Sensor Drift Calibration via Spatial Correlation Model in Smart Building

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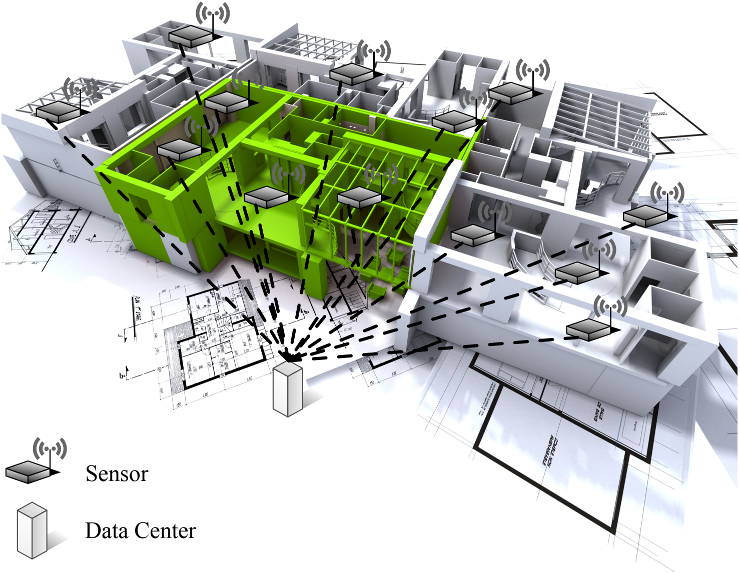
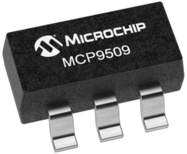
Smart Building and *Cyber-Physical System*



Temperature Sensor

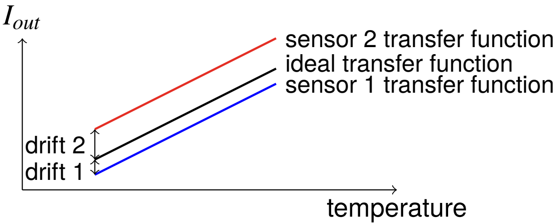
- Errors exist in senor output;
- Manufacturing defect, noise, aging...
- Cost varies significantly.

Part Number	Temp. Range	Accuracy	Price
SMT172	$-45 \sim 130$ °C	± 0.25 °C	\$ 35.13
AD590JH	$-50 \sim 150$ °C	± 0.5 °C	\$ 17.91
TMP100	$-55 \sim 125$ °C	± 2.0 °C	\$ 1.79
MCP9509	$-40 \sim 125$ °C	± 4.5 °C	\$ 0.88
LM335A	$-40 \sim 100$ °C	± 5.0 °C	\$ 0.75



Problem Formulation of Sensor Drift Calibration

- Several low-cost sensors are deployed to sense in-building temperatures;
- The sensor output deviates by a time-invariant drift.



Sensor Drift Calibration

Given the measurement values sensed by all sensors during several time-instants, drifts will be accurately estimated and calibrated.

Basic Model

Spatial Correlation Model:

- Defines a linear correlation among different temperature values;
- drift-free model: $x_i^{(k)} \approx \sum_{j=1, j\neq i}^{n} a_{i,j} x_j^{(k)} + a_{i,0}, \qquad k = 1, 2, \cdots, m_0.$
- drift-with model: $\hat{x}_i^{(k)} + \epsilon_i \approx \sum_{j=1, j\neq i}^{n} \hat{a}_{i,j} (\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0}, \qquad k = 1, 2, \cdots, m.$

Input:

- $\hat{x}_i^{(k)}$: the measurement value sensed by ith sensor at kth time-instant.
- $a_{i,j}$: the drift-free model coefficient.

Output:

- ϵ_i: a time-invariant drift calibration.

