth than any $\hat{b_i} < b_i^*$. For the same reason, if $\hat{x_i} < x_i^*$, strategy b_i^* performs better than $\hat{b_i}$ and strategy $\hat{b_i}$ cannot be a Nash strategy. Therefore, the second necessary condition for $\{\hat{b_i}, \hat{x_i}\}$ to be a Nash Equilibrium is

$\hat{x}_i = x_i^*$ for each player i.

This condition $\hat{x}_i = x_i^*\ \forall i$ implies that the "resource height" $\hat{x}_i/C_i = x_i^*/C_i = W_s/\sum_{j=1}^N C_j$, which is a constant for all players. By the water-filling mechanism, we know the initial "resource level" $\hat{b}_i/C_i$ must also be the same for each player i. Therefore, $\{\hat{b}_i\}$ should only be in the form of $\hat{b}_i = (1+\delta)b_i^*$ for all i with some constant $\delta > 0$. But this cannot be a Nash equilibrium. Because any player can unilaterally change $\hat{b}_i = (1+\delta)b_i^*$ to be smaller in order to gain $\hat{x}_i > x_i^*$. For example, if any player i unilaterally changed its strategy $\hat{b}_i = (1+\delta)b_i^*$ to be $\hat{b}_i = (1+\delta/2)b_i^*$, its bandwidth is increased by $\frac{1}{2}\delta h C_i(1 - C_i/\sum_{j=1}^N C_j)$, where $h = \mathcal{W}_s/\sum_{j=1}^N C_j$. ∎

Another important property of our protocol is that it can avoid one form of collusion attack.

Definition *2: κ-collusion* occurs when a subset of competing nodes $\mathcal{N}_\kappa$ use strategy profile $b_i \neq b_i^*\ \forall i \in \mathcal{N}_\kappa$, and achieve $\sum_{i\in\mathcal{N}_\kappa} x_i > \sum_{i\in\mathcal{N}_\kappa} x_i^*$.

Theorem *7: Assuming that all honest competing nodes use the Nash equilibrium strategy $b_i^* = \mathcal{W}_s C_i/\sum_{j=1}^N C_j$, the RBM-IU mechanism in the source node avoids κ-collusion.*

Proof: Suppose some players are not honest. But when honest players play their Nash equilibrium strategy $b_i^* = \mathcal{W}_s C_i/\sum_{j=1}^N C_j$, by Lemma 2 honest players are guaranteed to have $x_i^* = b_i^*$. Therefore, the aggregated resource received by the dishonest players are $\mathcal{W}_s - \sum_{\text{honest}} x_i^*$, which cannot exceed what they could have gained in the Nash equilibrium. ∎

B. Practical Competition Game Protocol

In the above sub-section, we show the interaction between the source node and all its competing nodes can be modeled as a competition game which has a Nash equilibrium solution. This solution assigns each competing node the amount of resource *proportional* to their contributions, efficiently utilizes all resource at the source node, and it also prevents collusion among group of competing nodes.

Although the theoretical competition game provides these attractive properties, there are gaps to fill so as to realize this theoretical competition game into an incentive P2P network. In particular, one needs to address the following issues:

- **I1** The information of contribution $\vec{C}$ and the amount of resource $\mathcal{W}_s$ is assumed to be common knowledge, how can this be implemented in a P2P system?
- **I2** In real life, a competing node, say $\mathcal{N}_i$, has its maximum download capacity, say w_i (in unit of bps). Also, due to the intermittent network congestion, the actual assigned bandwidth allocation x_i maybe less than the actual received bandwidth x_i'. These two factors will change the Nash equilibrium derived under the theoretical competitive game.
- **I3** In a dynamic environment like a P2P network, new competing node may arrive and request for file download, while existing competing node may leave due to the termination of its file transfer. Under these situations, how can the system reach the equilibrium point according to the change of the number of competing nodes. (More challenges are addressed in [5].)

To address these issues, let us first consider the behavior of the source node. Based on a given strategy profile $\vec{b}$ and contribution values $\vec{C}$, the source node carries out the RBM-IU for bandwidth resource distribution. The *justification* that the source node is willing to use this mechanism is that the allocation result is *Pareto optimal* (based on Theorem 3). Although other Pareto optimal solutions may exist, the source node has no incentive to switch to any other mechanism because no other resource allocation are more efficient. This implies that following the RBM-IU mechanism, the source node can maximize its contribution value **(i.e. amount of upload traffic)** so it can enjoy better service for its future file download request. But without perfect information for all competing nodes, the game solution may oscillate and induce resource wastage. In order for the source node to maximize its contribution, it has the *incentive* to help all competing nodes to reach the Nash equilibrium. In our practical game protocol, the source node will signal a competing node, say $\mathcal{N}_i$, the value of $S_i = \mathcal{W}_s C_i/\sum_{j=1}^N C_j$, when $\mathcal{N}_i$ initiates its request for file download. This information exchange is at low cost because: (1) the signal is sent only *once* for each competing node's arrival; (2) the signal value is computed on flight and it does not need global information of the contribution values of all nodes in a P2P networks. Hence, the issue **I1** is resolved.

For the behavior of the competing nodes, let us see how the signals sent by the source node may help the game to reach its equilibrium. Suppose that a competing node, say $\mathcal{N}_i$, has the maximum download capacity of w_i and a *signal variable* s_i. Initially, s_i encodes the signal value sent by the source node, i.e., $s_i = S_i = \mathcal{W}_s C_i/\sum_{j=1}^N C_j$. The competing node $\mathcal{N}_i$ sends its initial bidding message $b_i = \min\{w_i, s_i\}$ to the source node. After each round of data transfer, $\mathcal{N}_i$ measures x_i', the amount of bandwidth resource it receives from the source node and stores it as the current signal value s_i, i.e., $s_i = x_i'$, To start the next round of data transfer, $\mathcal{N}_i$ sends a new bidding message $b_i = min\{w_i, s_i\}$ to the source node. This bidding strategy assumes that the source node uses the RBM-IU mechanism, so all competing nodes feedback their strategies so as to reach the Nash equilibrium. In the bidding message, competing nodes inform the source node (1) its download bandwidth limit, and (2) whether there is any congestion along the data transfer path.

