# CENG4480 Homework 1

**Due**: Oct. 21, 2018

- **Small-Signal Gain:** For given amp circuits, small changes of input $\Delta V_{in}$ will cause output change of $\Delta V_{out}$. Small-signal gain is defined by $\frac{\Delta V_{out}}{\Delta V_{in}}$.

**Q1** (10%) Given a non-inverting amplifier as shown in Figure 1, $R_1 = 3R_2$ and $A_0 = 1000$, calculate the exact finite gain. Then determine the gain difference if the circuit is expected to have an ideal gain under $A_0 = \infty$.

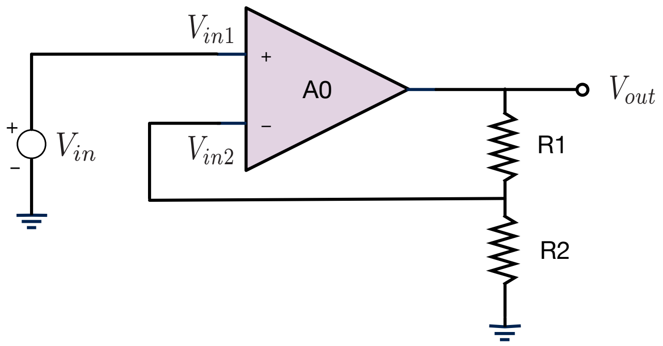


Figure 1: Non-inverting Amplifier.

**A1** From the proterties of Op Amplifier,

$$V_{out} = A_0(V_{in1} - V_{in2}) \tag{1}$$

Given that,

$$V_{in2} = \frac{R_2}{R_1 + R_2} V_{out} \tag{2}$$

Substituting into (1) we have,

$$G_{real} = \frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \frac{R_2}{R_1+R_2} A_0} \tag{3}$$

Besides,

$$G_{ideal} = (1 + \frac{R_1}{R_2}) \tag{4}$$

Substituting data into Eqs. (3) and (4),

$$G_{real} = 3.98, G_{ideal} = 4 \tag{5}$$

Thus, real circuit gain has a $0.5\%$ difference from ideal gain.

**Q2** (10%) An op-amp exhibits the following nonlinear characteristic:

$$V_{out} = \alpha \arctan[\beta(V_{in1} - V_{in2})]. \tag{6}$$

Determine the small-signal gain of the op amp in the case $V_{in1} \approx V_{in2}$. (Hint: use Taylor expansion of arctan and definition of aforementioned small-signal gain.)

**A2** Taylor expansion of arctan has the following form:

$$\arctan(Z) = Z - \frac{1}{3}Z^3 + \frac{1}{5}Z^5 - ... \tag{7}$$

When $V_{in1} \approx V_{in2}$,

$$V_{out} \approx \alpha\beta(V_{in1} - V_{in2}) \tag{8}$$

Thus, small-signal gain is,

$$\frac{dV_{out}}{d(V_{in1} - V_{in2})} = \alpha\beta \tag{9}$$

**Q3** (10%) In the circuit of Figure 2, $R_1 = R_2 = R' = R_f = R = 100\text{k}\Omega$ and $C = 1\mu\text{F}$. Assume the op-amps are ideal.

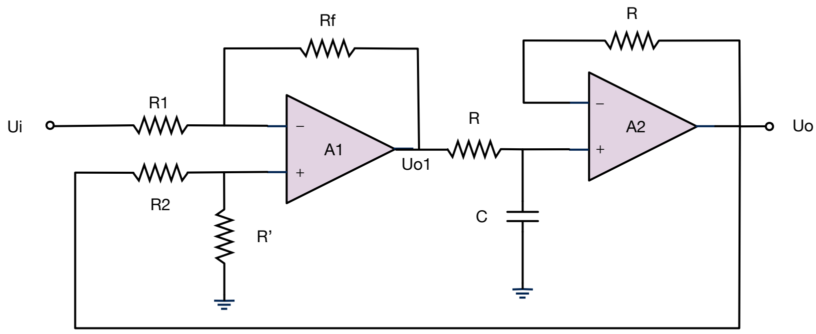


Figure 2: Voltage Follower.

1. (6%) The relationship between $U_i$ and $U_o$ ($U_{o1}$ is unknown).
2. (4%) Assume that when the time $t = 0$, $U_o = 0V$ and $U_i$ jumps from $0V$ to $-1V$. How long will the $U_o$ take to change from $0V$ to $6V$?

