



Ph.D. Thesis
Defense

Web Mining Techniques for Query Log Analysis and Expertise Retrieval

Hongbo Deng

Department of Computer Science and Engineering
The Chinese University of Hong Kong

Date: Sep 2, 2009

Rich Web Data

香港中文大學
The Chinese University of Hong Kong

Web pages
One trillion
unique URLs

Yahoo! My Yahoo! Mail More
Make Y! My Homepage Hi, Hongbo Sign Out Help
YAHOO! ANSWERS Search WEB SEARCH

Question answer
Yahoo! answer

Web mining
techniques

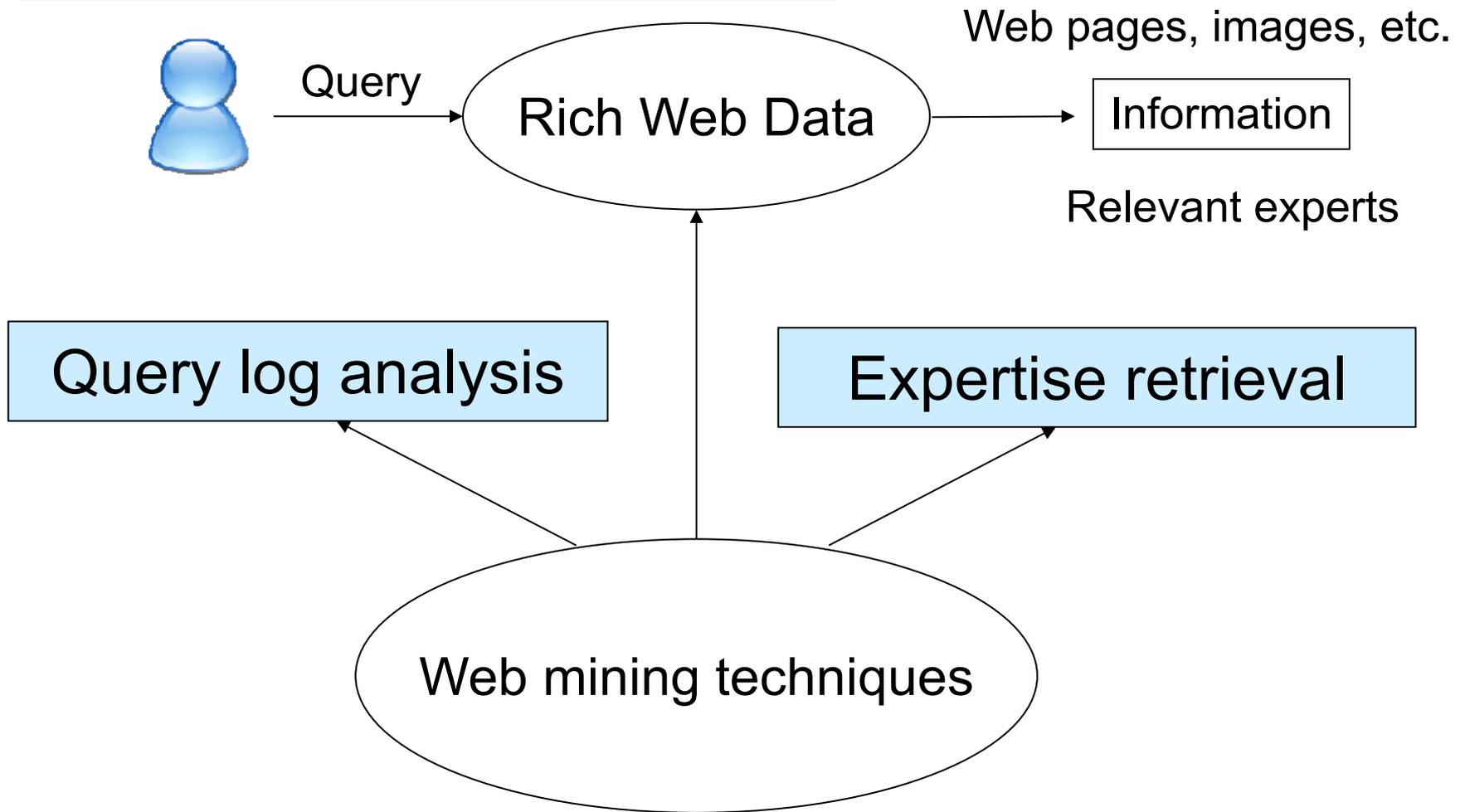
Christos Faloutsos
List of Ask o
370
269
268
267
266
265
264
263
262
261
260
DBLP data

QueryTime	ItemRank	ClickURL
100218	evaluating internal controls a local government managers guide	2006-03-07 21:51:18
100217		
100216		
100215		
100214		
100213		
100212		
100211		
100210		
100209		
100208		
100207		
100206		
100205		
100204		
100203		
100202		
100201		
100200		
100199		
100198		
100197		
100196		
100195		
100194		
100193		
100192		
100191		
100190		
100189		
100188		
100187		
100186		
100185		
100184		
100183		
100182		
100181		
100180		
100179		
100178		
100177		
100176		
100175		
100174		
100173		
100172		
100171		
100170		
100169		
100168		
100167		
100166		
100165		
100164		
100163		
100162		
100161		
100160		
100159		
100158		
100157		
100156		
100155		
100154		
100153		
100152		
100151		
100150		
100149		
100148		
100147		
100146		
100145		
100144		
100143		
100142		
100141		
100140		
100139		
100138		
100137		
100136		
100135		
100134		
100133		
100132		
100131		
100130		
100129		
100128		
100127		
100126		
100125		
100124		
100123		
100122		
100121		
100120		
100119		
100118		
100117		
100116		
100115		
100114		
100113		
100112		
100111		
100110		
100109		
100108		
100107		
100106		
100105		
100104		
100103		
100102		
100101		
100100		
100099		
100098		
100097		
100096		
100095		
100094		
100093		
100092		
100091		
100090		
100089		
100088		
100087		
100086		
100085		
100084		
100083		
100082		
100081		
100080		
100079		
100078		
100077		
100076		
100075		
100074		
100073		
100072		
100071		
100070		
100069		
100068		
100067		
100066		
100065		
100064		
100063		
100062		
100061		
100060		
100059		
100058		
100057		
100056		
100055		
100054		
100053		
100052		
100051		
100050		
100049		
100048		
100047		
100046		
100045		
100044		
100043		
100042		
100041		
100040		
100039		
100038		
100037		
100036		
100035		
100034		
100033		
100032		
100031		
100030		
100029		
100028		
100027		
100026		
100025		
100024		
100023		
100022		
100021		
100020		
100019		
100018		
100017		
100016		
100015		
100014		
100013		
100012		
100011		
100010		
100009		
100008		
100007		
100006		
100005		
100004		
100003		
100002		
100001		
100000		

Query logs
DBLP query log

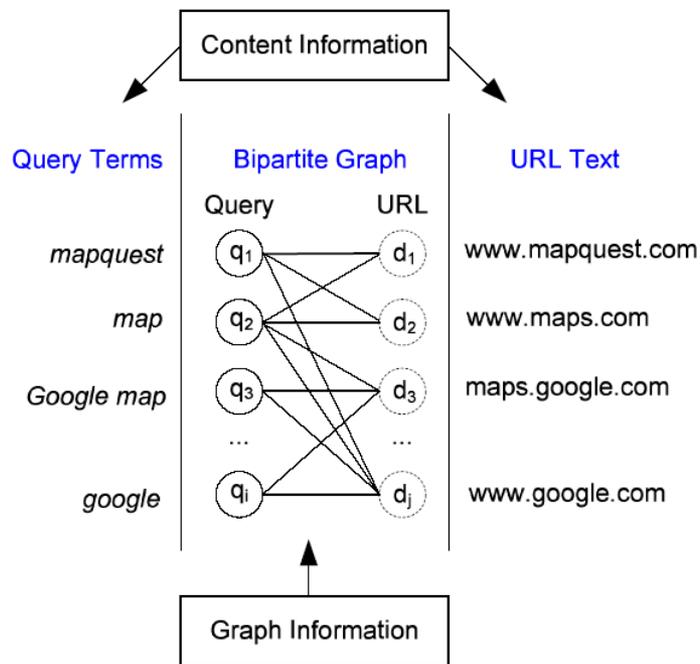


Overview

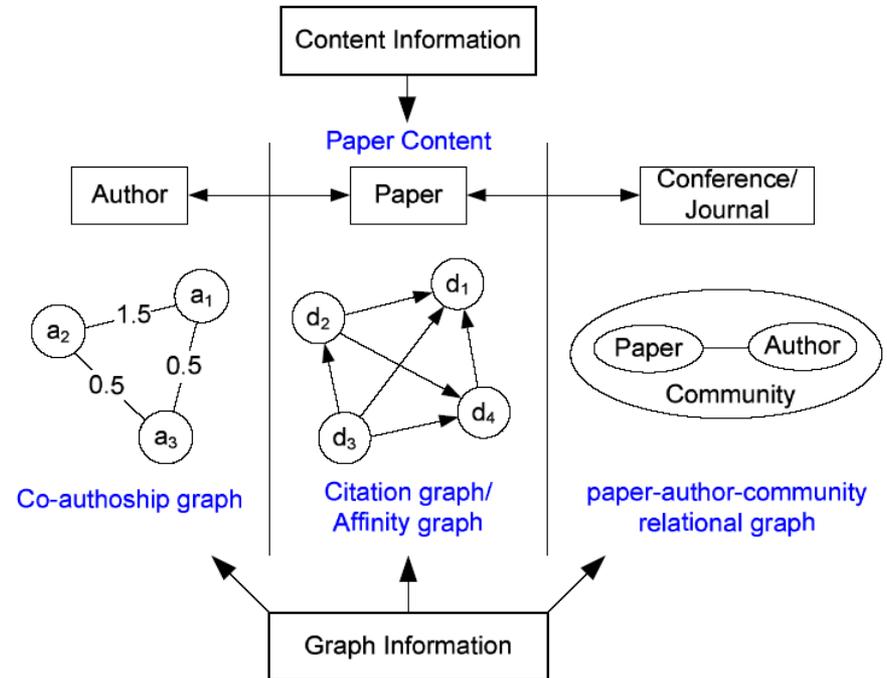


Objective

- Combine the content and the graph information
- Leverage IR, link analysis, ML in a unified way



The query log data



The expertise retrieval data



Outline

- Background: Web Mining Techniques
 - Information retrieval, link analysis, machine learning
- Modeling Bipartite Graph for Query Log Analysis
 - Entropy-biased Models [w/ King-Lyu, SIGIR'09]
 - Co-HITS Algorithm [w/ Lyu-King, KDD'09]
- Modeling Expertise Retrieval
 - Baseline and Weighted Model [w/ King-Lyu, ICDM'08]
 - Graph-based Re-ranking Model [w/ Lyu-King, WSDM'09]
 - Enhanced Models with Community [w/ King-Lyu, CIKM'09]
- Conclusion and Future Work



Background

- Information retrieval models
 - Vector space model
 - Probabilistic model
 - Language model
- Web link analysis
 - PageRank: a link represents a vote
 - HITS: good hubs points to good authorities
 - Other variations
- Machine learning
 - Semi-supervised learning
 - Graph-based regularization framework



Modeling Bipartite Graph for Query Log Analysis

- Many Web data can be modeled as **bipartite graphs**

Query log data:

Query-URL
bipartite graph

Netflix data:

User-Movie
bipartite graph

DBLP data:

Author-Paper
bipartite graph

How to weigh the
edges of the graph?

How to combine the
graph with other info.?

It is very essential to model bipartite graphs for mining these data types.



Query Log Analysis

- Query log analysis – improve search engine’s capabilities
 - Query suggestion
 - Query classification
 - Targeted advertising
 - Ranking

The screenshot shows a Yahoo! search interface with the query 'sunnyvale ca'. The search bar includes navigation links (Web, Images, Video, Local, Shopping, more), a search button, and options for 'Options' and 'Customize'. The search results are displayed in a dark-themed box with a red border. The results are organized into three columns:

- Left Column (Suggestions):** city of sunnyvale ca, 701 first ave sunnyvale ca, starbucks sunnyvale ca, sunnyvale ca zip code, sunnyvale ca map.
- Middle Column (Explore related concepts):** Sunnyvale, California, Sunnyvale Real Estate, City of Sunnyvale, population.
- Right Column (Related Concepts):** SAN JOSE, CA, Santa Clara County, San Jose, Fremont High School.