Further Assumption

Likelihood:

$$\mathcal{P}(\hat{\mathbf{x}}|\hat{\mathbf{a}},\boldsymbol{\epsilon}) \propto \exp(-\frac{\delta_0}{2}\sum_{i=1}^{n}\sum_{k=1}^{m}[\hat{x}_i^{(k)} + \epsilon_i - \sum_{j=1,j\neq i}^{n} \hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) - \hat{a}_{i,0}]^2).$$

Prior for all model coefficients (Bayesian Model Fusion [Wang+,TCAD15]):

$$\mathcal{P}(\hat{\mathbf{a}}) \propto \exp\left(-\sum_{i=1}^{n}\sum_{j=0,j\neq i}^{n}\frac{\lambda}{2a_{i,j}^2}(\hat{a}_{i,j} - a_{i,j})^2\right).$$

Mathematic Formulation based on MAP

Maximum-a-posteriori:

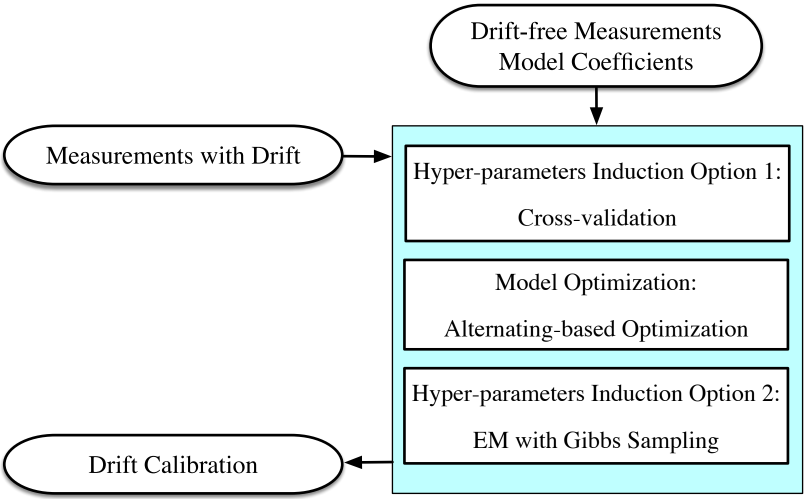
$$\min_{\hat{\mathbf{a}},\epsilon} \quad \delta_0 \sum_{i=1}^{n} \sum_{k=1}^{m} [\hat{x}_i^{(k)} + \epsilon_i - \sum_{j=1,j\neq i}^{n} \hat{a}_{i,j}(\hat{x}_j^{(k)} + \epsilon_j) - \hat{a}_{i,0}]^2$$

$$+ \lambda \sum_{i=1}^{n} \sum_{j=0,j\neq i}^{n} \frac{1}{a_{i,j}^2}(\hat{a}_{i,j} - a_{i,j})^2 + \delta_\epsilon \sum_{i=1}^{n} \epsilon_i^2.$$

Challenges:

- How to handle this Formulation
- How to determine hyper-parameters

Overall Flow



Alternating-based Optimization

Require: Sensor measurements $\hat{\mathbf{x}}$, prior $\mathbf{a}$ and hyper-parameters λ, δ_0, $\delta_{\boldsymbol{\epsilon}}$.

```
Initialize â ← a and ϵ ← 0;
repeat
    for i ← 1 to n do
        Fix ϵ, solve linear equations (1) using Gaussian elimination to update â_i;
    end for
    Fix â, solve linear equations (2) using Gaussian elimination to update ϵ;
until Convergence
return â and ϵ.
```

$$\delta_0 \sum_{k=1}^{m} (\hat{x}_t^{(k)} + \epsilon_t) \left[\sum_{j=1}^{n} \hat{a}_{i,j} (\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0} \right] + \lambda \frac{(\hat{a}_{i,t} - a_{i,t})}{a_{i,t}^2} = 0, \quad (1)$$

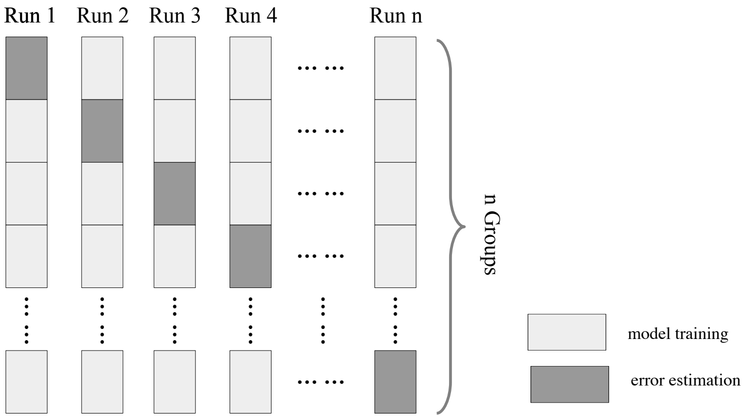
$$\delta_0 \sum_{i=1}^{n} \sum_{k=1}^{m} \left[\hat{a}_{i,t} (\sum_{j=1}^{n} \hat{a}_{i,j} (\hat{x}_j^{(k)} + \epsilon_j) + \hat{a}_{i,0}) \right] + \delta_{\boldsymbol{\epsilon}} \epsilon_t = 0, \quad (2)$$

