CENG4480 Homework 2

Due: Nov. 11, 2019

Q1 (10%) The circuit shown in Figure 1 represents a simple n-bit digital-to-analog converter (DAC). Each switch is controlled by the corresponding bit of the digital number if the bit is 1 the switch is up; if the bit is 0 the switch is down. Assume the unsigned decimal number 12_{10} is inputted to a four-bit DAC based on the aforementioned architecture. Given that $R_F = R_0/16$, $R_i = R_0/2^i$, logic 0 corresponds to 0 V, and logic 1 corresponds to 4.8 V, please answer the following questions:

(1) What is the output of the DAC?

(2) What is the maximum voltage that can be outputted from the DAC?

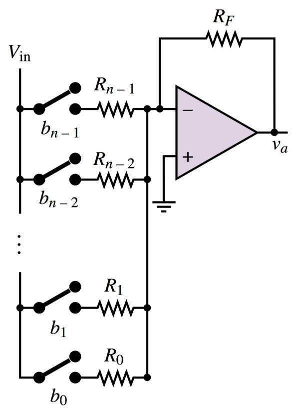

Figure 1: n-bit DAC.

Q2 (15%) Assume the full-scale value of the analog input voltage to a particular analog-to-digital converter (ADC) is 10 V. Please answer the following three questions:

(1) If this is a 3-bit device, what is the resolution (i.e. smallest step size) of the output?
(2) Given the resolution Δv and the full-scale value of an ADC v_{range}, try to give a general formulation to calculate the number of bits n.
(3) How many comparators are needed in a 5-bit flash ADC? Try to give some pros and cons on flash ADC.

Q3 (10%) A simple Infra-Red Sensor system to detect passing human is presented as in Figure 2. A and B are IR Sensors which will generate different output voltages for different infra-red intensity, and higher voltage level corresponds to high light intensity.

(1) Explain how this system works for counting passing pedestrians.

(2) To increase counting accuracy, usually B is covered with materials that can reflect infra-red light. Explain why.

Q4 (10%) Please answer the following questions:

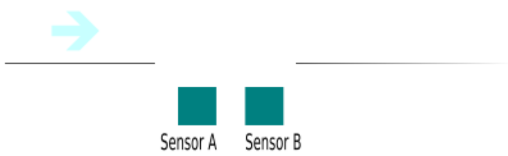

Figure 2: IR-System.

(1) Exemplify the working principles of sensors that measure: (a) Flow; (b) Force; (c) Temperature.

(2) Elaborate motion sensors you know.

Q-5 (15%) Please try to give the discrete incremental PID formulations. Some notations are given:

- $u(t)$ is the output of a controller in the tth measurement interval.

- $e(t)$ is the error between the target value and measurement value in the tth measurement interval. And the error is measured every T time interval (T is small enough).

- The PID parameters, K_p, K_i and K_d, are all set.
(Hint: incremental means $\Delta u(t) = u(t) - u(t-1)$.)

Q6 (10%)

Assume two normal random variables X_1, X_2, and $X_1 \sim N(\mu_1, \sigma_1)$, $X_2 \sim N(\mu_2, \sigma_2)$. A new random variable X_3 is the weighted sum of these two normal random variables (i.e. $aX_1 + bX_2$). Please answer the following two questions:

(1) If X_1 andX_2 are independent, try to deduce the expectation and variance of X_3.

(2) If X_1 andX_2 are not independent and the covariance $Cov(X_1, X_2) = \sigma_{12}$, try to deduce the expectation and variance of X_3.

Q7 (10%) Given a linear system

$$\begin{cases} \boldsymbol{x}_t = \boldsymbol{A}_{t-1}\boldsymbol{x}_{t-1} + \boldsymbol{\omega}_{t-1}, \\ \boldsymbol{z}_t = \boldsymbol{B}_t\boldsymbol{x}_t + \boldsymbol{v}_t, \\ \boldsymbol{v}_t = \boldsymbol{C}_{t-1}\boldsymbol{v}_{t-1} + \boldsymbol{n}_{t-1}, \end{cases} \tag{1}$$

where $\boldsymbol{\omega}_t$ and $\boldsymbol{n}_t$ are independent and obey Gaussian distribution zero-mean and co-variance $\boldsymbol{Q}_t$ and $\boldsymbol{R}_t$, respectively. Please give the estimate equation and measurement equation of the system.

Q8 (20%) Assume the liner estimate system equation is $\mathbf{x}_{t+1} = \mathbf{A}\mathbf{x}_t + \mathbf{w}_t$. Given a second-autoregression random series:

$$x(t) = \rho_1 x(t-1) + \rho_2 x(t-2) + \omega_t \tag{2}$$

Kalman Filter is used to estimate $x(t)$ (Here $x(t)$ is a scalar, $var(\omega_t)$ is the variance of the noise). Please answer the following two questions:

(1) Try to give the expressions of state transition matrix $\mathbf{A}$ and noise vector $\mathbf{w}_t$. (Hint: the state α_t in time t equals to $\left[\begin{array}{c} x_t \\ x_{t-1} \end{array}\right]$)

(2) Suppose $\hat{\alpha}_t$ denotes the prediction of the t state based on the information of $t-1$ state, and P_t is the variance of prediction error , which equals to $\mathbb{E}\left[(\alpha_t - \hat{\alpha}_t)(\alpha_t - \hat{\alpha}_t)^\top\right]$. Try to derive the mathematical relationship between P_t and P_{t-1}.