Below the search results, there are sponsored results. One sponsored result is highlighted with a red box:

- Sunnyvale Apartment Rentals:** Looking for a Great Location with Large Floor Plans? Contact Us Today. www.AvalonCommunities.com/sunnyvale

Other search results include:

- Sunnyvale, CA Homes:** Search 1000's of Beautiful Homes. Photos, Prices, Tours, Maps & More. SiliconValleyHomeSearcher.com
- Home - City of Sunnyvale, California:** www.ci.sunnyvale.ca.us - Cached
- Sunnyvale Public Library Home - City of Sunnyvale, California:** www.ci.sunnyvale.ca.us/library - Cached
- Sunnyvale, California - Wikipedia, the free encyclopedia:** Geography | Climate | History | Demographics. Sunnyvale is a city in Santa Clara County, California, United States. It is one of the major cities that make up the Silicon Valley. As of the 2000 census, the city population was 131,760. en.wikipedia.org/wiki/Sunnyvale,_CA - 204k - Cached

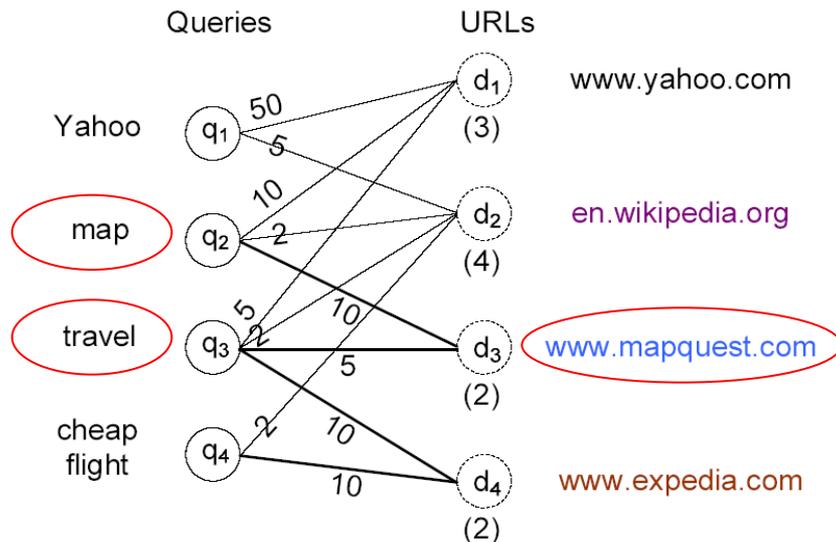
The page also shows a search status bar: '1 - 10 of 70,100,000 for sunnyvale ca (About) - 0.15 s | SearchScan BETA On'.



Click Graph

- Click graph – an important technique
 - A bipartite graph between queries and URLs
 - Edges connect a query with the URLs
 - Capture some semantic relations, e.g., “map” and “travel”

How to utilize and model the click graph to represent queries?



Traditional model based on the *raw click frequency (CF)*

Propose two kinds of models

- Entropy-biased framework
- Co-HITS algorithm



Outline

- Part I: Entropy-biased Framework for Modeling Click Graphs
 - Motivation and Preliminaries
 - Traditional Click Frequency Model
 - Entropy-biased Model
 - Experimental Results
 - Summary



Motivation

Is a single click on different URLs equally important?



- Basic idea
 - Various query-URL pairs should be treated differently
- Intuition
 - Common clicks on **less frequent but more specific** URLs are of **greater value** than common clicks on **frequent and general** URLs



Preliminaries

Query instance: $\langle q, d, u \rangle$

Query: $Q = \{q_1, q_2, \dots, q_M\}$

URL: $D = \{d_1, d_2, \dots, d_N\}$

User: $U = \{u_1, u_2, \dots, u_K\}$

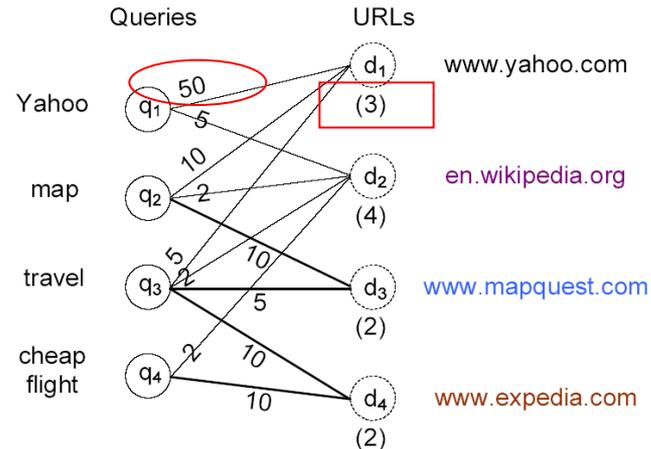


Table 1: Table of Notation.

Symbol	Meaning
C	$M \times N$ query-URL matrix
c_{ij}	Click frequency between query q_i and URL d_j , with the entry (i, j) of the matrix C
$u_{f_{ij}}$	User frequency between q_i and d_j
$n(d_j)$	Number of queries associated with URL d_j
$idf(d_j)$	Importance of a certain URL d_j
$p(d_j q_i)$	Transition probability from q_i to d_j
$p(q_i d_j)$	Transition probability from d_j to q_i
P_{q2d}	An $M \times N$ query-URL probability matrix
P_{d2q}	An $N \times M$ URL-query probability matrix

Click frequency matrix

C	d_1	d_2	d_3	d_4
q_1	50	5	0	0
q_2	10	2	10	0
q_3	5	2	5	10
q_4	0	2	0	10



Traditional Click Frequency Model

- Transition probability: Normalize the *click frequency* (CF)

From query to URL:

$$p(d_j|q_i) = \frac{c_{ij}}{cf(q_i)}$$

$$P_{q2d}: \vec{q}_i = \langle P_{q2d}(i, 1), \dots, P_{q2d}(i, N) \rangle$$

From URL to query:

$$p(q_i|d_j) = \frac{c_{ij}}{cf(d_j)}$$

$$P_{d2q}: \vec{d}_j = \langle P_{d2q}(j, 1), \dots, P_{d2q}(j, M) \rangle$$

Click frequency matrix

C	d_1	d_2	d_3	d_4
q_1	50	5	0	0
q_2	10	2	10	0
q_3	5	2	5	10
q_4	0	2	0	10



CF transition probabilities

P_{q2d}	d_1	d_2	d_3	d_4
q_1	0.909	0.091	0	0
q_2	0.455	0.091	0.455	0
q_3	0.227	0.091	0.227	0.455
q_4	0	0.167	0	0.833

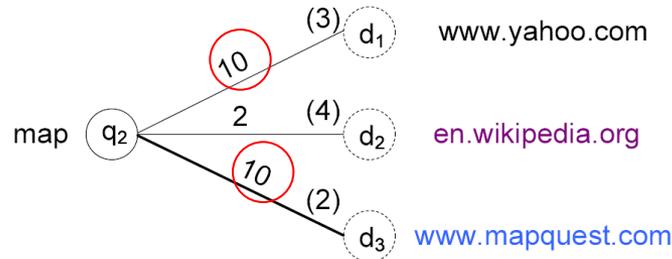
- Measure the similarity between queries

- The most similar query: q_2 (“map”) → q_1 (“Yahoo”)
- More reasonable: q_2 (“map”) → q_3 (“travel”)

Entropy-biased
model



Entropy-biased Model



It would be more reasonable to weigh these two edges differently

□ The more general and highly ranked URL

- Connect with more queries
- Increase the ambiguity and uncertainty

□ The entropy of a URL:

$$E(d_j) = - \sum_{i \in Q} p(q_i | d_j) \log p(q_i | d_j)$$

Transition probability from a URL to a query

■ Suppose

$$p(q_i | d_j) = \frac{1}{n(d_j)} \longrightarrow \text{The number of queries that connected with } d_j$$

Query frequency

■ Tend to be proportional to the $n(d_j)$

$$E(d_j) = \log n(d_j)$$



Entropy → Discriminative Ability

□ Entropy increase, discriminative ability decrease

- Be inversely proportional to each other
- A URL with a high query frequency is less discriminative overall

□ Inverse query frequency

- Measure the discriminative ability of the URL

$$iqf(d_j) = \frac{\log |Q|}{\text{Constant}} - \frac{\log n(d_j)}{\text{Entropy}} = \log \frac{|Q|}{n(d_j)}$$

■ Benefits

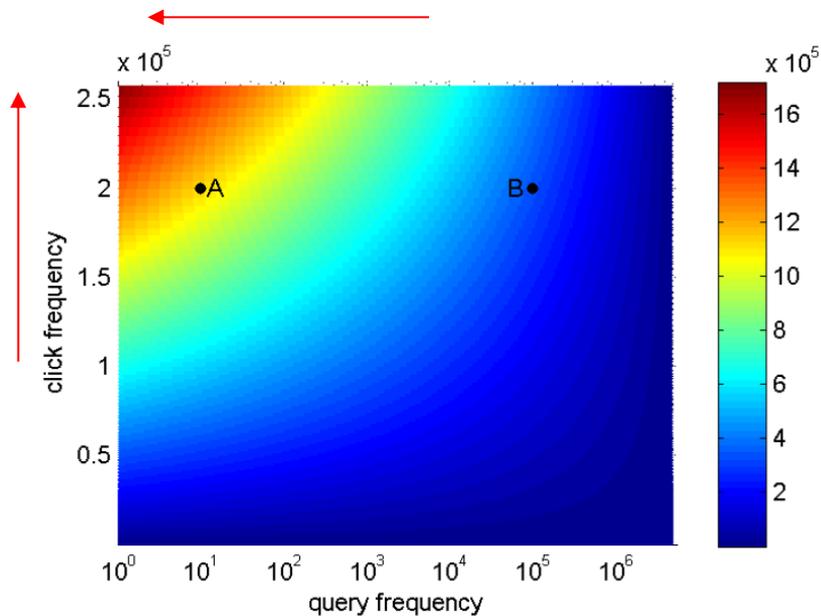
- Reduce the influence of some heavily-clicked URLs
- Balance the bias of clicks for those highly ranked URLs
- Incorporate with other factors to tune the model



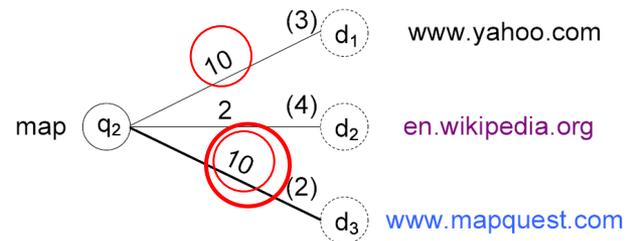
CF-IQF Model

- Incorporate the IQF with the click frequency

$$cfiqf(q_i, d_j) = c_{ij} \cdot iqf(d_j)$$



- A high click frequency
- A low query frequency
- “A” is weighted higher than “B”



The surface specified by the click frequency, query frequency and cfqf, with color specified by the cfqf value. The color is proportional to the surface height.



CF-IQF Model

□ Transition probability

$$p'_c(d_j|q_i) = \frac{cfiqf(q_i, d_j)}{cfiqf(q_i)}$$

Click frequency matrix

C	d_1	d_2	d_3	d_4
q_1	50	5	0	0
q_2	10	2	10	0
q_3	5	2	5	10
q_4	0	2	0	10



CF-IQF transition probabilities

P'_{q_2d}	d_1	d_2	d_3	d_4
q_1	1	0	0	0
q_2	0.293	0	0.707	0
q_3	0.122	0	0.293	0.586
q_4	0	0	0	1

The most similar query
 q_2 (“map”) \rightarrow q_1 (“Yahoo”)

The most similar query
 q_2 (“map”) \rightarrow q_3 (“travel”)