The behavior of competing nodes described above is an attempt to resolve the issue of **I2** and **I3**. However, one can show that using this protocol, the system may *not* be able to reach the Nash equilibrium. Consider the following illustrative example, initially the source node $\mathcal{N}_s$ has resource $\mathcal{W}_s = 6$ and it has one competing node $\mathcal{N}_1$ with $w_1 = 10$ and $C_1 = 1$. The source node sends $\mathcal{N}_1$ a signal of $S_1 = 6$, therefore, the initial bidding message from $\mathcal{N}_1$ is $b_1 = \min\{10, 6\} = 6$ and the resource allocation is $x_1 = 6$ (which is a Nash equilibrium point). Afterward, a new competing node $\mathcal{N}_2$ arrives with $w_2 = 1$ and $C_2 = 1$. The source node sends $\mathcal{N}_2$ a signal of $S_2 = 3$, therefore, the initial bidding message from $\mathcal{N}_2$ is $b_2 = \min\{1, 3\} = 1$. The final resource allocation is $\vec{x} = [5, 1]$ (which is also a Nash equilibrium point). Now a new competing node $\mathcal{N}_3$ arrives with $w_3 = 10$ and $C_3 = 1$. The source node sends $\mathcal{N}_3$ a signal of $S_3 = 2$, therefore, the initial bidding message from $\mathcal{N}_3$ is $b_3 = \min\{10, 2\} = 2$. The final resource allocation is $\vec{x} = [3, 1, 2]$. Note that this equilibrium point is *not* a Nash equilibrium since there is some degree of unfairness between the two homogeneous nodes $\mathcal{N}_1$ and $\mathcal{N}_3$, and $\mathcal{N}_3$ could have received a higher bandwidth if it increases its bidding. Another scenario which shows the final resource allocation is not a Nash equilibrium is that some of the competing nodes may suffer from the network congestion such that $x'_i < x_i$. When these nodes feedback their new biddings $b_i = x'_i$ for resource allocation, some resource at the source node will not be utilized and may remains idle. This condition continues even if these competing nodes are relieved from the network congestion at a later time. In other words, they cannot gain back the amount of resource they could have obtained in the Nash equilibrium. In summary, each competing node needs to behave more *aggressively* in order to get the proper amount of resource and also help the system to reach the new Nash equilibrium efficiently.

To properly resolve issues **I2** and **I3**, we propose the following extension protocol. Each competing node, say $\mathcal{N}_i$, enhances its bidding by sending

$$b_i \quad = \quad \min\{w_i, (1+\delta)s_i\} \tag{7}$$

where δ is a small positive constant for all competing nodes. The functionality of reporting a slightly larger bidding value is to explore the possibility of whether there is some idle resource at the source node. The Nash equilibrium result $\vec{x}^*$ in the theoretical model does not change except that the strategy profile is changed to be $\vec{b}^* = (1+\delta)\vec{x}^*$. In case there are idle resource or temporarily unfair resource allocation in the system, competing nodes which gain a smaller amount resource can increase their biddings and push the system to the new Nash equilibrium point. Hence, their subsequent bidding values will increase, eventually, a new equilibrium is made when each competing node bids $b_i = min\{w_i, (1+\delta)s_i\}$ and receives $x'_i = s_i$.

From now on, we assume all competing nodes in the incentive P2P network send the bidding message according to Eq. (7). Obviously, all competing nodes interact with the source node will achieve a different allocation result in equilibrium as compared with the Nash equilibrium in the theoretical context. We classify these competing nodes into three categories at the equilibrium points. When the bidding is $b_i = w_i$, then the competing node receives $x'_i = w_i$, and the allocated resource must be $x_i = w_i$. This implies the competing node does not encounter any network congestion. When the bidding is $b_i = (1+\delta)x'_i$, there are two cases to consider: (1) There is a bottleneck (with available bandwidth v_i) along the path of the competing node and the source node. Therefore, no matter how large the contribution value of the competing node or its bidding value, the competing node can only receive v_i amount of bandwidth resource. So we have $b_i = (1+\delta)x_i{}' = (1+\delta)v_i$. (2) The competing node competes with other competing nodes for the resource at the source node, therefore, the bottleneck is on the source node side. So we know $b_i = (1+\delta)x_i{}' = (1+\delta)x_i$. Suppose the above three categories of competing nodes in equilibrium are defined in the sets $\mathcal{N}_\alpha$, $\mathcal{N}_\beta$ and $\mathcal{N}_\gamma$ respectively, we have the following results:

Lemma *3: For any equilibrium of the dynamics game,*

$$x_i/C_i = x_j/C_j$$

for all $\mathcal{N}_i, \mathcal{N}_j \in \mathcal{N}_\gamma$.

Proof: For competing nodes $\mathcal{N}_i \in \mathcal{N}_\gamma$, the bottleneck is on the source node side. Following the equilibrium condition $b_i = (1+\delta)x_i$ and $x'_i = x_i$ for each competing node in $\mathcal{N}_\gamma$, the final "resource allocation level" in the RBM-IU mechanism should be $(x_i + b_i)/C_i = (2+\delta)x_i/C_i$ for all competing node in $\mathcal{N}_i \in \mathcal{N}_\gamma$. ■

Lemma *4: For any equilibrium of the dynamics game,*

$$x_i/C_i + \frac{1}{2}\delta x_i/C_i \geq x_j/C_j$$

for all $\mathcal{N}_i \in \mathcal{N}_\gamma$ *and* $\mathcal{N}_j \in \mathcal{N}_\alpha \cup \mathcal{N}_\beta$.

