CENG 3420
Computer Organization & Design

Lecture 05: Logic Basis

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(Textbook: Chapter 2.4)

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Overview

1. Numeral System
2. Logic Gates (Optional)

Numeral System

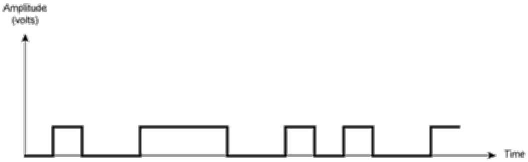
Analog vs. Digital

Analog Signal

- Vary in a smooth way over time
- Analog data are continuous valued
  - Example: audio, video

Digital Signal

- Maintains a constant level then changes to another constant level (generally operate in one of the two states)
- Digital data are discrete valued
  - Example: computer data



Number Systems

- An ordered set of symbols, called digits, with relations defined for addition, subtraction, multiplication, and division
- **Radix** or **base** of the number system is the total number of digits allowed in the number system
- Commonly used numeral systems

System Name	Decimal	Binary	Octal	Hexadecimal
Radix	10	2	8	16
First seventeen positive integers	0	0	0	0
	1	1	1	1
	2	10	2	2
	3	11	3	3
	4	100	4	4
	5	101	5	5
	6	110	6	6
	7	111	7	7
	8	1000	10	8
	9	1001	11	9
	10	1010	12	A
	11	1011	13	B
	12	1100	14	C
	13	1101	15	D
	14	1110	16	E
	15	1111	17	F
	16	10000	20	10

Example

- In the 2009 film Avatar, Na'vi race employs an octal numeral system.

Conversion from Decimal Integer

- Step 1: Divide the decimal number by the radix (number base)
- Step 2: Save the remainder (first remainder is the least significant digit)
- Repeat steps 1 and 2 until the quotient is zero
- Result is in reverse order of remainders

EX: L02-1

- EX1: Convert 36_8 to binary value
- EX2: Convert 36_{10} to binary value

Unsigned Binary Representation

Hex	Binary	Decimal
0x00000000	0…0000	0
0x00000001	0…0001	1
0x00000002	0…0010	2
0x00000003	0…0011	3
0x00000004	0…0100	4
0x00000005	0…0101	5
0x00000006	0…0110	6
0x00000007	0…0111	7
0x00000008	0…1000	8
0x00000009	0…1001	9
	…	
0xFFFFFFFC	1…1100	$2^{32} - 4$
0xFFFFFFFD	1…1101	$2^{32} - 3$
0xFFFFFFFE	1…1110	$2^{32} - 2$
0xFFFFFFFF	1…1111	$2^{32} - 1$

2^{31} 2^{30} 2^{29} . . . 2^{3} 2^{2} 2^{1} 2^{0} bit weight

31 30 29 . . . 3 2 1 0 bit position

1 1 1 . . . 1 1 1 1 bit

1 0 0 0 . . . 0 0 0 0 - 1

2^{32} - 1

	2's binary	decimal
-2^3 =	1000	-8
$-(2^3 - 1)$ =	1001	-7
	1010	-6
	1011	-5
	1100	-4
	1101	-3
	1110	-2
	1111	-1
	0000	0
	0001	1
	0010	2
	0011	3
	0100	4
	0101	5
	0110	6
$2^3 - 1$ =	0111	7

complement all the bits

0101

and add a 1

0110

1011

and add a 1

1010

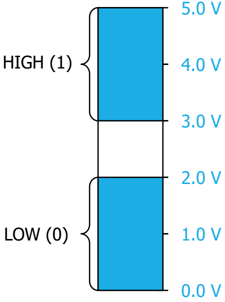
complement all the bits

EX: L02-2

- For an n-bit signed binary numeral system, what's the largest positive number and the smallest negative number?

- Active HIGH
  - High voltage means On
- Active LOW
  - Low voltage means Off

Logic 0	Logic 1
False	True
Off	On
LOW	HIGH
No	Yes
Open switch	Closed switch



Addition & Subtraction

- Just like in grade school (carry/borrow 1s)

```
  0111         0111         0110
+ 0110       - 0110       - 0101
-------      ------       -------
```

- Two's complement operations are easy: do subtraction by negating and then adding

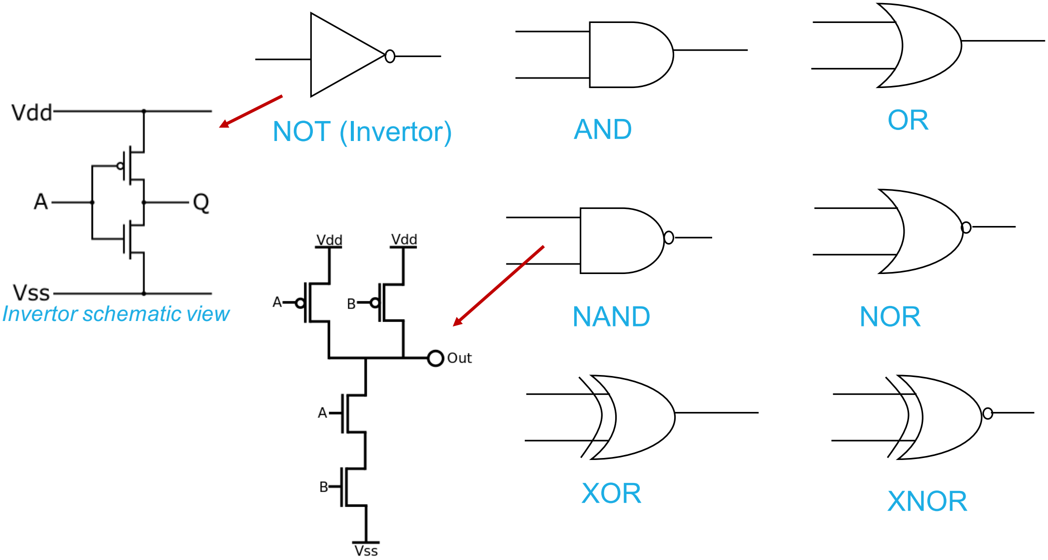
```
  0111    ->      0111
- 0110    ->    + 1010
------          -------
```

- Overflow (result too large for finite computer word). E.g., adding two n-bit numbers does not yield an n-bit number

```
  0111
+ 0001
------
```

Logic Gates (Optional)

Logic Gates



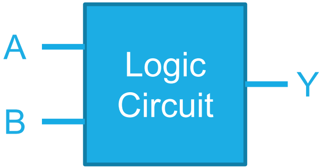
- What is the schematic view of an AND gate?

EX: L02-3

- Please draw NOR gate schematic view

Truth Table

- A means for describing how a logic circuit's output depends on the logic levels present at the circuit's inputs
- The number of input combinations will equal 2^N for an N-input truth table



Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

EX: L02-4

- Determine the true table of a three-input AND gate