UF Model and UF-IQF Model

- Drawback of CF model
 - Prone to spam by some malicious clicks (if a single user clicked on a certain URL thousands of times)
- UF model
 - Utilize **user frequency** instead of **click frequency**
 - Improve the resistance against malicious clicks
- UF-IQF model

$$ufiqf(q_i, d_j) = uf_{ij} \cdot iqf(d_j)$$

$$p'_u(d_j|q_i) = \frac{ufiqf(q_i, d_j)}{ufiqf(q_i)}$$



Connection with TF-IDF

□ TF-IDF

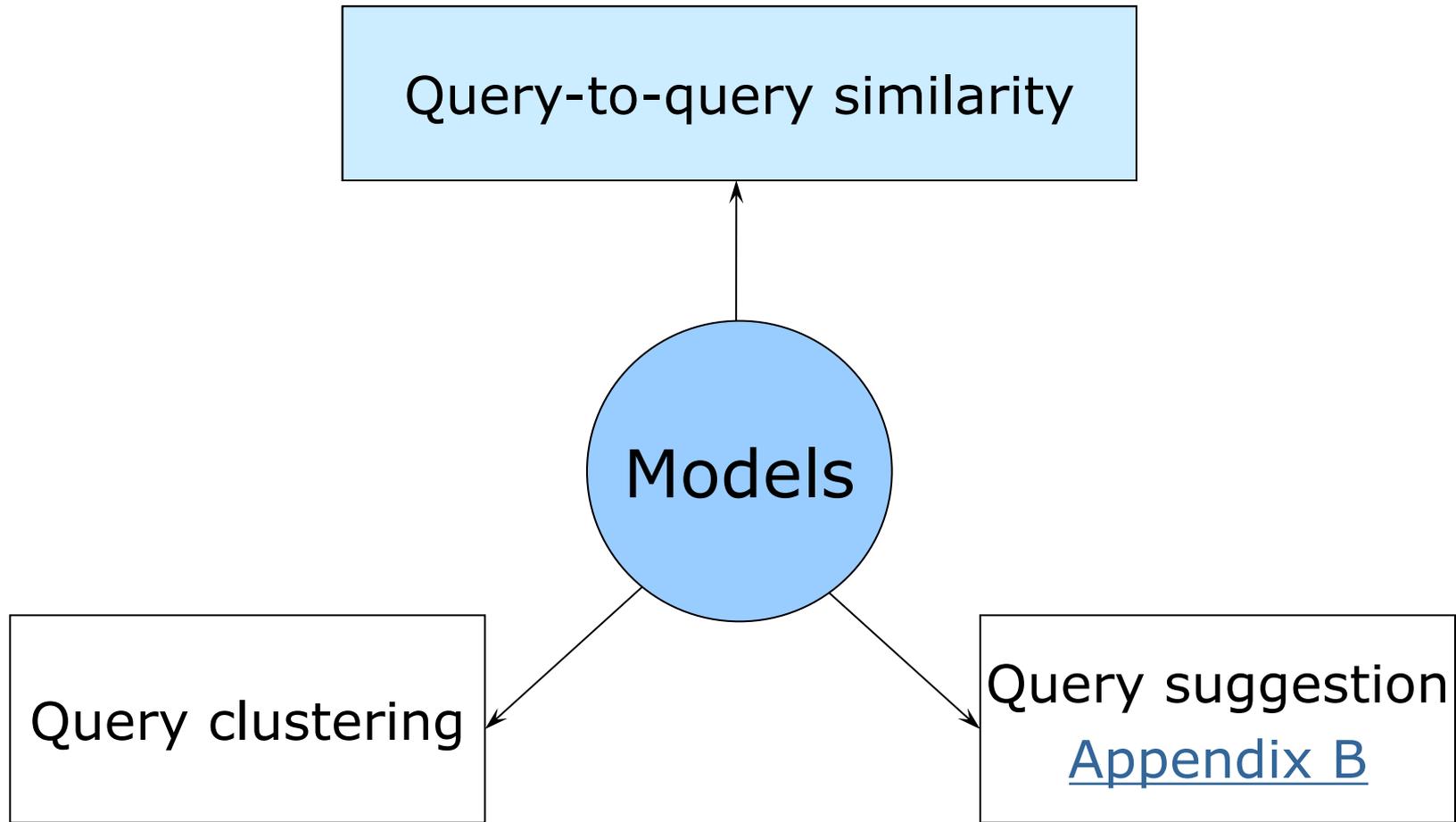
- Successfully used in vector space model for text retrieval
- Try to interpret IDF based on binary independence retrieval (BIR), information entropy and LM
- TF-IDF has never been explored to bipartite graphs

□ Entropy-biased framework (CF-IQF)

- IQF is new
- CF-IQF is a simplified version of entropy-biased model
- Share the key point to tune the importance of an edge
- The scheme can be applied to other bipartite graphs



Mining Query Log on Click Graph



Similarity Measurement

□ Cosine similarity

$$\text{Cos}(\theta) = \frac{\vec{q}_i \cdot \vec{q}_j}{\|\vec{q}_i\| \|\vec{q}_j\|}$$

□ Jaccard coefficient

$$J(\vec{q}_i, \vec{q}_j) = \frac{\sum_{n \in N} |P_{q2d}(i, n) \cap P_{q2d}(j, n)|}{\sum_{n \in N} |P_{q2d}(i, n) \cup P_{q2d}(j, n)|}$$

where $P_{q2d}(i, n)$ denotes the n -th value of \vec{q}_i

□ The similarity results are reported and analyzed



Experimental Evaluation

□ Data collection

■ AOL query log data

Table 2: Samples of the AOL query log dataset.

UserID	Query	Time	Rank	ClickURL
2722	yahoo	2006-04-25 13:03:23	1	http://www.yahoo.com
121537	map	2006-05-25 18:28:58	1	http://www.mapquest.com
123557	travel	2006-03-13 01:09:53	2	http://www.expedia.com
1903540	cheap flight	2006-05-15 00:31:43	1	http://www.cheapflights.com

□ Cleaning the data

- Removing the queries that appear less than 2 times
- Combining the near-duplicated queries
- 883,913 queries and 967,174 URLs
- 4,900,387 edges



Evaluation: ODP Similarity

- A simple measure of similarity among queries using ODP categories (query → category)

- Definition: $Sim(Ca_i, Ca_r) = \frac{|P(Ca_i, Ca_r)|}{\max(|Ca_i|, |Ca_r|)}$

- Example: 3/5

- Q1: “United States” → “Regional > North America > United States”

- Q2: “National Parks” → “Regional > North America > United States > Travel and Tourism > National Parks and Monuments”

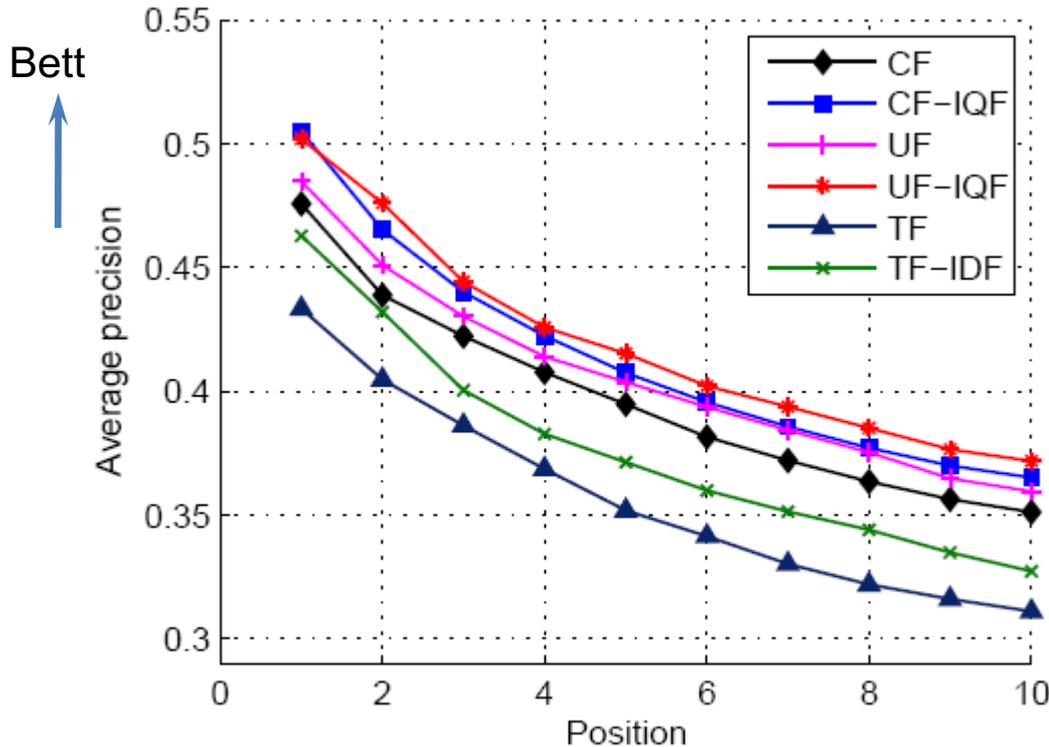
- Precision at rank n (P@n): $P@n = \frac{\sum_{i=1}^n Sim(q_i, q_r)}{n}$

- 300 distinct queries



Experimental Results

□ Query similarity analysis



(a) Cosine similarity

Results:

1. CF-IQF is better than CF
UF-IQF > UF
2. UF is better than CF
UF-IQF > CF-IQF
3. TF-IDF is better than TF

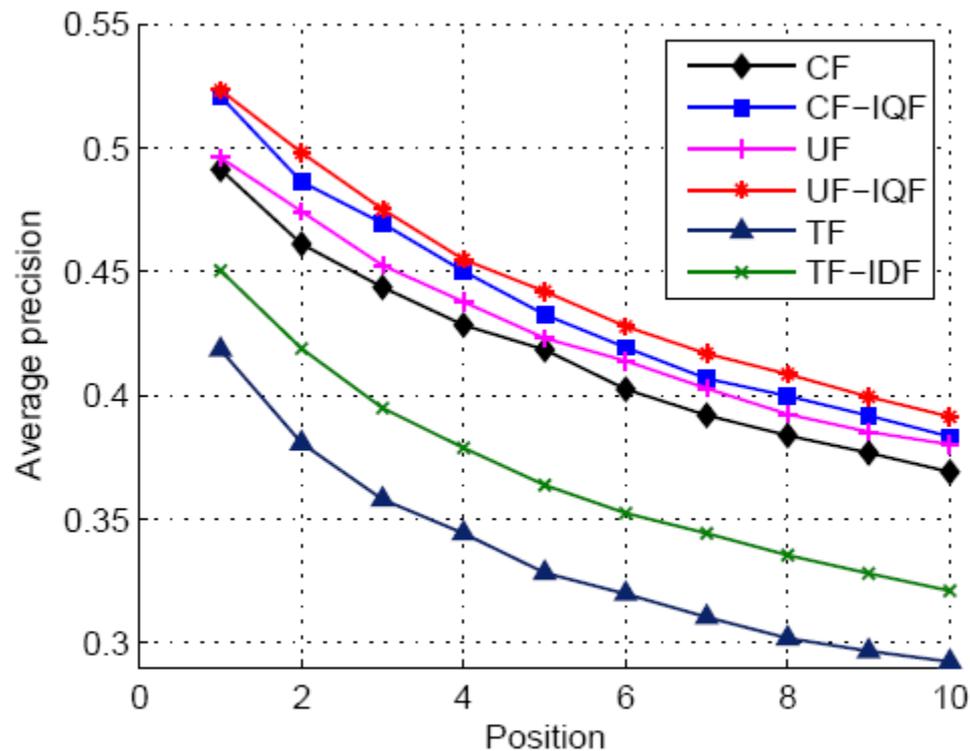


Experimental Results

□ Query similarity analysis

4. Jaccard coefficient

The improvements are consistent with the Cosine similarity



(b) Jaccard coefficient



Experimental Results

□ Query similarity analysis

Table 4: Comparison of different methods by P@1 and P@10. We also show the percentage of relative improvement in the lower part.

Method	Cosine		Jaccard	
	P@1	P@10	P@1	P@10
CF	0.476	0.351	0.491	0.369
CF-IQF	0.505	0.365	0.521	0.383
UF	0.485	0.360	0.500	0.380
UF-IQF	0.502	0.372	0.523	0.391
TF	0.433	0.311	0.418	0.292
TF-IDF	0.463	0.327	0.450	0.321
CF-IQF/CF	6.12%	3.96%	6.01%	3.84%
UF-IQF/UF	3.52%	3.38%	5.50%	2.92%
UF-IQF/CF	5.49%	5.86%	6.51%	6.01%
TF-IDF/TF	6.78%	5.21%	7.63%	9.79%
CF/TF	9.76%	12.91%	17.41%	26.23%
UF/TF	11.85%	15.61%	18.53%	30.02%
CF-IQF/TF-IDF	9.09%	11.57%	15.65%	19.39%
UF-IQF/TF-IDF	8.44%	13.61%	16.19%	21.89%

5. UF-IQF achieves best performance in most cases.

6. CF and UF models > TF
CF-IQF, UF-IQF > TF-IDF

The click graph catches more semantic relations between queries than the query terms



Summary of Part I

- Introduce the **inverse query frequency** (IQF) to measure the discriminative ability of a URL
- Propose the framework of the **entropy-biased model** for the click graph
 - IQF + CF, IQF + UF
 - Formal model to distinguish the variation on different query-URL pairs in the click graph.
- Experimental results show the **improvements** of the proposed models are consistent and promising



Outline

- Part II: Co-HITS Algorithm
 - Motivation
 - Co-HITS Algorithm
 - Iterative Framework
 - Regularization Framework
 - Experimental Results
 - Summary



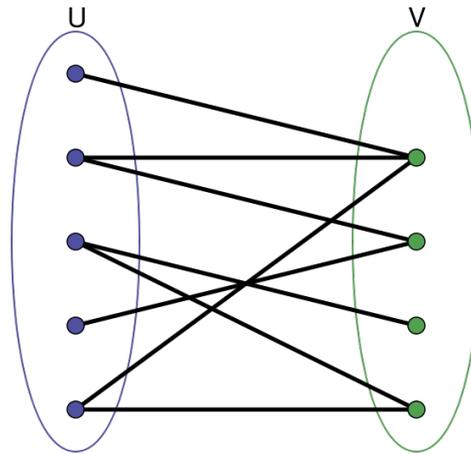
Motivation

Two kinds of information

IR Models for **Content**

- VSM
- Language Model
- etc.

Relevance



Link Analysis for **Graph**

- HITS
- PageRank
- etc.

Semantic relations

Incorporate **Content** with **Graph**

- Personalized PageRank (PPR)
- Linear Combination
- etc.