Proof: Suppose we have competing node $\mathcal{N}_i \in \mathcal{N}_\gamma$ and $\mathcal{N}_j \in \mathcal{N}_\alpha \cup \mathcal{N}_\beta$. For some competing node $\mathcal{N}_j$, the bottleneck is at the client side or intermediate links. The final resource allocation level in the RBM-IU mechanism, which is $(x_i + b_i)/C_i$, must be higher than or equal to the resource allocation level of any $\mathcal{N}_j$, which is $(x_j + b_j)/C_j$. Therefore, we have

$$(x_i + b_i)/C_i \geq (x_j + b_j)/C_j.$$

From the equilibrium condition (1) $b_j = x_j = x'_j = w_j$ for $\mathcal{N}_j \in \mathcal{N}_\alpha$, (2) $b_j = x_j = (1+\delta)x'_j = v_j$ for $\mathcal{N}_j \in \mathcal{N}_\beta$ and (3) $b_i = (1+\delta)x_i = (1+\delta)x'_i$ for $\mathcal{N}_i \in \mathcal{N}_\gamma$. Therefore, we have $(2+\delta)x_i/C_i \geq 2x_j/C_j$. ■

Theorem *8: The dynamic game equilibrium described above has the bandwidth allocation solution :*

$$x_i = \begin{cases} w_i & \text{if } x_i \in \mathcal{N}_\alpha \\ v_i & \text{if } x_i \in \mathcal{N}_\beta. \\ \dfrac{C_i}{\sum\limits_{j \in \mathcal{N}\gamma} C_j}\left((\mathcal{W}_s - \sum\limits_{j\in\mathcal{N}_\alpha} w_j - \sum\limits_{j\in\mathcal{N}_\beta} v_j)\right) & \text{if } x_i \in \mathcal{N}_\gamma. \end{cases} \tag{8}$$

In addition, it becomes a Nash equilibrium solution when δ *approaches zero.*

Proof: $x_i = w_i$ if $x_i \in \mathcal{N}_\alpha$ and $x_i = v_i$ if $x_i \in \mathcal{N}_\beta$ follow directly from the equilibrium condition. Since all competing nodes in $\mathcal{N}_\gamma$ are not saturated, they use up all the remaining resource $\mathcal{W}_s - \sum_{j\in\mathcal{N}_\alpha} w_j - \sum_{j\in\mathcal{N}_\beta} v_j$. Follow Lemma 3, the last equation holds.

When δ approaches zero, the strategy profile in equilibrium approaches:

$$b_i = \begin{cases} w_i & \text{if } x_i \in \mathcal{N}_\alpha \\ v_i & \text{if } x_i \in \mathcal{N}_\beta. \\ \dfrac{C_i}{\sum\limits_{j \in \mathcal{N}\gamma} C_j}(\mathcal{W}_s - \sum\limits_{j\in\mathcal{N}_\alpha} w_j - \sum\limits_{j\in\mathcal{N}_\beta} v_j) & \text{if } x_i \in \mathcal{N}_\gamma. \end{cases} \tag{9}$$

By Lemma 4, $x_i/C_i \geq x_j/C_j$ for all $\mathcal{N}_i \in \mathcal{N}_\gamma$ and $\mathcal{N}_j \in \mathcal{N}_\alpha \cup \mathcal{N}_\beta$. Physically, it implies all $\mathcal{N}_i \in \mathcal{N}_\gamma$ have the final resource "water level" higher than or equal to that of nodes in $\mathcal{N}_\alpha \cup \mathcal{N}_\beta$. This strategy profile with the solution is a Nash equilibrium: (1) For competing nodes $\mathcal{N}_j \in \mathcal{N}_\alpha \cup \mathcal{N}_\beta$, they gain the maximum resource so that no other better strategy for them to deviate. (2) For competing nodes $\mathcal{N}_i \in \mathcal{N}_\gamma$, they won't bid $b'_i < b_i$ since

x_i always less than or equal to b_i in RBM-IU. If they bid $b'_i > b_i$, consider the sub-game when $\mathcal{W}_s^{'} = \mathcal{W}_s - \sum_{j\in\mathcal{N}_\alpha} w_j - \sum_{j\in\mathcal{N}_\beta} v_j$ and the set of competing nodes is $\mathcal{N}_\gamma$. From Theorem 5, we know the strategy b_i is a Nash equilibrium which will not make one competing node better off after deviating from it. ∎

Remark: Although the equilibria in the dynamics game are not strictly Nash equilibria, they are close to Nash equilibria when δ is small. The allocation results from these equilibria are the same as the equilibrium allocation when $\delta = 0$. Therefore, we can regard the game reaching the Nash equilibria as if all player play the Nash's strategy profile.

V. Generalized Mechanism and Game

In last two sections, we discussed a specific RBM-IU mechanism and its corresponding resource competition game. In this section, we generalize the resource distribution mechanism with respect to incentive and utility. For incentive, we give a parametric manipulation of the contribution values C_i used, such that we can control the degree of incentive provided in the mechanism. For utility, we explore heterogeneous nodes which have diverse utility functions (i.e., not necessarily perceived by $U_i(x_i, b_i) = \log(x_i/b_i + 1)$ as assumed in Section III). We will analyze the properties of the competition games corresponding to the generalized mechanisms.

A. Generalized Mechanism with Incentive

Recall the mechanism RBM-I in Equation (3). We introduce incentive by distributing the resource linearly proportional to each competing node's contribution value C_i. In general, contribution values can be weighted by an exponent $r \geq 0$, and the resource distribution becomes:

$$x_{\hat{k}} = \min\left\{ b_{\hat{k}}, \frac{C_{\hat{k}}^r \left(\mathcal{W}_s - \sum_{\hat{i}=1}^{\hat{k}-1} x_{\hat{i}}\right)}{\sum_{j=\hat{k}}^N C_j^r} \right\} \quad \hat{k} = 1, \ldots, N. \tag{10}$$

It is easy to observe that when $r = 0$, this mechanism is equivalent to the mechanism RBM-I. On the other hand, when r tends to infinity, this mechanism becomes a strict priority service mechanism which serves the requests by their contribution values in descending order. Generally speaking, the larger value of r, the higher amount of the allocated resource this mechanism provides based on contribution values. Therefore, the parameter r provides some degree of freedom for the mechanism designer to balance the incentive and the fairness for the P2P system.

Similarly, by generalizing C_i to C_i^r, RBM-IU becomes:

$$\max \sum_{i=1}^N C_i^r \log\left(\frac{x_i}{b_i} + 1\right) \quad \text{s.t.} \sum_{i=1}^N x_i \leq \mathcal{W}_s, x_i \in [0, b_i] \ \forall\ i.$$

Because the new mechanism linearly weights each contribution by C_i^r, the implementation of the new mechanism can be easily extended by changing the original filling algorithm. In extending both the RBM-I and the RBM-IU, we make the filling rate of each competing node to be C_i^r instead of C_i.