Estimation of Hyper-parameters

Comparison of Estimation for Hyper-parameters

- **Unsupervised Cross-validation:**
  simple, accurate but time-consuming.
- **Monte Carlo Expectation Maximization:**
  fast, flexible but no-accurate.

Unsupervised Cross-validation



Training Set: $$\min_{\hat{\mathbf{a}},\epsilon} \quad \delta_0 \sum_{i=1}^{n}\sum_{k=1}^{m}[\hat{x}_i^{(k)}+\epsilon_i-\sum_{j=1,j\neq i}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)}+\epsilon_j)-\hat{a}_{i,0}]^2+\lambda\sum_{i=1}^{n}\sum_{j=0,j\neq i}^{n}\frac{1}{a_{i,j}^2}(\hat{a}_{i,j}-a_{i,j})^2+\delta_\epsilon\sum_{i=1}^{n}\epsilon_i^2.$$

Validation Set: $$\sum_{i=1}^{n}\sum_{k=1}^{m}[\hat{x}_i^{(k)}+\epsilon_i-\sum_{j=1,j\neq i}^{n}\hat{a}_{i,j}(\hat{x}_j^{(k)}+\epsilon_j)-\hat{a}_{i,0}]^2$$

$\hat{a}_{i,j}$ and ϵ_i

Monte Carlo Expectation Maximization

Maximum Likelihood Estimation:

$$\max_{\delta_{\epsilon},\delta_0,\lambda} \quad \mathcal{P}(\hat{\mathbf{x}}; \delta_0, \lambda, \delta_{\epsilon}).$$

Expectation Maximization

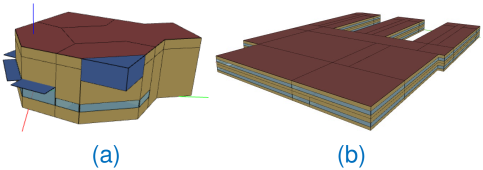
E-Step

$$Q(\Omega|\Omega^{\text{old}}) = \int\int \mathcal{P}(\hat{\mathbf{a}}, \boldsymbol{\epsilon}|\hat{\mathbf{x}}; \Omega^{\text{old}}) \ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}, \boldsymbol{\epsilon}; \Omega) d\hat{\mathbf{a}} d\boldsymbol{\epsilon}$$
$$\approx \frac{1}{L} \sum_{l=1}^{L} \ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}^{(l)}, \boldsymbol{\epsilon}^{(l)}; \Omega)$$

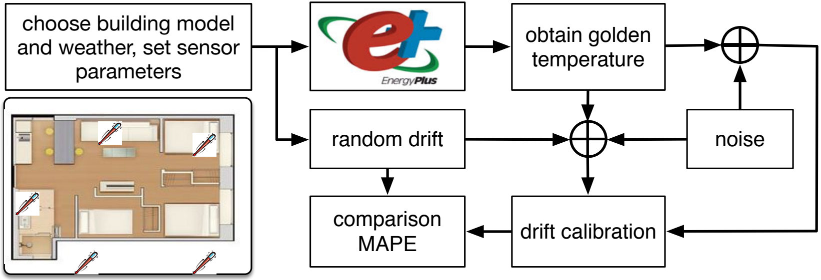
M-Step

$$\max_{\Omega} \quad \frac{1}{L} \sum_{l=1}^{L} \ln \mathcal{P}(\hat{\mathbf{x}}, \hat{\mathbf{a}}^{(l)}, \boldsymbol{\epsilon}^{(l)}; \Omega).$$