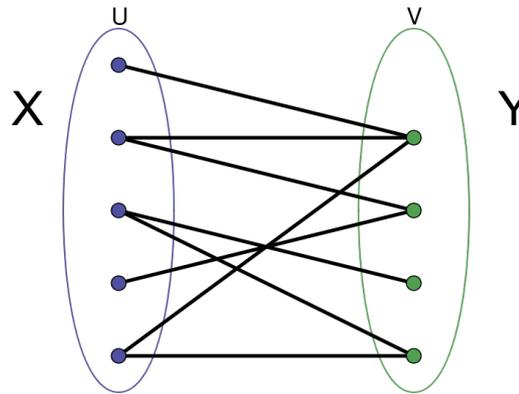


Preliminaries

Content

$$x_i^0 = f(q, u_i)$$

$$y_j^0 = f(q, v_j)$$



Graph

Explicit links:

$$W^{uv}$$

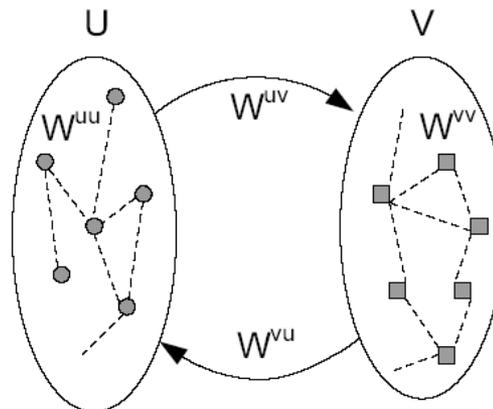
$$W^{vu}$$

Hidden links:

$$W^{uu}$$

$$W^{vv}$$

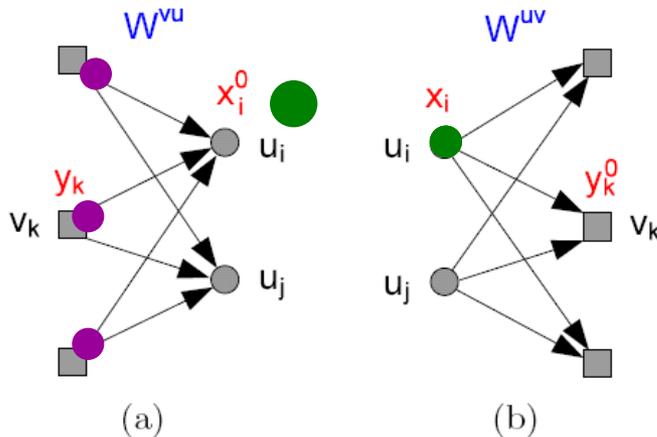
$$w_{ij}^{uu} = \sum_{k \in V} w_{ik}^{uv} w_{kj}^{vu}$$



Generalized Co-HITS

□ Basic idea

- Incorporate the bipartite graph with the content information from both sides
- Initialize the vertices with the relevance scores x^0, y^0
- Propagate the scores (mutual reinforcement)



$$\begin{aligned}
 x_i &= (1 - \lambda_u)x_i^0 + \lambda_u \sum_{k \in V} w_{ki}^{vu} y_k, \\
 y_k &= (1 - \lambda_v)y_k^0 + \lambda_v \sum_{j \in U} w_{jk}^{uv} x_j,
 \end{aligned}$$

Initial scores

Score propagation



Generalized Co-HITS

□ Iterative framework

$$\begin{aligned}
 x_i &= (1 - \lambda_u)x_i^0 + \lambda_u(1 - \lambda_v) \sum_{k \in V} w_{ki}^{vu} y_k^0 + \lambda_u \lambda_v \sum_{j \in U} \left(\sum_{k \in V} w_{jk}^{uv} w_{ki}^{vu} \right) x_j, \\
 &= (1 - \lambda_u)x_i^0 + \lambda_u(1 - \lambda_v) \sum_{k \in V} w_{ki}^{vu} y_k^0 + \lambda_u \lambda_v \sum_{j \in U} w_{ji}^{uu} x_j.
 \end{aligned}$$

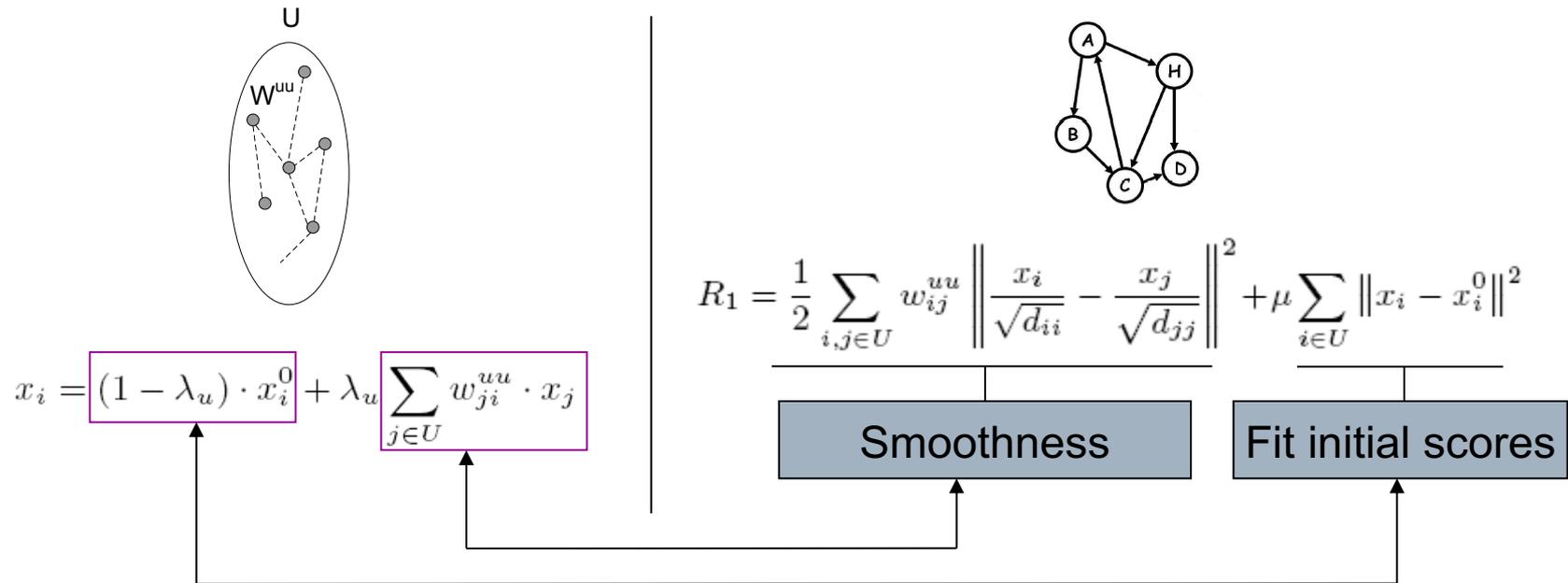
Iterative Framework		
λ_u	λ_v	Description
$= 0$	$\in [0, 1]$	Initial scores $x_i = x_i^0$
$= 1$	$= 1$	<u>Original HITS</u>
$\in (0, 1)$	$= 1$	<u>Personalized PageRank</u>
$\in (0, 1)$	$= 0$	<u>One-step propagation</u>
$\in (0, 1)$	$\in (0, 1)$	<u>General Co-HITS</u>

$$\begin{aligned}
 x_i &= \sum_{j \in U} w_{ji}^{uu} x_j \\
 x_i &= (1 - \lambda_u) \cdot x_i^0 + \lambda_u \sum_{j \in U} w_{ji}^{uu} \cdot x_j \\
 x_i &= (1 - \lambda_u) \cdot x_i^0 + \lambda_u \sum_{k \in V} w_{ki}^{vu} \cdot y_k^0
 \end{aligned}$$



Iterative \rightarrow Regularization Framework

- Consider the vertices on one side

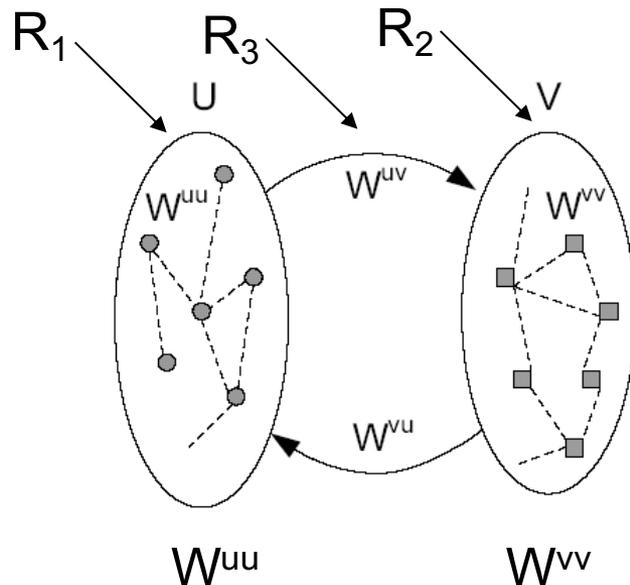


Personalized PR		Regularization Model	
$\lambda_u = 0$	Initial scores $x_i = x_i^0$	$\mu \rightarrow +\infty$	Initial scores $x_i = x_i^0$
$\lambda_u = 1$	Original HITS	$\mu = 0$	Only consider graph
$\lambda_u \in (0, 1)$	Personalized PageRank	$\mu \in (0, +\infty)$	Combine content and graph



Generalized Co-HITS

□ Regularization framework



Intuition: the highly connected vertices are most likely to have similar relevance scores.

$$R_1 = \frac{1}{2} \sum_{i,j \in U} w_{ij}^{uu} \left\| \frac{x_i}{\sqrt{d_{ii}}} - \frac{x_j}{\sqrt{d_{jj}}} \right\|^2 + \mu \sum_{i \in U} \|x_i - x_i^0\|^2$$

$$R_2 = \frac{1}{2} \sum_{i,j \in V} w_{ij}^{vv} \left\| \frac{y_i}{\sqrt{d_{ii}}} - \frac{y_j}{\sqrt{d_{jj}}} \right\|^2 + \mu \sum_{i \in V} \|y_i - y_i^0\|^2$$

$$R_3 = \frac{1}{2} \sum_{i \in U, j \in V} w_{ij}^{uv} \left\| \frac{x_i}{\sqrt{d_{ii}}} - \frac{y_j}{\sqrt{d_{jj}}} \right\|^2 + \frac{1}{2} \sum_{j \in V, i \in U} w_{ji}^{vu} \left\| \frac{y_j}{\sqrt{d_{jj}}} - \frac{x_i}{\sqrt{d_{ii}}} \right\|^2$$

$$R = \lambda_r (R_1 + \alpha R_2) + (1 - \lambda_r) R_3$$

$$\alpha = 1$$



Generalized Co-HITS

□ Regularization framework

The cost function:

$$R = \lambda_r(R_1 + R_2) + (1 - \lambda_r)R_3$$

Solution:

$$\begin{aligned} F^* &= \mu\beta(I - \mu\alpha S)^{-1}F^0, \\ S &= D^{-\frac{1}{2}}WD^{-\frac{1}{2}} \\ \mu\alpha &= \frac{1}{1 + \mu}, \text{ and } \mu\beta = \frac{\mu}{1 + \mu}, \end{aligned}$$

Optimization problem:

$$\begin{aligned} \min_F \quad & \frac{1}{2} \sum_{i,j=1}^{m+n} w_{ij} \left\| \frac{f_i}{\sqrt{d_{ii}}} - \frac{f_j}{\sqrt{d_{jj}}} \right\|^2 + \mu \sum_{i=1}^{m+n} \|f_i - f_i^0\|^2 \\ \text{s.t.} \quad & W = \begin{bmatrix} W^{uu} & \beta \cdot W^{uv} \\ \beta \cdot W^{vu} & W^{vv} \end{bmatrix} \\ & F = \begin{bmatrix} X \\ Y \end{bmatrix} \\ & \beta = (1 - \lambda_r)/\lambda_r, \end{aligned}$$

Regularization Framework	
$\mu\alpha, \lambda_r$	Description
$\mu\alpha = 0$	Initial scores $x_i = x_i^0$
$\mu\alpha = 1$	Corresponding to HITS
$\mu\alpha \in (0, 1)$	Regularization model
$\lambda_r = 1$	Single-sided regularization
$\lambda_r \in (0, 1)$	Double-sided regularization



Application to Query-URL Bipartite Graphs

- Bipartite graph construction
 - Edge weighted by the click frequency
 - Normalize to obtain the transition matrix
- Overall algorithm

Algorithm 1 Generalized Co-HITS Algorithm

Input: Given a query q and the bipartite graph

Perform:

1. Calculate the initial ranking scores based on the statistical language model and extract the top-ranked U_L and V_L as the seed sets;
2. Expand and extract the compact bipartite subgraph $\hat{G} = (\hat{U} \cup \hat{V}, \hat{E})$;
3. Get the weight matrix \hat{W} or \hat{S} , and normalize the corresponding initial scores F^0 ;
4. Solve Eq. (5) or Eq. (16) and get the final scores \hat{F}^* .

Output: Return the ranked queries



Experimental Evaluation

□ Data collection

■ AOL query log data

Table 2: Samples of the AOL query log dataset.