Lastly, this extension maintains the properties of all previous theorems and the corresponding resource competition game. Some generalized version of theorems are as follows:

Theorem *9: For any two competing nodes $\mathcal{N}_i, \mathcal{N}_j$, the generalized RBM-IU assigns the bandwidth x_i and x_j such that:*

$$\text{if } \frac{C_i^r}{b_i} \geq \frac{C_j^r}{b_j} \Longrightarrow U_i(x_i) \geq U_j(x_j). \tag{11}$$

Proof: The proof is similar to that of Theorem 2. ∎

Theorem *10: The strategy profile $b_i^* = \frac{\mathcal{W}_s C_i^r}{\sum_{j=1}^N C_j^r}$ for player $\mathcal{N}_i$, where $i = 1, \ldots, N$, is the unique Nash equilibrium.*

Proof: The proof is similar to that of Theorem 6. ∎

B. Generalized Mechanism with Utility

In RBM-IU, we assumed a special form of the perceived utility function: $U_i(x_i) = \log(\frac{x_i}{b_i} + 1)$. Although this form can be reasonable in practice, we would like to consider the more general situation in which the competing nodes have heterogeneous utility functions. We will first design a mechanism for the general situation, and then discuss the properties of the corresponding competition game (e.g., existence and characterization of any equilibrium solution).

Similar to RBM-U, our new mechanism tries to maximize the social utility. As in the context of resource bidding, the perceived utility function of node i should depend on the bidding b_i. Again, we use "utility" to indicate the "perceived utility" throughout this section. We design the new mechanism to solve the following distributed optimization problem:

$$\max \sum_{i=1}^N U_i(x_i, b_i) \qquad \text{s.t.} \sum_{i=1}^N x_i \leq \mathcal{W}_s \ \text{ and } \ x_i \in [0, b_i] \ \forall\ i.$$

The utility function of a node, say i, is denoted by $U_i(\cdot)$ and depends on x_i and b_i. Let us consider $U_i = x_i/b_i$. We can regard x_i/b_i (with range $[0,1]$) as node i's "fraction of satisfaction".

Theorem *11: There exists at least one Nash equilibrium in the competition game induced by the following mechanism:*

$$\max \sum_{i=1}^N U_i(\frac{x_i}{b_i}) \qquad \textit{s.t.} \sum_{i=1}^N x_i \leq \mathcal{W}_s \ \textit{ and } \ x_i \in [0, b_i] \ \forall\ i.$$

where $U_i(\cdot)$ is any concave function for all i.

Proof: Consider any strategy profile $\vec{b} = \{b_i, \vec{b}_{-i}\}$, where $\vec{b}_{-i}$ is any fixed strategy profile (of bidding messages) of all the players other than $\mathcal{N}_i$. The resource allocation $\mathcal{N}_i$ is a function of i's bidding value, denoted by $x_i(b_i)$. By the constraint $x_i \leq b_i$, when $x_i(\hat{b}_i) = \hat{b}_i$ for some $\hat{b}_i$, x_i may increase for node i only if i bids $b_i > \hat{b}_i$. Again, the mechanism solves the optimization problem by giving a resource increment to the competing node which currently has the largest marginal utility. Also, the marginal utility $dU_i/dx_i = U_i^{'}(\frac{x_i}{b_i})/b_i$ is decreasing when we increase b_i. Hence, we need to consider three cases of the function $x_i(b_i)$: (1) The marginal utility of player i when $x_i = 0$ for any $b_i > 0$ (i.e. $U_i^{'}(0)/b_i$) is less than that of any other player when the resource is used up. We have: $x_i = 0 \forall b_i > 0$. (2) The marginal utility when $x_i = W_s$ for any $b_i > 0$ (i.e. $U_i^{'}(W_s/b_i)/b_i$) is always the largest among all the players. We have: $x_i = min\{b_i, \mathcal{W}_s\} \forall b_i > 0$. (3) We increase b_i gradually from zero to infinity. x_i increases from zero when its marginal utility is among the largest and $x_i(b_i) = b_i$. After that, when we continue to increase b_i, the marginal utility decreases and x_i decreases until $x_i = 0$. When we have $x_i(\hat{b_i}) = 0$ for some $\hat{b_i} > 0$, we have $x_i(b_i) = 0 \ \forall \ b_i \geq \hat{b_i}$. From the single peak property of $x_i(b_i)$, we know that the upper-level contour set is convex. Therefore, the function is quasi-concave in b_i. By similar arguments used in Theorem 4, we know that there exists at least one Nash Equilibrium in the competition game. ■

Theorem *12: Suppose $\vec{b^*}$ and $\vec{x^*}$ is a Nash equilibrium in Theorem 11. For any $x_i^*, x_j^* > 0$, we have:*

$$U_i'(\frac{x_i^*}{b_i^*}) : U_j'(\frac{x_j^*}{b_j^*}) = b_i^* : b_j^*$$

Proof: Assume $U_i'(\frac{x_i^*}{b_i^*}) : U_j'(\frac{x_j^*}{b_j^*}) > b_i^* : b_j^*$ for some $x_i^*, x_j^* > 0$. Therefore, $U_i'(\frac{x_i^*}{b_i^*})/b_i^* > U_j'(\frac{x_j^*}{b_j^*})/b_j^*$ which implies the marginal utility of player i is higher than that of player j. Therefore, the resource distribution mechanism can increase the aggregated utility by shifting some resource from player j to player i. So the mechanism does not solve the maximization on the aggregated utility. We have the contradiction here. ■

Remarks: The implication of this theorem is that the bidding of each player in equilibrium should be proportional to their marginal utility (or shadow price) at that equilibrium point.

VI. Convergence Analysis

In this section, we investigate the convergence of the practical competition game described in Section IV. We show that by using a small positive value of δ, the solutions of the practical game converges to a neighborhood of the Nash equilibrium in the theoretical competition game. Without loss of generality, we assume all competing nodes have the same contribution values. We also focus on the non-trivial case when the aggregated demand is larger than the resource bandwidth, i.e. $\sum_{i=1}^{N} b_i > \mathcal{W}_s$.