**A3** 1. Because $A_2$ is a voltage follower ( the voltage on the capacitance $U_c = U_o$) , so we can obtain $U_{o1}$

$$U_{o1} = -\frac{R_f}{R_1}U_i + \left(1 + \frac{R_f}{R_1}\right)\frac{R'}{R_2 + R'}U_o = U_o - U_i \tag{10}$$

The voltage on the capacitance $U_c = U_o$, so the current of the capacitance is

$$i_C = \frac{U_{o1} - Uo}{R} = -\frac{U_i}{R} \tag{11}$$

And

$$U_o = \frac{1}{C}\int_{-\infty}^{t} i_c \text{d}t = -\frac{1}{RC}\int_{-\infty}^{t} U_i \text{d}t = -10\int_{-\infty}^{t} U_i \text{d}t \tag{12}$$

2. We can get $U_o = -10\int_0^t U_i \mathrm{d}t = 10t = 6$. So $t = 0.6sec$

**Q4** (15%) Determine the output voltage (i.e. the mathematical expression of $V_{out}(t)$) for the integrator circuit of Figure 3a if the input is a square wave of amplitude $\pm A$ and period $T$ shown in Figure 3b. Assume $T = 10ms$, $C_F = 1\mu F$, $R_s = 10k\Omega$ and ideal op-amp. The square wave starts at $t = 0$ and therefore $V_{out}(0) = 0$.

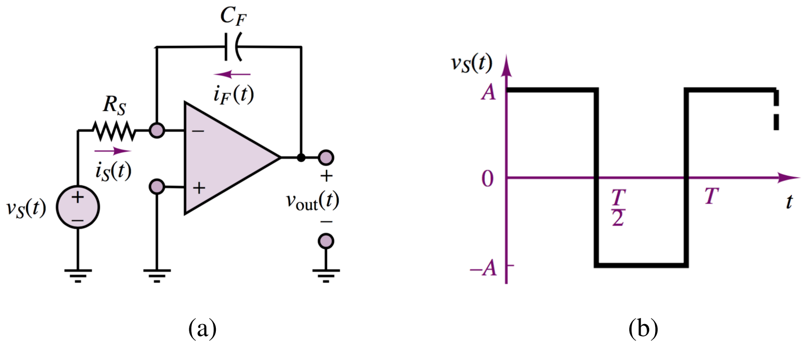


Figure 3: (a) Op-amp integrator; (b) Input of a square wave.

**A4** We write the expression for the output of the integrator:

$$\begin{aligned} v_{out}(t) &= -\frac{1}{R_s C_F}\int_{-\infty}^{t} v_s(t')dt' \\ &= -\frac{1}{R_s C_F}\left(\int_{-\infty}^{0} v_s(t')dt' + \int_{0}^{t} v_s(t')dt'\right) \\ &= -\frac{1}{R_s C_F}\int_{0}^{t} v_s(t')dt'. \end{aligned} \tag{13}$$

Next, we note that we can integrate the square wave in a piecewise fashion by observing that $v_s(t) = A$ for $0 \le t < T/2$ and $v_s(t) = -A$ for $T/2 \le t < T$. We consider the first half of the waveform:

$$\begin{aligned} v_{out}(t) &= -\frac{1}{R_s C_F}\int_{0}^{t} v_s(t')dt' \\ &= -100At \qquad 0 \le t < T/2, \end{aligned} \tag{14}$$

and

$$\begin{aligned} v_{out}(t) &= v_{out}(\frac{T}{2}) - \frac{1}{R_s C_F}\int_{T/2}^{t} v_s(t')dt' \\ &= -100A\frac{T}{2} + 100A(t - \frac{T}{2}) \\ &= -100A(T - t) \qquad T/2 \le t < T. \end{aligned} \tag{15}$$

Since the waveform is periodic, the above result will repeat with period $T$, as shown in Figure 4.

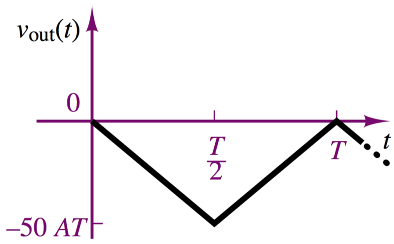


Figure 4: The sketch of output of integrator.