UserID	Query	Time	Rank	ClickURL
2722	yahoo	2006-04-25 13:03:23	1	http://www.yahoo.com
121537	map	2006-05-25 18:28:58	1	http://www.mapquest.com
123557	travel	2006-03-13 01:09:53	2	http://www.expedia.com
1903540	cheap flight	2006-05-15 00:31:43	1	http://www.cheapflights.com

□ Cleaning the data

- Removing the queries that appear less than 2 times
- Combining the near-duplicated queries
- 883,913 queries and 967,174 URLs
- 4,900,387 edges
- 250,127 unique terms



Evaluation: ODP Similarity

- A simple measure of similarity among queries using ODP categories (query → category)

- Definition:
$$Sim(Ca_i, Ca_r) = \frac{|P(Ca_i, Ca_r)|}{\max(|Ca_i|, |Ca_r|)}$$

- Example: 3/5

- Q1: “United States” → “Regional > North America > United States”

- Q2: “National Parks” → “Regional > North America > United States > Travel and Tourism > National Parks and Monuments”

- Precision at rank n (P@n):
$$P@n = \frac{\sum_{i=1}^n Sim(q_i, q_r)}{n}$$

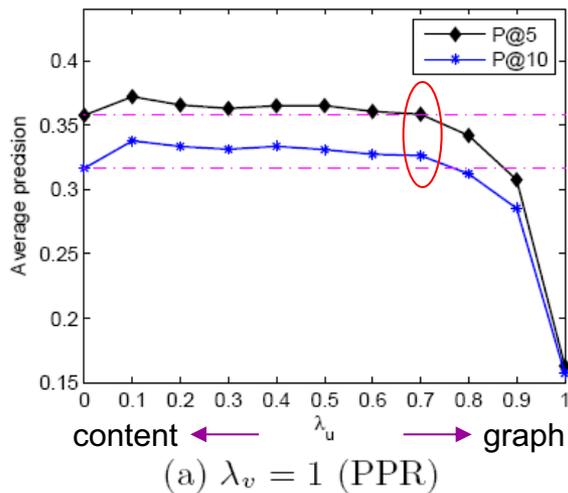
- 300 distinct queries



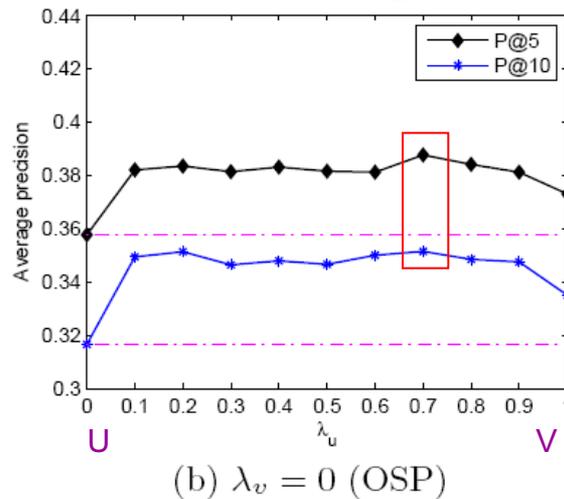
Experimental Results

Comparison of iterative framework

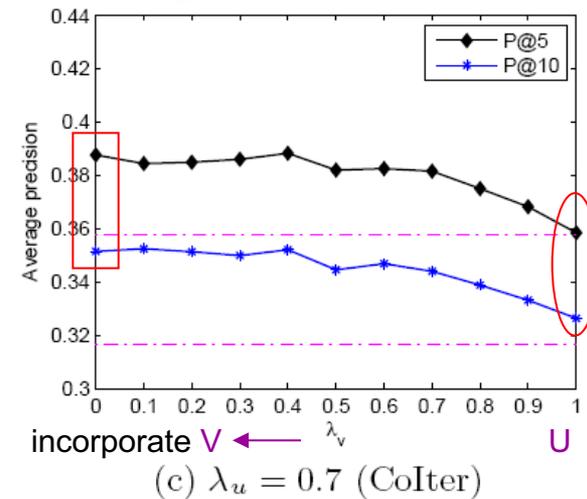
personalized PageRank



one-step propagation



general Co-HITS



Iterative Framework

λ_u	λ_v	Description
$= 0$	$\in [0, 1]$	Initial scores $x_i = x_i^0$
$= 1$	$= 1$	Original HITS
$\in (0, 1)$	$= 1$	Personalized PageRank
$\in (0, 1)$	$= 0$	One-step propagation
$\in (0, 1)$	$\in (0, 1)$	General Co-HITS

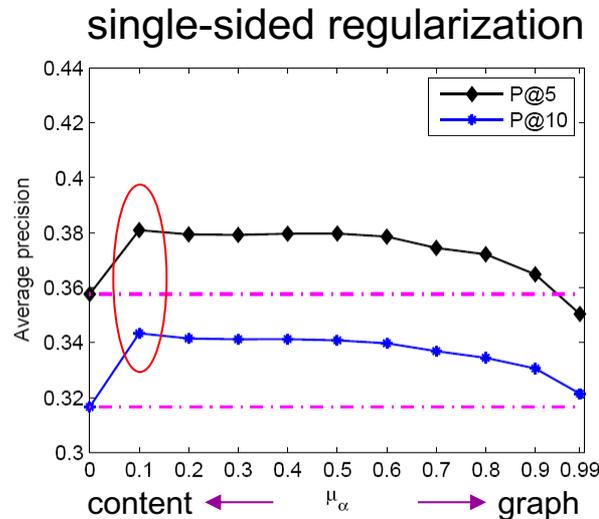
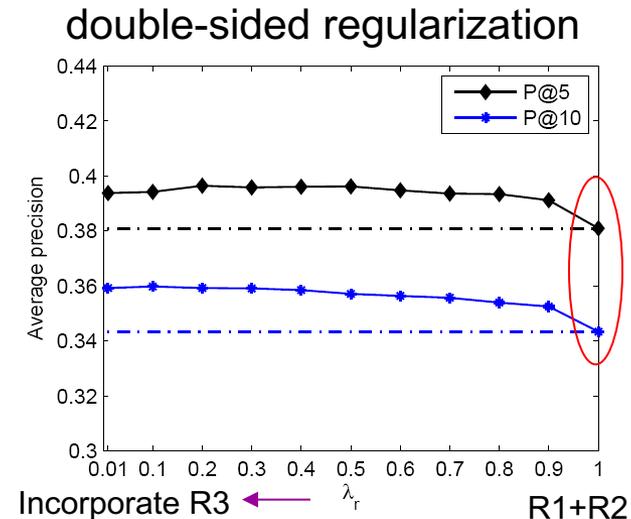
Result 1:

The initial relevance scores from both sides provide valuable information. The improvements of OSP and Colter over the baseline (the dashed line) are promising when compared to the PPR.



Experimental Results

Comparison of regularization framework

(a) $\lambda_r = 1$ (SiRegu)(b) $\mu_\alpha = 0.1$ (CoRegu)

Regularization Framework	
μ_α, λ_r	Description
$\mu_\alpha = 0$	Initial scores $x_i = x_i^0$
$\mu_\alpha = 1$	Corresponding to HITS
$\mu_\alpha \in (0, 1)$	Regularization model
$\lambda_r = 1$	Single-sided regularization
$\lambda_r \in (0, 1)$	Double-sided regularization

Result 2:

SiRegu can improve the performance over the baseline. CoRegu performs better than SiRegu, which owes to the newly developed cost function R_3 .

Moreover, CoRegu is relatively robust.



Experimental Results

□ Detailed results

Table 3: Comparison of different methods by P@5 and P@10. The mean precisions and the percentages of relative improvements are shown in the table.

Method	Para		Evaluation metrics	
Iter	λ_u	λ_v	P@5	P@10
Baseline	0	\times	0.358 (0%)	0.317 (0%)
PPR-0.1	0.1	1	0.372 (4.0%)	0.338 (6.7%)
OSP-0.7	0.7	0	0.388 (8.4%)	0.351 (11.0%)
CoIter-0.4	0.7	0.4	0.388 (8.6%)	0.352 (11.2%)
Regu	λ_r	μ_α	P@5	P@10
SiRegu-0.1	1	0.1	0.381 (6.5%)	0.343 (8.5%)
CoRegu-0.5	0.5	0.1	0.396 (10.8%)	0.357 (12.8%)

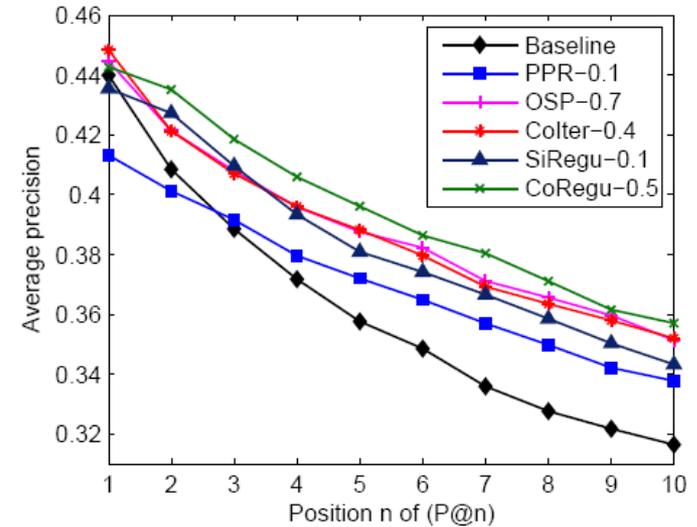


Figure 5: Comparison of six models.

Result 3:

The CoRegu-0.5 achieves the best performance. It is very essential and promising to consider the double-sided regularization framework for the bipartite graph.



Summary of Part II

- Propose the generalized **Co-HITS algorithm**
 - Incorporate the bipartite graph with the content information from both sides

- Investigate two different frameworks
 - Iterative: include **HITS** and **personalized PageRank** as special cases
 - Regularization: build the **connection** with HITS, develop **new** cost functions

- Experimental results
 - CoRegu is **more robust**, achieves the **best performance**



Outline

- Background: Web Mining Techniques
 - Information retrieval, link analysis, machine learning
- Modeling Bipartite Graph for Query Log Analysis
 - Entropy-biased Models [w/ King-Lyu, SIGIR'09]
 - Co-HITS Algorithm [w/ Lyu-King, KDD'09]
- Modeling Expertise Retrieval
 - Baseline and Weighted Model [w/ King-Lyu, ICDM'08]
 - Graph-based Re-ranking Model [w/ Lyu-King, WSDM'09]
 - Enhanced Models with Community [w/ King-Lyu, CIKM'09]
- Conclusion and Future Work



Modeling Expertise Retrieval

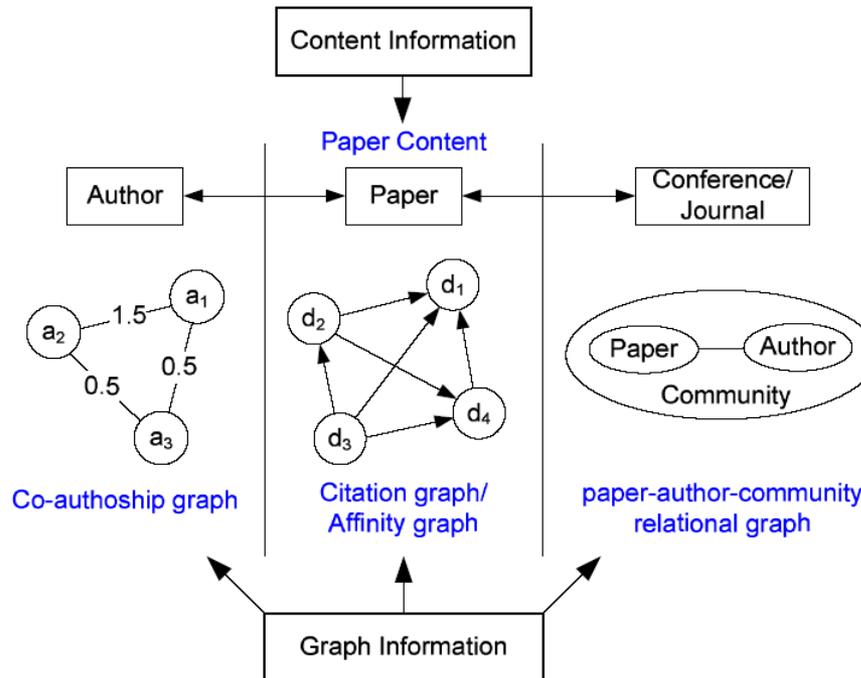
- Expertise retrieval (Expert finding) task:
 - Identify people with relevant expertise for a given query
 - A high-level information retrieval
 - DBLP bibliography and its supplemental data

The screenshot shows the 'ExpertFinding' website interface. At the top, there is a search bar with the text 'Information Retrieval' entered and a 'Search' button. Below the search bar, it says 'Based on DBLP Data' and 'Search: Show related articles'. The main content area is titled 'Expert Lists' and shows 'Results 1 - 10 of 1000 for I... (ds)'. The results list four experts: W. Bruce Croft, Gerard Salton, Norbert Fuhr, and C. J. van Rijsbergen. Each entry includes a link to their profile and a list of associated databases: DBLP, Google, and Google Scholar. A callout box with the text 'Information Retrieval' points to the search input field.