We start from a simple example shown in Figure 4. There

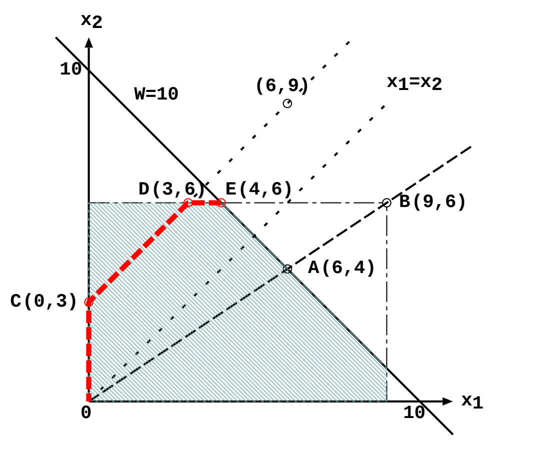


Fig. 4. Convergence illustration where $N = 2$, $\mathcal{W}_s = 10$ and $\delta = 0.5$.

are two competing nodes and a source node with bandwidth resource $\mathcal{W}_s$ = 10Mb/s. Suppose at some moment the resource allocation is at point A $(6, 4)$. Each competing nodes sends a bidding value $b_i = (1 + \delta)x_i$, where $\delta = 0.5$. We have the new bidding at point B $(9, 6)$. The shaded area in the figure is the feasible region for the new allocation, which physically implies that (1) each competing node's allocation will be non-negative and no larger than its bidding value, and (2) the aggregate allocation will not exceed the total bandwidth resource $\mathcal{W}_s$.

The mechanism progressively moves the resource allocation from the origin to point C $(0, 3)$, giving the bandwidth resource to the competing nodes offering smaller bidding values. After that, the mechanism shares the bandwidth resource evenly until the second competing node reaches its bidding value at point D $(3, 6)$. The mechanism then completes the allocation process at point E $(4, 6)$. Therefore, the allocation result oscillates between points A and E near the equilibrium point $(5, 5)$. We can imagine that the smaller the value of δ, the shorter the corresponding convergence diameter.

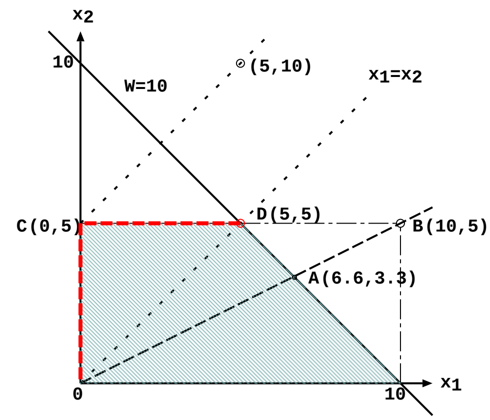


Fig. 5. Convergence illustration where $N = 2$, $\mathcal{W}_s = 10$ and $\delta = 0.5$.

Also, notice that by choosing an initial condition according to the value δ, we can achieve the Nash equilibrium solution. In Figure 5, the initial allocation is at point A $(6, 6, 3.3)$ and the following bidding is at point B $(10, 5)$. This bidding pair leads the two players to reach the Nash equilibrium solution D $(5, 5)$.

Theorem *13: For two players with bidding $b_i = (1 + \delta)x_i$, the allocation solution converges to the neighborhood of equilibrium point $(\frac{\mathcal{W}_s}{2}, \frac{\mathcal{W}_s}{2})$, where $|x_i - \frac{\mathcal{W}_s}{2}| \leq \frac{\delta}{2}\frac{\mathcal{W}_s}{2}$ for $i = 1, 2$.*

Proof: We define:

$$e(t) = |x_1(t) - \frac{\mathcal{W}_s}{2}| = |x_2(t) - \frac{\mathcal{W}_s}{2}| \quad \texttt{for} \quad t = 0, 1, 2 \ldots$$

$$\text{and } \limsup e \equiv lim_{t\to\infty} \sup e(t) \equiv \inf_{t\geq 0}\{\sup\{e(k)|k \geq t\}\}.$$

Without loss of generality, we assume $x_1 > x_2$ at some time t. Hence, we have $(x_1(t), x_2(t)) = (\frac{\mathcal{W}_s}{2} + e(t), \frac{\mathcal{W}_s}{2} - e(t))$ and $(b_1(t), b_2(t)) = ((1 + \delta)x_1(t)), (1 + \delta)x_2(t))$. Consequently, we gain the resource allocation at time $t + 1$ as follows:

1. If $e(t) \leq \frac{\delta}{2(1+\delta)}\frac{\mathcal{W}_s}{2}$, then $x_1(t + 1) = \frac{\mathcal{W}_s}{2} - (1 + \delta)e(t)$ and $x_2(t + 1) = \frac{\mathcal{W}_s}{2} + (1 + \delta)e(t)$.

2. If $e(t) > \frac{\delta}{2(1+\delta)}\frac{\mathcal{W}_s}{2}$, then $x_1(t+1) = (1-\delta)\frac{\mathcal{W}_s}{2} + (1+\delta)e(t)$ and $x_2(t + 1) = (1 + \delta)\frac{\mathcal{W}_s}{2} - (1 + \delta)e(t)$.

In the first case, $e(t+1) = (1+\delta)e(t) > e(t)$. In the second case, when $e(t) \in (\frac{\delta}{2(1+\delta)}\frac{\mathcal{W}_s}{2}, \frac{\delta}{2+\delta}\frac{\mathcal{W}_s}{2})$, we have $e(t+1) > e(t)$; otherwise, $e(t+1) \leq e(t)$.

Therefore, $e(t+1) > e(t)$ only if $e(t) < \frac{\delta}{2+\delta}\frac{\mathcal{W}_s}{2}$ for all t. It is easy to show that we also have

$$\sup\{e(t+1)|e(t) < \frac{\delta}{2+\delta}\frac{\mathcal{W}_s}{2}\} = \frac{\delta}{2}\frac{\mathcal{W}_s}{2} \quad \blacksquare$$

One can extend this theorem to more than two competing nodes.