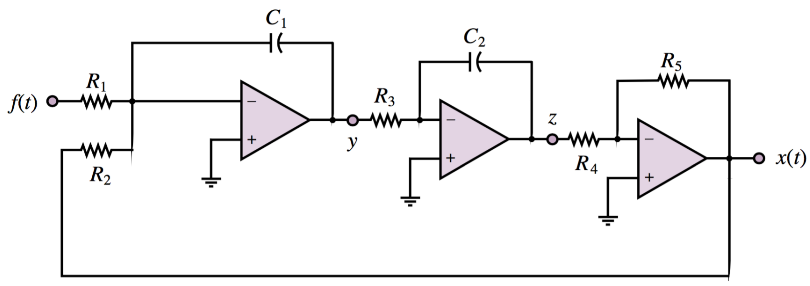


Figure 5: Analog computer simulation of unknown system.

**Q5** (20%) Assume op-amps are ideal. Given $R_1 = 0.4\text{M}\Omega$, $R_2 = R_3 = R_5 = 1\text{M}\Omega$, $R_4 = 2.5\text{k}\Omega$ and $C_1 = C_2 = 1\mu F$, derive the differential equation corresponding to the analog computer simulator of Figure 5, i.e. the mathematical relationship between $f$ and $x$. Note that $f(t)$ is input signal, y and z are outputs of corresponding op-amps.

**A5** We start the analysis from the right-hand side of the circuit, to determine the intermediate variable $z$ as a function of $x(t)$:

$$x = -\frac{R_5}{R_4}z = -400z. \tag{16}$$

Next, we move to the left to determine the relationship between y and z:

$$z = -\frac{1}{R_3C_2}\int y(t')dt', \tag{17}$$

or

$$y = -\frac{dz}{dt}. \tag{18}$$

Finally, we determine y as a function of x and f:

$$\begin{aligned} y &= -\frac{1}{R_2C_1}\int x(t')dt' - \frac{1}{R_1C_1}\int f(t')dt' \\ &= -\int [x(t') + 2.5f(t')]dt', \end{aligned} \tag{19}$$

or

$$\frac{dy}{dt} = -x - 2.5f. \tag{20}$$

Substituting the expressions into one another and eliminating the variables y and z, we obtain the differential equation in x:

$$\begin{aligned} x &= -400z \\ \frac{dx}{dt} &= -400\frac{dz}{dt} = 400y \\ \frac{d^2x}{dt^2} &= 400\frac{dy}{dt} = -400(x + 2.5f). \end{aligned} \tag{21}$$

**Q6** (10%) Let us consider the Schmitt Trigger shown in Figure 6

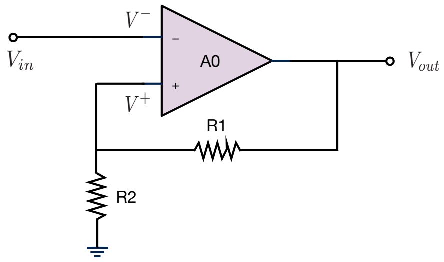


Figure 6: Schmitt Trigger.

1. (5%) Due to the manufacturing defects, a parasitic resister $R_3$ occurs between the output node and ground, calculate the reference voltages.
2. (5%) If the parasitic device is a capacitor $C$, sketch $v_{out}$ versus $v_{in}$. Label the key coordinates on the curve.

**A6** 1. According to the properties of comparator, when $v_{in}$ is small, $v_{out} = v_{sat}$ and

$$\frac{v^+}{R_2} = \frac{v_{out} - v^+}{R_1}, \tag{22}$$

i.e.,

$$v^+ = \frac{R_2}{R_1 + R_2} v_{sat}. \tag{23}$$

Similarly, if $v_{in}$ is large, we have

$$v^+ = -\frac{R_2}{R_1 + R_2} v_{sat}. \tag{24}$$

Therefore two reference voltages are given by $\frac{R_2}{R_1+R_2} v_{sat}$ and $-\frac{R_2}{R_1+R_2} v_{sat}$.

2. $v_{out}$ start to change when $v_{in}$ reaches references above. However, due to the existing capacitor, voltage cannot change immediately (Changes fast, then slowly).