Expert Name	DBLP	Google	GoogleScholar
W. Bruce Croft	- 5.668803	- Google	- GoogleScholar
Gerard Salton	- 5.366938	- Google	- GoogleScholar
Norbert Fuhr	- 4.315518	- Google	- GoogleScholar
C. J. van Rijsbergen	- 4.155306	- Google	- GoogleScholar



Overview of Expertise Retrieval



Part III:

- Baseline model
- Weighted language model
- Graph-based re-ranking

Part IV:

- Enhanced models with community-aware strategies



Expertise Modeling

□ Expert finding

- $p(ca|q)$: what is the probability of a candidate ca being an expert given the query topic q ?
- Rank candidates ca according to this probability.

□ Approach:

- Using Bayes' theorem,

$$p(ca|q) = \frac{p(ca, q)}{p(q)}$$

where $p(ca, q)$ is joint probability of a candidate and a query, $p(q)$ is the probability of a query.

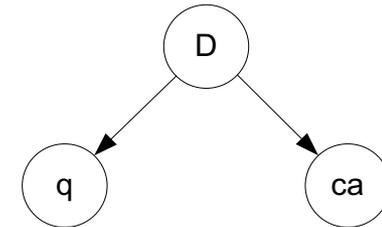
$$p(ca|q) \propto p(ca, q),$$



Baseline Model (Document-based Model)

□ The probability $p(ca, q)$:

$$\begin{aligned} p(ca, q) &= \sum_{d \in D} p(d) p(ca, q | d) \\ &= \sum_{d \in D} p(d) \underbrace{p(q | d)}_{\text{Language Model}} \underbrace{p(ca | d, q)}_{\text{Conditionally independent}} \end{aligned}$$



Baseline model

Language Model

Conditionally independent

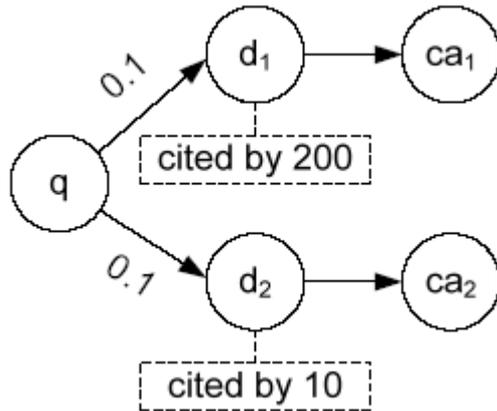
$$\begin{aligned} p(q | \theta_d) &= \prod_{t \in q} p(t | \theta_d)^{c(t, q)} \\ p(t | \theta_d) &= (1 - \lambda) p(t | d) + \lambda p(t). \end{aligned}$$

$$\begin{aligned} p(ca | d, q) &= p(ca | d_q) \\ p(ca | d) &= \begin{cases} \frac{1}{n_d}, & (ca \text{ is the author of } d) \\ 0, & (\text{otherwise}) \end{cases} \end{aligned}$$

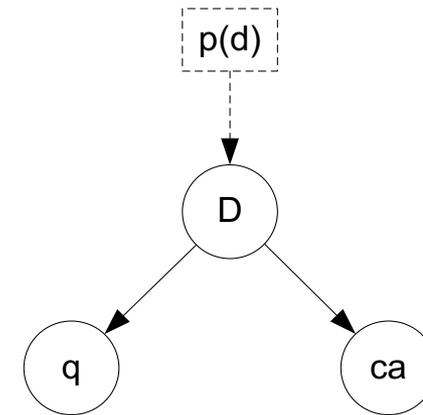
- Find out **documents** relevant to the query
- Aggregate the **expertise of an expert candidate** from the associated documents



Weighted Model



A query example



Weighted model

$$\begin{aligned}
 p(ca, q) &= \sum_{d \in D} p(d) p(ca, q|d) \\
 &= \sum_{d \in D} p(d) p(q|d) p(ca|d, q)
 \end{aligned}$$

$$p(d) = \frac{w_d}{C} \propto w_d,$$

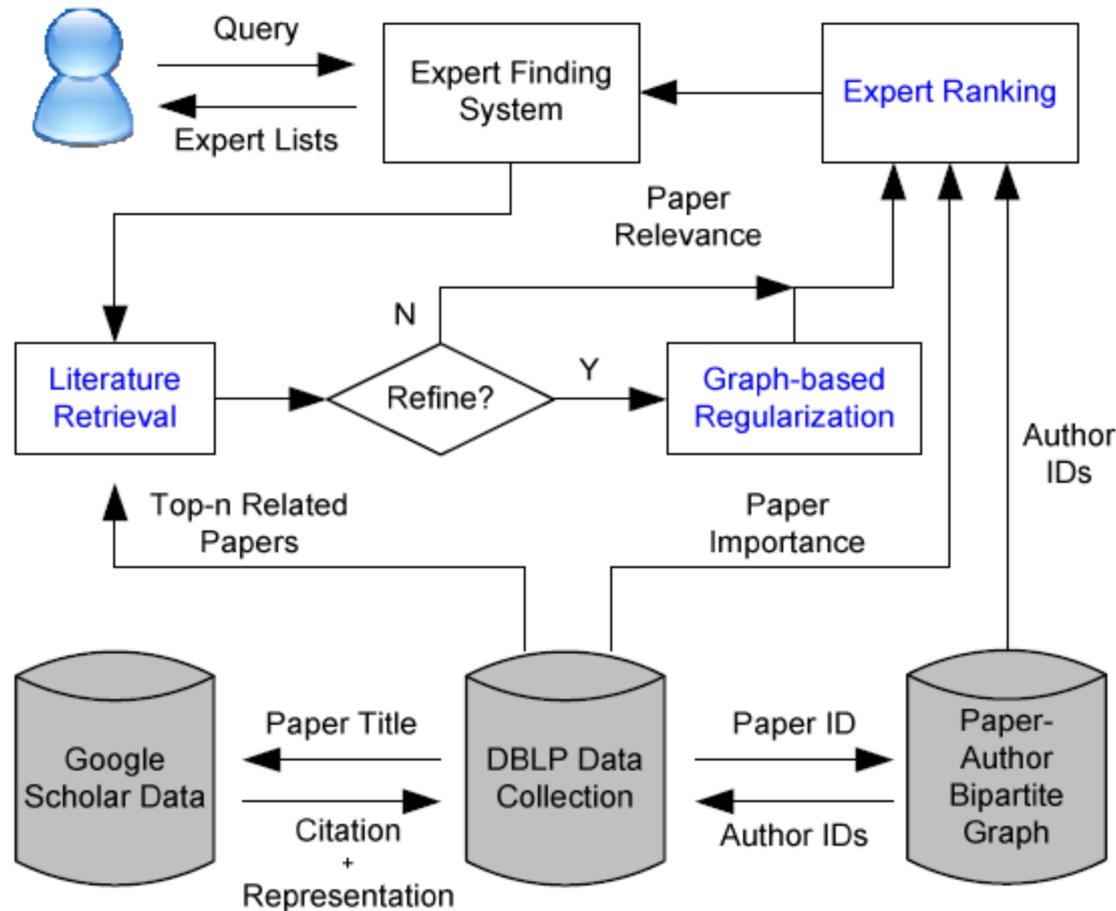
$$w_d = \begin{cases} 1, & (B1) \\ \log(10 + c_d), & (B2) \end{cases}$$

The final estimation of the weighted language model is

$$p_l(q, ca) \stackrel{rank}{=} \sum_{d \in D} w_d \left\{ \prod_{t \in q} (p(t|\theta_d))^{n(t,q)} \right\} p(ca|d).$$



General Expert Finding System



The schematic of general expert finding systems.



Graph-based Re-ranking

- Key issue for expert finding:
 - To retrieve the most relevant documents along with the relevance scores
- Intuition
 - **Global consistency:** Similar documents are most likely to have similar ranking scores with respect to a query
- Regularization framework

$$R(F, q, G) = \underbrace{\frac{1}{2} \sum_{i,j=1}^n w_{ij} \left\| \frac{f(d_i, q)}{\sqrt{D_{ii}}} - \frac{f(d_j, q)}{\sqrt{D_{jj}}} \right\|^2}_{\text{Global consistency}} + \underbrace{\mu \sum_{i=1}^n \|f(d_i, q) - f^0(d_i, q)\|^2}_{\text{Fit initial scores}}$$

Parameter



Graph-based Re-ranking

□ Optimization problem

$$F^* = \arg \min_{F \in \mathbb{R}^{+n}} R(F, q, G)$$

□ A closed-form solution

$$\begin{aligned} F^* &= \mu_\beta (I - \mu_\alpha S)^{-1} F^0, \\ S &= D^{-\frac{1}{2}} W D^{-\frac{1}{2}}, \\ \mu_\alpha &= \frac{1}{1+\mu}, \text{ and } \mu_\beta = \frac{\mu}{1+\mu}, \end{aligned}$$

□ Connection with other methods

- $\mu_\alpha \rightarrow 0$, return the initial scores
- $\mu_\alpha \rightarrow 1$, a variation of PageRank-based model
- $\mu_\alpha \in (0, 1)$, combine both information simultaneously



Combination of Different Methods

Model	w_d	Refine	Meaning
LM(bas)	B1 ^a	N ^c	baseline model
LM(w)	B2 ^b	N	weighted language model
LM(r)	B1	Y ^d	LM(bas) with graph-based regularization
LM(w+r)	B2	Y	LM(w) with graph-based regularization

^a uniform weight ($w_d = 1$)

^b common logarithm weight ($w_d = \log(10 + c_d)$)

^c without graph-based regularization

^d with graph-based regularization



Experimental Setup

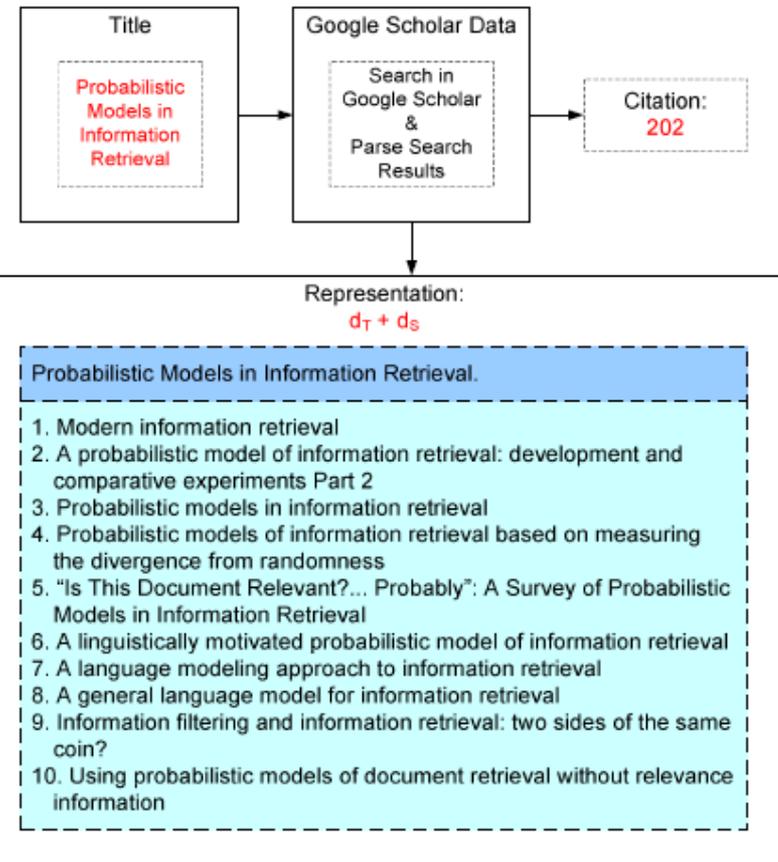
□ DBLP collection and representation

A sample of the DBLP XML records

```
<article mdate="2003-11-24" key="journals/cj/Fuhr92">
  <author>Norbert Fuhr</author>
  <title>Probabilistic Models in Information Retrieval.</title>
  <pages>243-255</pages>
  <year>1992</year>
  <volume>35</volume>
  <journal>Comput. J.</journal>
  <number>3</number>
  <url>db/journals/cj/cj35.html#Fuhr92</url>
</article>
```

Statistics of the DBLP collection

Property	#of entities
Number of papers	925,293
Number of authors	574,369
Number of terms	308,651



Experimental Setup

□ Assessments

- Manually created the ground truth through the method of pooled relevance judgments
- 17 query topics and 17 expert lists

□ Evaluation metrics

- Precision at rank n ($P@n$)
- MAP
- Bpref (Appendix D)

Topic	#Expert
Boosting	56
Information Extraction	20
Intelligent Agents	29
Machine Learning	42
Natural Language Processing	43
Planning	34
Semantic Web	45
Support Vector Machine	31
Ontology Alignment	55
Probabilistic Relevance Model	13
Information Retrieval	23
Language Model For Information Retrieval	12
Face Recognition	21
Semi Supervised Learning	21
Reinforcement Learning	17
Privacy Preservation	17
Kernel Methods	22