VII. Numerical Examples

In this section, we present numerical results to illustrate the performance and the incentive property of our resource distribution protocol. In particular, we show that our protocol can properly adapt to dynamic join/leave of competing nodes, and to various conditions of network congestion.

Example A (Incentive Resource Distribution): We consider a source node $\mathcal{N}_s$ with resource $\mathcal{W}_s = 2$ Mb/s. There are four competing nodes $\mathcal{N}_1$ to $\mathcal{N}_4$. Their maximum download bandwidths are $\vec{w} = [2, 1.5, 1, 0.5]$ (in Mb/s). The arrival times of $\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3$ and $\mathcal{N}_4$ are t =20, 40, 60 and 80 s, respectively. Unless stated otherwise, the propagation delay between a competing node and the source node $\mathcal{N}_s$ is one second and all competing nodes use $\delta = 0.1$ in Equation 7. We consider three scenarios, each using different contribution values for the four competing nodes. In **A.1**, we have $\vec{C} = [100, 100, 100, 100]$; in **A.2**, we have $\vec{C} = [400, 300, 200, 100]$; in **A.3**, we have $\vec{C} = [400, 100, 200, 300]$. Figure 6 illustrates the *instantaneous* bandwidth allocation for all the competing nodes for $t \in [0, 100]$. One can make the following observations:

- Figure 6(a) shows that when all nodes have the same contribution value, they will eventually get a *fair share* (i.e., even distribution) of the bandwidth resource. For example, for $t \in [20, 40]$, $\mathcal{N}_1$ gets all of $\mathcal{W}_s$'s resource of 2 Mb/s since it is the only competing node and $w_1 = 2$Mb/s.

For $t \in [40, 60]$, the resource is evenly shared by $\mathcal{N}_1$ and $\mathcal{N}_2$ since they have the same contribution values. When all the four competing nodes are present ($t \in [80, 100]$), each node will get a resource amount $x = 0.5$ Mb/s.

- Figure 6(b) shows that the bandwidth assignment is *proportional* to the contribution value of a competing node. When all four competing nodes are present ($t \in [80, 100]$), the resource allocation vector is $\vec{x} = [0.8, 0.6, 0.4, 0.2]$ (Mb/s). Hence, RBM-IU provides service differentiation, such that nodes have incentive to share information and to provide services.
- Figure 6(c) shows that the protocol will not waste any resource at the source node. Given $\vec{C} = [400, 100, 200, 300]$, the resource distribution should be $\vec{x} = [0.8, 0.2, 0.4, 0.6]$ (Mb/s). But since the maximum download bandwidth of $\mathcal{N}_4$ is $w_4 = 0.5$ Mb/s only, the remaining resource (0.1 Mb/s) will be distributed *proportionally* to $\mathcal{N}_1$, $\mathcal{N}_2$ and $\mathcal{N}_3$. The final resource distribution is $\vec{x} = [0.86, 0.21, 0.43, 0.5]$ (Mb/s).

In summary, these examples show that the RBM-IU can provide incentive service differentiation and will efficiently utilize resources at the source node.

Example B (Adaptivity to dynamic join/leave of competing nodes): We consider a source node $\mathcal{N}_s$ with resource $\mathcal{W}_s = 2$ Mb/s. There are four competing nodes $\mathcal{N}_1$ to $\mathcal{N}_4$ with contributions $\vec{C} = [400, 300, 200, 100]$ and maximum download bandwidths $\vec{w} = [2, 1.5, 1, 0.5]$ (in Mb/s). There is a propagation delay of one second between a competing node and the source node.

We consider two scenarios of arrival and departure patterns: **B.1:** $\mathcal{N}_1$ arrives and departs at $t = 40$ and $t = 160$, $\mathcal{N}_2$ arrives and departs at $t = 60$ and $t = 100$, $\mathcal{N}_3$ arrives and departs at $t = 80$ and $t = 120$, and $\mathcal{N}_4$ arrives and departs at $t = 20$ and $t = 140$. **B.2:** $\mathcal{N}_1$ arrives and departs at $t = 20$ and $t = 100$, $\mathcal{N}_2$ arrives and departs at $t = 80$ and $t = 120$, $\mathcal{N}_3$ arrives and departs at $t = 60$ and $t = 140$, and $\mathcal{N}_4$ arrives and departs at $t = 40$ and $t = 160$. Figure 7 illustrates the instantaneous bandwidth allocation for time $t \in [0, 180]$. One can make the following observations:

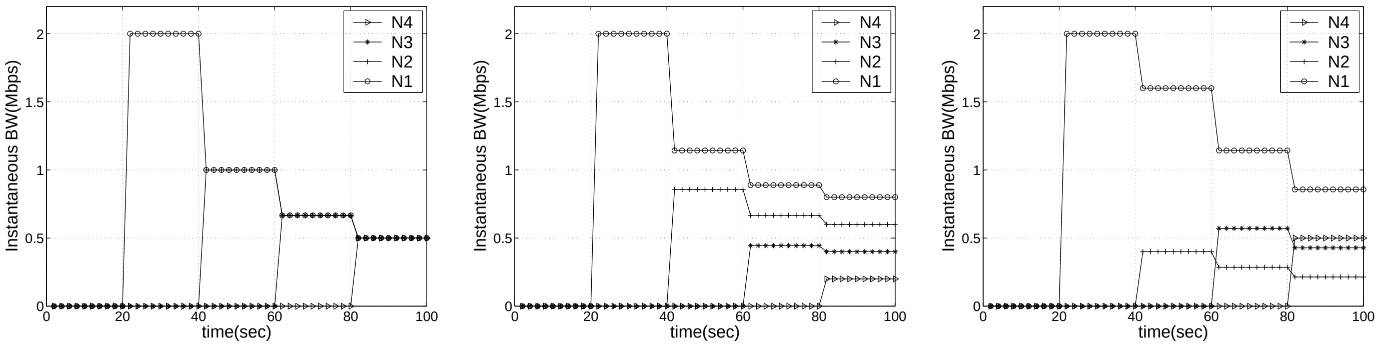
- The protocol can assign the proper amount of resource to competing nodes without wastage. For example, for time $t \in [20, 40]$, Figure 7(a) shows that $\mathcal{N}_4$ obtains 0.5 Mb/s (since this is its maximum download bandwidth). But for the same time period, Figure 7(b) shows that $\mathcal{N}_1$ can get 2.0 Mb/s, its maximum download bandwidth and the full resource of the source node.
- Both Figures 7(a) and (b) show that the protocol can fully utilize the source resources. For example, for period $t \in [40, 120]$, the source node distributes the resource proportionally to the contribution values of the competing nodes. The assignment is independent of the number of competing nodes and their arrival patterns.
- The protocol can reach the same equilibrium point, *independent* of the arrival and departure sequences of **B.1** or **B.2**. For example, consider the time period $t \in [80, 100]$. The resource distribution for both cases is $\vec{x} = [0.8, 0.6, 0.4, 0.2]$ (in Mb/s), which is also the Nash equilibrium point.