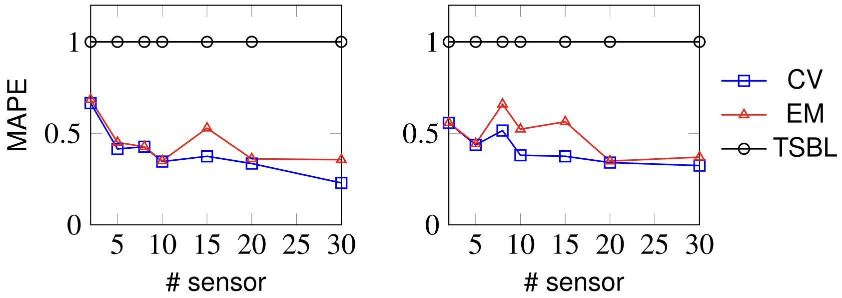
Benchmark



Benchmark: (a) Hall; (b) Secondary School.



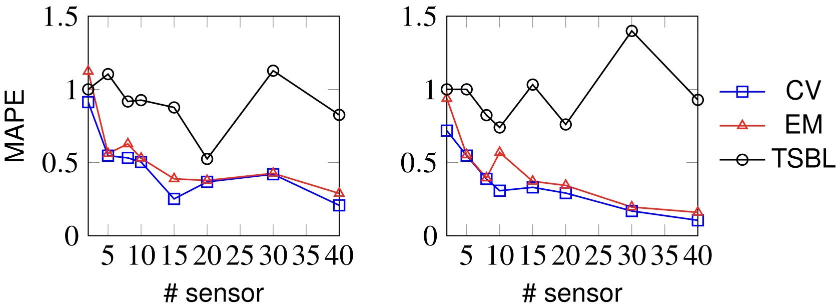
Accuracy



(a) (b)

Drift variance is set to (a) 2.25; (b) 2.78; Benchmark: (a,b) Hall;.

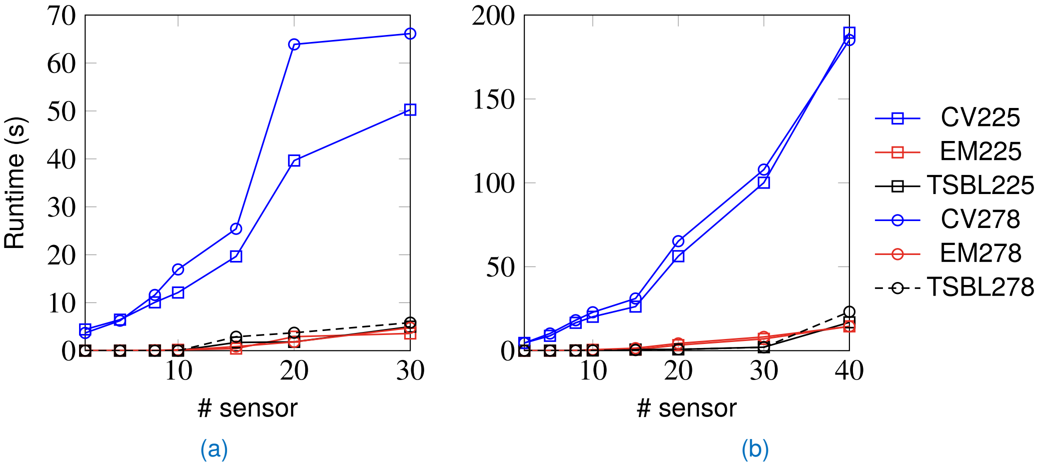
Accuracy



(a) (b)

Drift varians set to (a) 2.25; (b) 2.78; Benchmark: (a,b) Secondary School.

Runtime



Runtime vs. # sensor (a) Hall; (b) Secondary School.

Conclusion

- A sensor spatial correlation model has been proposed to perform drift calibration
- MAP estimation is then formulated as a non-convex problem with three hyper-parameters, which is handled by the proposed alternating-based method.
- Cross-validation and EM with Gibbs sampling are used to determine hyper-parameters, respectively.
- Experimental results show that on benchmarks simulated from EnergyPlus, the proposed framework with EM can achieve a robust drift calibration and better trade-off between accuracy and runtime.

Thank You