Experimental Results

“Title”	P@5	P@10	P@20	R-prec	MAP	bpref
LM(bas)	61.18	51.18	44.71	40.30	27.27	33.20
LM(w)	72.94	60.59	48.53	43.22	31.91	36.79
	+19.2	+18.4	+8.55	+7.25	+17.0	+10.8
“Title+GS”	P@5	P@10	P@20	R-prec	MAP	bpref
LM(bas)	72.94	64.12	47.94	43.98	33.06	38.16
LM(w)	81.18	65.29	53.24	47.93	37.10	41.60
	+11.3	+1.84	+11.0	+8.98	+12.2	+9.01

- Weighted model LM(w) outperforms baseline model LM(bas)



Experimental Results

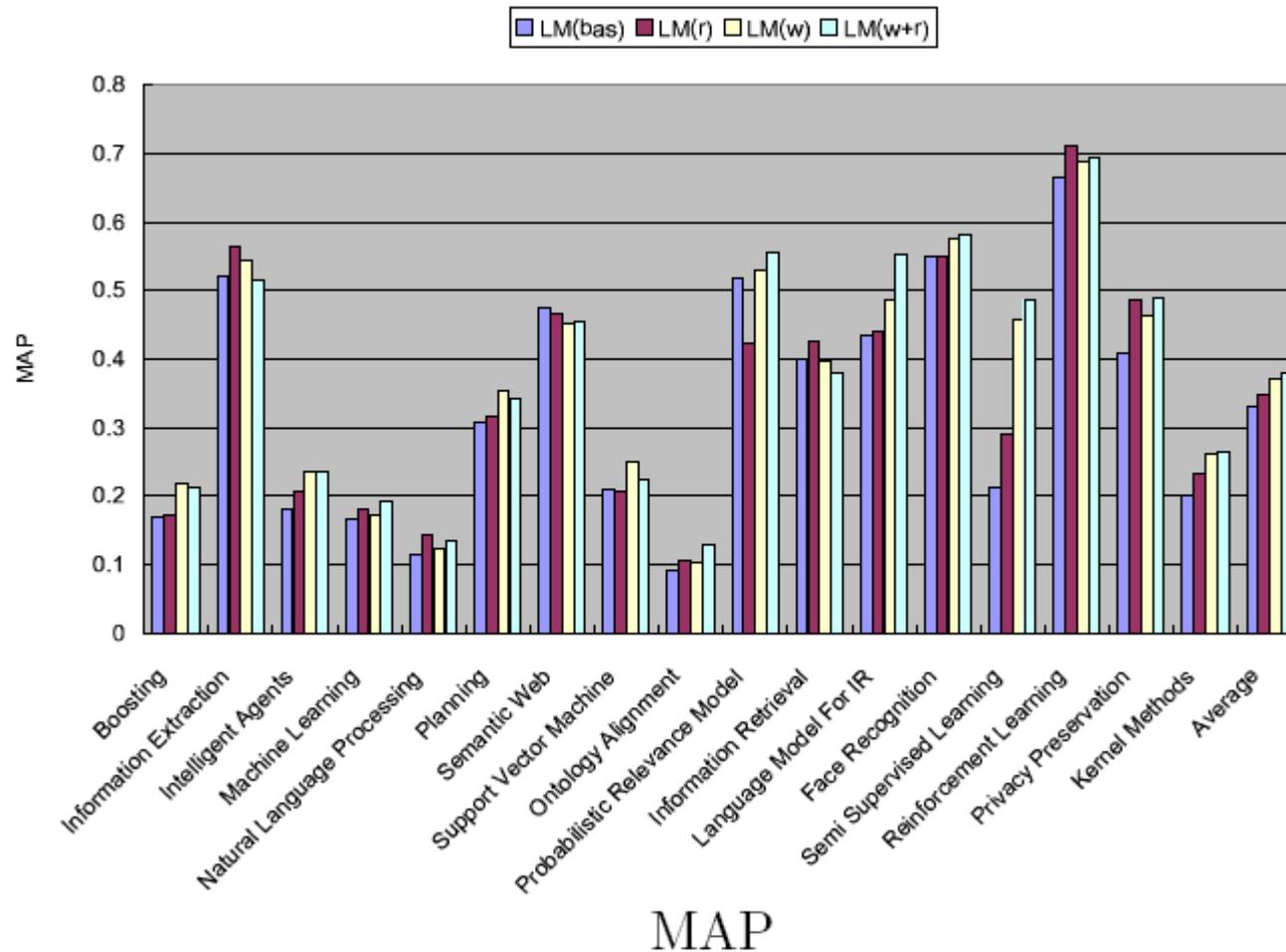
Comparison of different methods (%). The percentages of relative improvements are shown in the lower part

Method	P@5	P@10	P@20	R-prec	MAP	bpref
LM(bas)	72.94	64.12	47.94	43.98	33.06	38.16
LM(r) ($\mu_\alpha = 0.5$)	77.65	65.29	51.18	46.25	34.86	39.97
LM(w)	81.18	65.29	53.24	47.93	37.10	41.60
LM(w+r) ($\mu_\alpha = 0.5$)	82.35	68.24	55.59	48.88	37.89	42.60
LM(r) / LM(bas)	+6.45%	+1.83%	+6.75%	+5.15%	+5.42%	+4.75%
LM(w+r) / LM(w)	+1.45%	+4.50%	+4.42%	+1.97%	+2.13%	+2.40%
LM(w) / LM(bas)	+11.29%	+1.84%	+11.04%	+8.98%	+12.22%	+9.01%
LM(w+r) / LM(bas)	+12.90%	+6.42%	+15.95%	+11.13%	+14.61%	+11.63%

- The performance can be boosted with Graph-based regularization



Experimental Results



Summary of Part III

- Present the weighted model for expert finding
 - Take into account both the relevance scores and the importance of the documents

- Investigate and integrate the graph-based regularization method with the weighted model

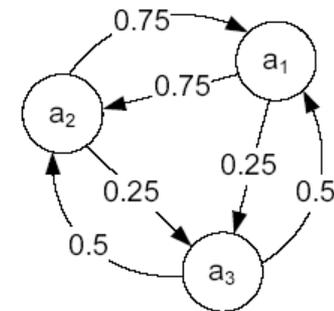
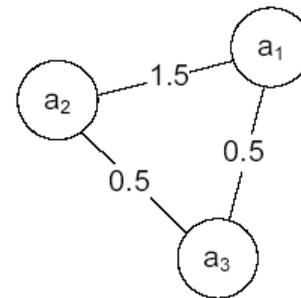
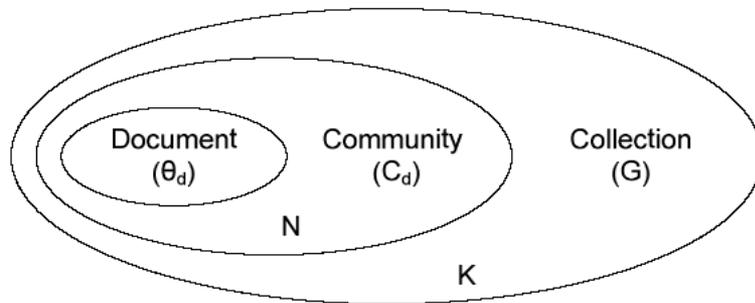
- Experimental results are presented to show the effectiveness of proposed models
 - The performance is further boosted by refining the relevance scores of the documents



Summary of Other Contributions

□ Enhancing Expertise Retrieval

- Communities could provide valuable insight and distinctive information
- A new smoothing method using the **community context**
- Ranking refinement based on **community co-authorship**
- Details in [Appendix A](#)



Conclusion

Model	Data	Techniques
Entropy-biased Model	Bipartite Graph	LA + IR
Co-HITS Algorithm	Bipartite Graph + Content	LA + IR + ML
Weighted Language Model	Content + Citation	IR
Graph-based Re-ranking Model	Content + Affinity Graph	LA + IR + ML
Enhanced Model with Communities	Content + Community	LA + IR + ML



Future Work

- Query log analysis
 - Personalization
 - General click graph, user's click graph, session info.
 - Incorporate with other information
 - Query-flow model, user modeling
- Expertise retrieval on the Web
 - Beyond a particular domain or intranet
 - Identify relevant experts/trusted people
 - Create a global expert and friend recommendation
- Apply to other applications
 - Entity retrieval
 - Online social media search
 - ...



Selected Publications

- **Hongbo Deng**, Irwin King, Michael R. Lyu. "Entropy-biased Models for Query Representation on the Click Graph." *Proceedings of the 32nd Annual ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR 2009)*. Pages 339-346, Boston, MA, July 19-23, 2009. [Acceptance rate = $78/494 = 15.8\%$]
- **Hongbo Deng**, Michael R. Lyu and Irwin King. "A Generalized Co-HITS Algorithm and Its Application to Bipartite Graphs." *Proceedings of the 15th ACM SIGKDD Conference on Knowledge Discovery and Data Mining (KDD 2009)*. Pages 239-248, Paris, France, June 28th - July 1st, 2009. [Acceptance rate = $105/561 = 18.7\%$]
- **Hongbo Deng**, Irwin King, Michael R. Lyu. "Enhancing Expertise Retrieval Using Community-aware Strategies." *Proceedings of the 18th ACM Conference on Information and Knowledge Management (CIKM 2009)*. Hong Kong, China, Nov. 2-6, 2009. Short paper. [Acceptance rate = $294/847 = 34.7\%$]
- **Hongbo Deng**, Michael R. Lyu and Irwin King. "Effective Latent Space Graph-based Re-ranking Model with Global Consistency." *Proceedings of the 2nd ACM International Conference on Web Search and Data Mining (WSDM 2009)*. Pages 212-221, Barcelona, Spain, Feb. 9-12, 2009. [Acceptance rate = $29/170 = 17\%$]
- **Hongbo Deng**, Irwin King and Michael R. Lyu. "Formal Models for Expert Finding on DBLP Bibliography Data." *Proceedings of the 8th IEEE International Conference on Data Mining (ICDM 2008)*. Pages 163-172, Pisa, Italy, Dec. 15-19, 2008. [Acceptance rate = $70/723 = 10\%$]



Q&A

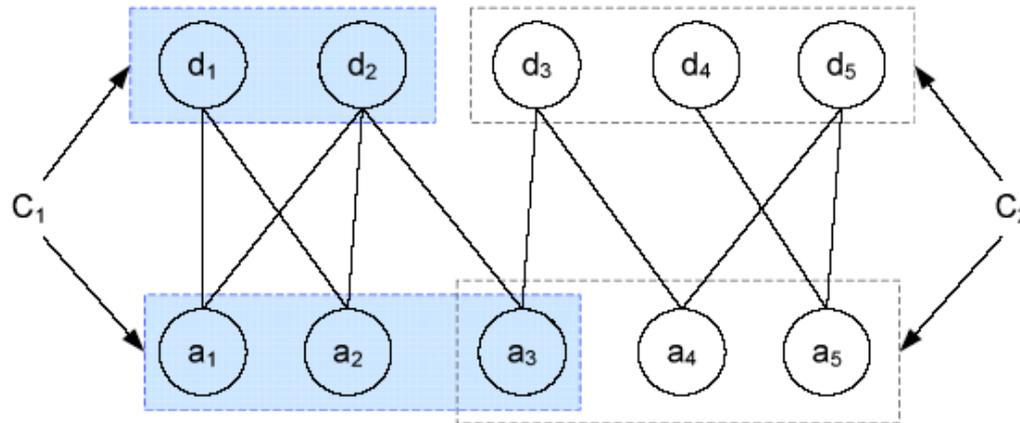
Thanks!