**Q7** (10%) Compute and sketch the output voltage of the op-amp in Fig. 8. Given $R_S = 1\text{k}\Omega$, $R_F = 10\text{k}\Omega$, $R_L = 1\text{k}\Omega$, $V_S^+ = 15\text{V}$, $V_S^- = -15\text{V}$, $v_s(t) = 2\sin(1000t)$. Repeat the problem if $V_S^+ = 20\text{V}$ and $V_S^- = -20\text{V}$. Assume the op-amp is supply voltage-limited.

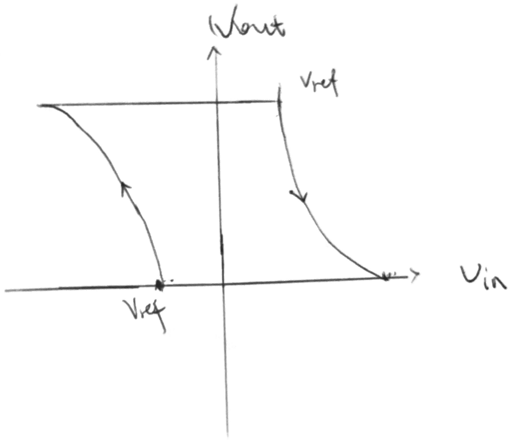


Figure 7: A6(2).

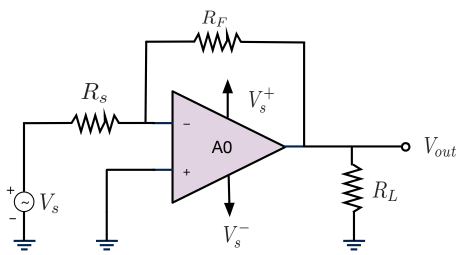


Figure 8: Inverting Amplifier.

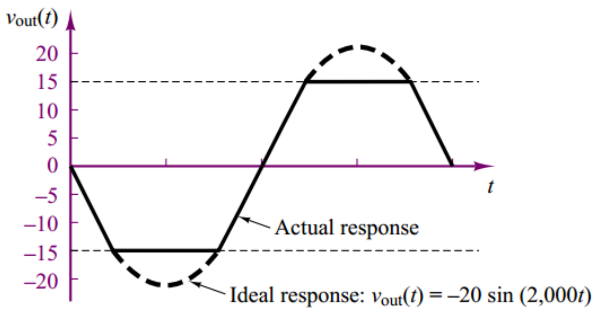


Figure 9: A7. Sketch of output voltage.

**A7** The result will be the solid curve with $V_S^+ = 15$V and $V_S^- = -15$V (actual response); If $V_S^+ = 20$V and $V_S^- = -20$V, the result will be dashed curve (ideal response).

**Q8** (15%) Determine the closed-loop voltage gain as a function of frequency (i.e. $A(j\omega) = \frac{V_{out}(j\omega)}{V_s(j\omega)}$) for the op-amp circuit of Fig. 10. Assume the op-amp is ideal. Given only $R_1$, $R_2$ and $\omega_0$, $R_2C = \frac{L}{R_1} = \omega_0$. (Hint: the impedance of a inductor $L$ equals to $j\omega L$. )

**A8** The expression for the gain of the filter of the filter of Fig. 10 can be determined by using

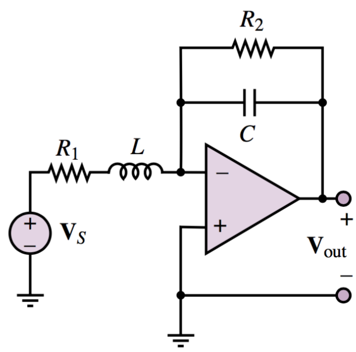


Figure 10: A second-order low-pass filter.

equation:

$$A(j\omega) = \frac{V_{out}(j\omega)}{V_s(j\omega)} = -\frac{Z_F(j\omega)}{Z_s(j\omega)}, \tag{25}$$

where

$$Z_F(j\omega) = R_2 \,||\, j\omega C = \frac{R_2}{1 + j\omega C R_2} = \frac{R_2}{1 + j\omega\omega_0}, \tag{26}$$

and

$$Z_s(j\omega) = R_1 + j\omega L = R_1 (1 + j\omega\omega_0). \tag{27}$$

Thus, the gain of the filter is:

$$A(j\omega) = -\frac{R_2}{R_1}\frac{1}{(1 + j\omega\omega_0)^2}. \tag{28}$$