In summary, these examples show that the protocol is adaptive to the arrival and departure sequence, and it provides service differentiation to different competing nodes having different contribution values.

Example C (Adaptivity to network congestion): We consider one source node $\mathcal{N}_s$ with resource $\mathcal{W}_s = 2$ Mb/s. At time $t = 0$, there are four competing nodes $\mathcal{N}_1$ to $\mathcal{N}_4$ in the system. These nodes have contribution values $\vec{C} = [400, 300, 200, 100]$ and maximum download bandwidths of $\vec{w} = [2, 1.5, 1, 0.5]$ (in Mb/s). There is a propagation delay of one second from each competing node to the source node. In this example, we consider the dynamic congestion situation. In particular, the congestion occurs along the communication path between $\mathcal{N}_1$ and the source node $\mathcal{N}_s$. Congestion occurs twice, at time $t = [30, 40]$ and $t = [50, 60]$. During the congestion, the available bandwidth along the communication path is reduced to 400 kb/s.

Figure 8 illustrates the instantaneous bandwidth allocation of all four competing nodes for time $t \in [0, 100]$. One can make the following observations:

- At time $t = 0$, the system starts at Nash equilibrium with resource allocation of $\vec{x} = [0.8, 0.6, 0.4, 0.2]$ (in Mb/s).
- Between time $t \in [30, 40]$ (or $t = [50, 60]$), since there is



(**a**) (**b**) (**c**)

Fig. 6. Instantaneous bandwidth allocations: (a) $\vec{C} = [100, 100, 100, 100]$; (b) $\vec{C} = [400, 300, 200, 100]$; (c) $\vec{C} = [400, 100, 200, 300]$.

network congestion, the competing node $\mathcal{N}_1$ receives less transfer bandwidth from the source node. Other competing nodes $\mathcal{N}_2$ to $\mathcal{N}_4$ can discover this idle bandwidth resource of 0.4 Mb/s via their bidding messages. The source node $\mathcal{N}_s$ will distribute this excessive bandwidth resource to the other three competing nodes proportionally to their contribution values. New Nash equilibria are reached ($t \in [35-40]$ and $t \in [55-60]$).

- When the congestion disappears, the competing node $\mathcal{N}_1$ can gain back its proper resource amount of $x_1 = 0.8$ Mb/s. Also, the new Nash equilibrium can be quickly reached and the final resource allocation is $\vec{x} = [0.8, 0.6, 0.4, 0.2]$ Mb/s.

In summary, this example shows that the protocol is adaptive to network congestion. During network congestion, the resource at the source node will not be wasted but rather distributed proportionally to other competing nodes.

Example D (Relationship between the step size δ and the equilibrium allocation): We consider one source node $\mathcal{N}_s$ with resource $\mathcal{W}_s = 2$ Mb/s. At time $t = 0$, there are four competing nodes $\mathcal{N}_1$ to $\mathcal{N}_4$ in the system. In particular, node $\mathcal{N}_1$ leaves the system at time 30. These nodes have contribution values $\vec{C} = [400, 300, 200, 100]$ and maximum download bandwidths of $\vec{w} = [2, 1.5, 1, 0.5]$ (in Mb/s). There is a propagation delay of one second from each competing node to the source node.

We consider four scenarios, each using different step size values δ for the four competing nodes. In **D.1**, we have $\vec{\delta} = [0.01, 0.01, 0.01, 0.01]$; in **D.2**, we have $\vec{\delta} = [0.1, 0.1, 0.1, 0.1]$; in **D.3**, we have $\vec{\delta} = [0.01, 0.01, 0.1, 0.01]$; in **D.4**, we have $\vec{\delta} = [0.01, 0.1, 0.01, 0.1]$. Figure 9 illustrates the *instantaneous* bandwidth allocation for all the competing nodes for $t \in [0, 120]$. One can make the following observations:

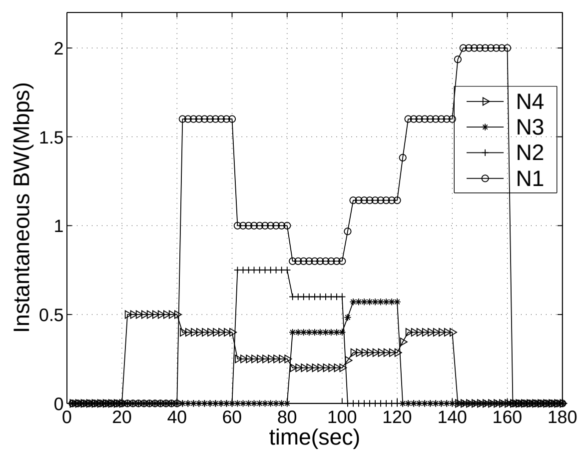
- In Figure 9(a) and (b), all competing nodes have the same value of δ (0.01 and 0.1 respectively). They show the same equilibrium allocation for $\mathcal{N}_2$,$\mathcal{N}_3$ and $\mathcal{N}_4$ after $\mathcal{N}_1$ leaves the system. The difference in these two scenarios is that from the time $\mathcal{N}_1$ leaves the system, it takes around 50 seconds to reach the new equilibrium in (a) but 5 seconds in (b). It is intuitive that larger δ value makes faster convergence to the new equilibrium point.
- Figure 9(c) shows the scenario when all competing nodes have δ value 0.01 except node $\mathcal{N}_3$ has $\delta_3 = 0.1$. From $t = 30$, node $\mathcal{N}_3$ increases its resource much faster than $\mathcal{N}_2$ and $\mathcal{N}_4$, and even gain more resource than it gains in equilibrium from time 35 to 80. On the other hand, by comparing Figure 9(c) with (a), we find both the original equilibrium and the new equilibrium are different. In both equilibria, node $\mathcal{N}_3$ gains less resource in (c) than in (a). Theoretically, when all competing nodes have the same δ value, the actual equilibrium coincides with the theoretical Nash equilibrium. But larger δ value makes less resource in actual equilibrium.
- Figure 9(d) shows the other way around when $\mathcal{N}_3$ has less δ value that other competing nodes. In this case, node SN_3 reaches its new equilibrium slower, but gains more resource than that of the theoretical Nash equilibrium.