Appendix A: Enhancing Expertise Retrieval

□ Motivation

- Communities could provide valuable insight and distinctive information



An example graph with two communities

□ Community-aware strategies

- A new smoothing method using the **community context**
- Ranking refinement based on **community co-authorship**



Document-based Model

□ Key challenge

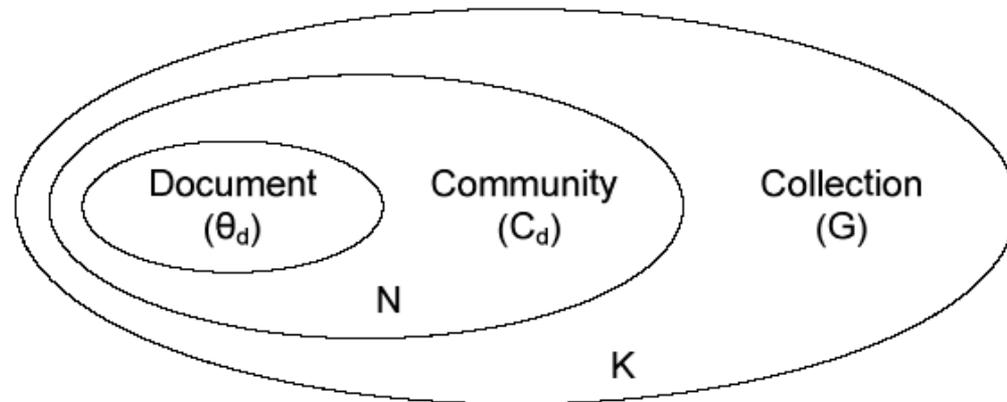
- Compute the **relevance** between query and document

□ Statistical language model

- Smoothing $p(t|\theta_d)$ with the community language model $p(t|C_d)$ instead of the collection language model $p(t|G)$

$$p(t|\theta_d) = (1 - \lambda) \frac{n(t, d)}{|d|} + \lambda p(t|G)$$

$$p(t|\theta_d) = (1 - \lambda) \frac{n(t_i, d)}{|d|} + \lambda p(t_i|C_d)$$



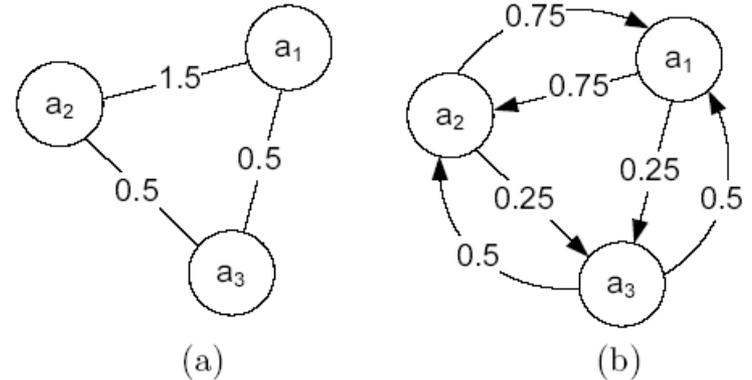
Discovering Authorities in a Community

□ Co-authorship frequency

$$f_{ij} = \sum_{k=1}^N \frac{\delta_i^k \delta_j^k}{n_k - 1}$$

□ Normalized weight

$$w_{ij} = \frac{f_{ij}}{\sum_{k=1}^n f_{ik}}$$



Co-authorship graph with: (a) co-authorship frequency, and (b) normalized weight

□ AuthorRank

- Measure the authority for the authors within a community

$$p(a_i|C_k) = (1 - \alpha) \frac{1}{N_a(C_k)} + \alpha \sum_{j=1}^{N_a(C_k)} w_{ij} \cdot p(a_j|C_k)$$



Community-Sensitive AuthorRank

□ Relevance

$$p(C_k|q) = \frac{p(C_k) \cdot p(q|C_k)}{p(q)} \propto p(C_k) \prod_{t_i \in q} p(t_i|C_k)$$

□ The quantity $p(C_k)$

$$p(C_k) \propto N_a(C_k) \cdot \log(10 + N_c(C_k))$$

□ Community Sensitive AuthorRank

$$p(a_i|q) \propto \sum_k p(C_k)p(q|C_k)p(a_i|C_k)$$

- Suppose C_k be a “virtual” document, it becomes the document-based model
- Capture the high-level and general aspects for a query



Ranking Refinement Strategy

- Two kinds of ranking results
 - Rd : capture more specific and detailed aspects
 - Rc : reflect more general and abstract aspects
- Measure the similarity and diversity

$$J = \frac{|\vec{R}d \cap \vec{R}c|}{|\vec{R}d \cup \vec{R}c|}$$

- Ranking refinement

$$S(a_i) = \frac{1}{Rd(a_i)} + \delta(a_i) \cdot J \cdot \frac{1}{\hat{R}c(a_i)}$$



Experimental Results

□ Comparison of Document-based Models

Method	P@10	P@20	P@30	R-prec	MAP	bpref
DM(b)	0.5353	0.45	0.3726	0.4316	0.2897	0.3524
DM(bc)	0.5588	0.4647	0.3824	0.4417	0.3015	0.3621
DM(w)	0.6882	0.5029	0.4235	0.4845	0.3633	0.4159
DM(wc)	0.6882	0.5265	0.4314	0.4943	0.3771	0.4279
DM(bc)/DM(b)	+4.40%	+3.27%	+2.63%	+2.34%	+4.09%	+2.78%
DM(wc)/DM(w)	0%	+4.68%	+1.85%	+2.03%	+3.79%	+2.89%
DM(w)/DM(b)	+28.57%	+11.76%	+13.68%	+12.26%	+25.43%	+18.02%
DM(wc)/DM(b)	+28.57%	+16.99%	+15.79%	+14.53%	+30.19%	+21.44%



Experimental Results

□ Comparison of Enhanced Models

Method	P@10	P@20	P@30	R-prec	MAP	bpref
EDM(b)	0.5882	0.4971	0.4196	0.4716	0.3228	0.38933
EDM(bc)	0.5941	0.5059	0.4275	0.4803	0.3342	0.39879
EDM(w)	0.7059	0.55	0.4608	0.5317	0.403	0.45839
EDM(wc)	0.7118	0.5677	0.4628	0.5332	0.4089	0.46241
EDM(b)/DM(b)	+9.89%	+10.46%	+12.63%	+9.28%	+11.44%	+10.49%
EDM(bc)/DM(bc)	+6.31%	+8.86%	+11.79%	+8.75%	+10.85%	+10.12%
EDM(w)/DM(w)	+2.56%	+9.36%	+8.79%	+9.75%	+10.92%	+10.22%
EDM(wc)/DM(wc)	+3.42%	+7.82%	+7.27%	+7.86%	+8.43%	+8.06%



Experimental Results

□ Discussion and Detailed Results

The detailed results of the community-sensitive AuthorRank for the query “machine learning.”

journals/ML	conf/ICML	conf/NIPS	journals/JMLR	conf/ECML
Pat Langley	Andrew W. Moore	Terrence J. Sejnowski	Michael I. Jordan	Saso Dzeroski
Robert E. Schapire	Sridhar Mahadevan	Michael I. Jordan	Yoram Singer	Johannes Frnkranz
Manfred K. Warmuth	Thomas G. Dietterich	Geoffrey E. Hinton	Tong Zhang	Gerhard Widmer
Thomas G. Dietterich	Prasad Tadepalli	Peter Dayan	Francis R. Bach	Ivan Bratko
Yoram Singer	Michael L. Littman	Christof Koch	Olivier Bousquet	Enric Plaza
Ryszard S. Michalski	Pat Langley	Klaus-Robert Mller	Klaus-Robert Mller	Pavel Brazdil
Michael J. Pazhani	Andrew McCallum	Zoubin Ghahramani	Bernhard Schlkopf	Birgit Tausend
Dana Angluin	Thorsten Joachims	Michael Mozer	Andr Elisseeff	Stephen Muggleton
Avrim Blum	Satinder P. Singh	Bernhard Schlkopf	Koby Crammer	Floriana Esposito
Leo Breiman	Michael I. Jordan	Satinder P. Singh	Ingo Steinwart	Stan Matwin



Experimental Results

DM(wc)	Authorities	EDM(wc)
Pat Langley	Pat Langley	Pat Langley
Thomas G. Dietterich	Robert E. Schapire	Thomas G. Dietterich
Sumio Watanabe	Manfred K. Warmuth	Sumio Watanabe
David E. Goldberg	Yoram Singer	David E. Goldberg
Tom M. Mitchell	Thomas G. Dietterich	Avrim Blum
Avrim Blum	Michael I. Jordan	Tom M. Mitchell
Ivan Bratko	Satinder P. Singh	Sanjay Jain
Donald Michie	Sanjay Jain	Ivan Bratko
Carl H. Smith	John Shawe-Taylor	Donald Michie
J. Ross Quinlan	Michael J. Pazzani	Michael I. Jordan



Summary

- Investigate the smoothing method using community context instead of the whole collection
- Introduce the community-sensitive AuthorRank for determining the query-sensitive authorities
- Develop an adaptive ranking refinement strategy to aggregate the ranking results
- Experimental results shows a significant improvement over the baseline method
- Return from [Appendix A](#)



Appendix B: Graph-based Random Walk

□ Query-to-query graph

- The transition probability from q_i to q_j

$$p(q_j|q_i) = \sum_{k \in D} p(d_k|q_i)p(q_j|d_k)$$

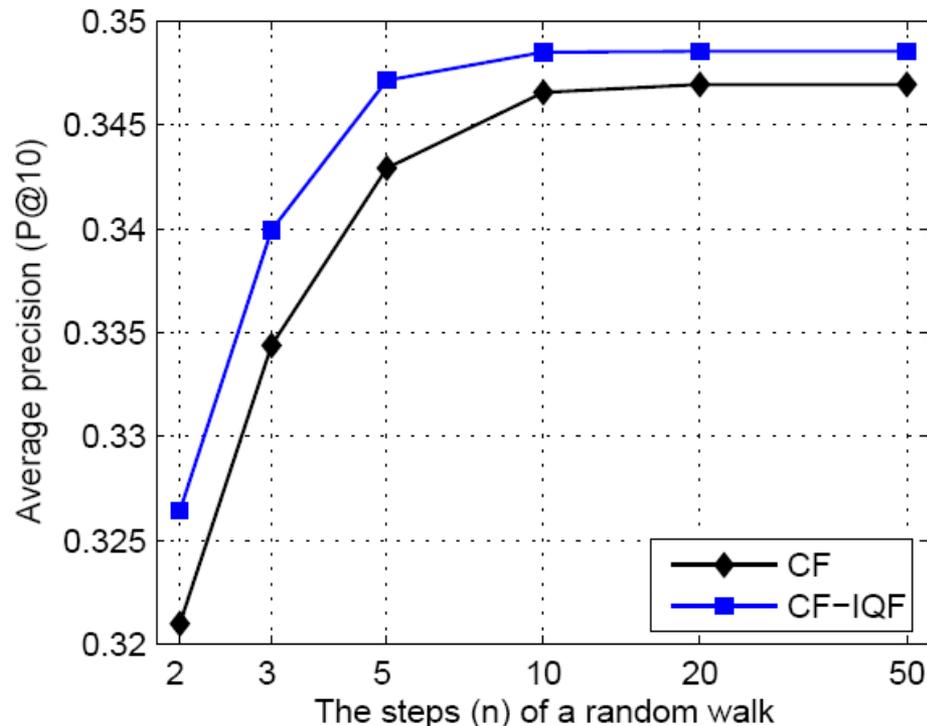
□ The personalized PageRank

$$R_j^{n+1} = (1 - \alpha)R_j^{(0)} + \alpha \cdot \sum_i p(q_j|q_i)R_i^n$$



Experimental Results

□ Random Walk Evaluation



Results:

1. With the increase of n , both models improve their performance.
2. CF-IQF model always performs better than the CF model.



Experimental Results

□ Random Walk Evaluation

CF model	CF-IQF model
Query = aa	
american airlines	american airlines
alcoholics anonymous	alcoholics anonymous
aa.com	aa.com
airlines	airlines
Query = east texas real estate	
google	east texas acreage
east texas acreage	tyler real estate
texas real estate	tyler texas realtors
tyler real estate	texas real estate
Query = home gym equipment	
home gyms	home gyms
gym equipment	gym equipment
treadmills	treadmills
buy.com	edge 329 upright exercise bike

In general, the results generated by the CF and the CF-IQF models are similar, and mostly semantically relative to the original query, such as “American airline”.

CF-IQF model can boost more relevant queries as suggestion and reduce some irrelevant queries.



Appendix C: Optimization Problem

Optimization problem:

$$\min_F \quad \frac{1}{2} \sum_{i,j=1}^{m+n} w_{ij} \left\| \frac{f_i}{\sqrt{d_{ii}}} - \frac{f_j}{\sqrt{d_{jj}}} \right\|^2 + \mu \sum_{i=1}^{m+n} \|f_i - f_i^0\|^2$$

$$s.t. \quad W = \begin{bmatrix} W^{uu} & \beta \cdot W^{uv} \\ \beta \cdot W^{vu} & W^{vv} \end{bmatrix}$$

$$F = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\beta = (1 - \lambda_r) / \lambda_r,$$

Differentiating and simplifying:

$$\frac{dR}{dF} \Big|_{F=F^*} = F^* - SF^* + \mu(F^* - F^0) = 0,$$

$$F^* - \frac{1}{1 + \mu} SF^* - \frac{\mu}{1 + \mu} F^0 = 0$$

Solution:

$$F^* = \mu_\beta (I - \mu_\alpha S)^{-1} F^0,$$

$$S = D^{-\frac{1}{2}} W D^{-\frac{1}{2}}$$

$$\mu_\alpha = \frac{1}{1 + \mu}, \text{ and } \mu_\beta = \frac{\mu}{1 + \mu},$$



Appendix D: Evaluation Metrics

- Precision at rank n ($P@n$):

$$P@n = \frac{\# \text{ relevant candidates in top } n \text{ results}}{n}$$

- Mean Average Precision (MAP):

$$AP = \frac{\sum_{n=1}^N (P@n * \text{rel}(n))}{R}$$

- Bpref: The score function of the number of non-relevant candidates

$$\text{bpref} = \frac{1}{R} \sum_{r=1}^N \left(1 - \frac{\#n \text{ ranked higher than } r}{R} \right)$$