In summary, these examples show that δ affects both the equilibrium solution and the convergence rate for reaching new equilibria. In general, the larger value of δ, the faster convergence to the new equilibrium with less resource gain at the equilibrium.

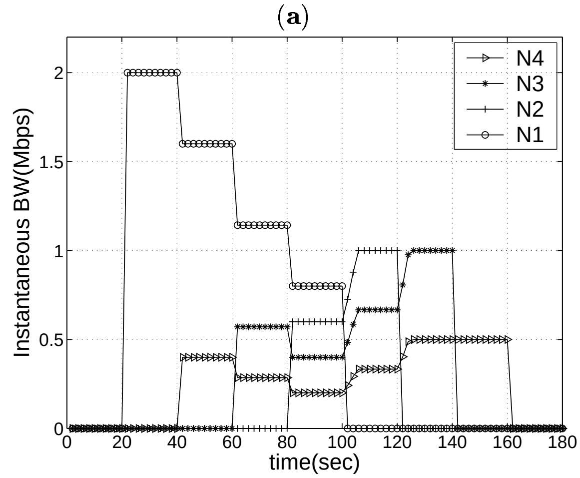
VIII. Conclusion

In this paper, we have presented a framework for building incentive P2P networks. The framework consists of the resource allocation mechanism RBM-IU and an interaction protocol for competing nodes to reach equilibria of the competition game induced by RBM-IU. Our solution is efficient: (1) RBM-IU can be implemented by a linear time algorithm, (2) the feedback bidding messages used by the competing nodes are simple, and (3) RBM-IU achieves Pareto-optimality allocation results. The robustness of the solution is evidenced by the fact that all the competing nodes can reach the equilibrium solutions of the competition game. The justification for the source node to use our protocol is its guarantee of the Pareto optimality. On the other hand, competing nodes are motivated to use the protocol because it guarantees Nash equilibrium. We show that the protocol can be extended to heterogeneous nodes with different utility functions. Convergence analysis is carried out to show the existence of the Nash equilibrium. Lastly, we also show that the protocol is adaptive to various nodes arrival and departure events, as well as in different forms of network congestion.

Acknowledgment: We like to thank the anonymous referees for their insightful and helpful comments.



$(\mathbf{a})$



$(\mathbf{b})$

Fig. 7. Instantaneous bandwidth allocations for arrival and departure patterns (a) B.1; (b) B.2.

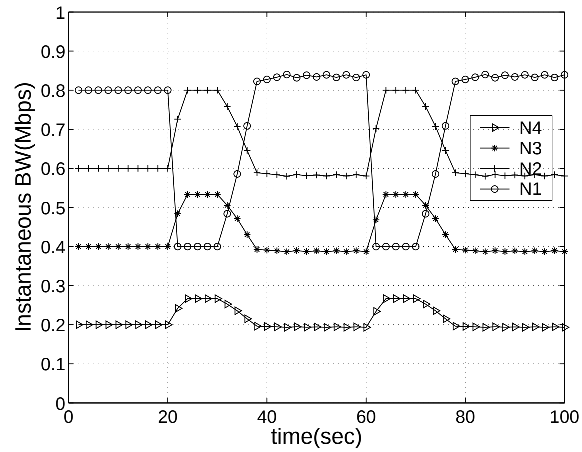
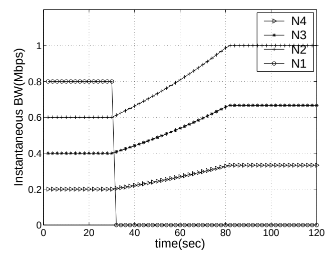
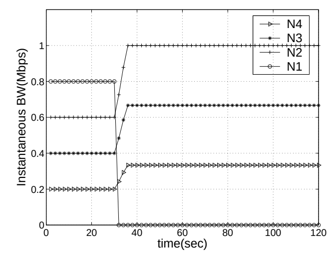


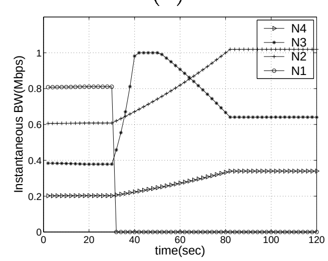
Fig. 8. Instantaneous bandwidth allocations for four competing nodes; congestion occurs at $t = [30, 40]$ and $t = [50, 60]$.



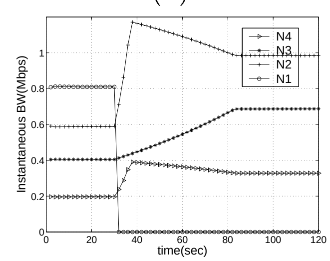
$(\mathbf{a})$



$(\mathbf{b})$



$(\mathbf{c})$



$(\mathbf{d})$

Fig. 9. Instantaneous bandwidth allocations: (a) $\vec{\delta} = [0.01, 0.01, 0.01, 0.01]$; (b) $\vec{\delta} = [0.1, 0.1, 0.1, 0.1]$; (c) $\vec{\delta} = [0.01, 0.01, 0.1, 0.01]$; (d) $\vec{\delta} = [0.01, 0.1, 0.01, 0.1]$.

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