Spatial Sigma Delta Modulation for Quantized MIMO Precoding

Wai-Yiu Keung

The Chinese University of Hong Kong

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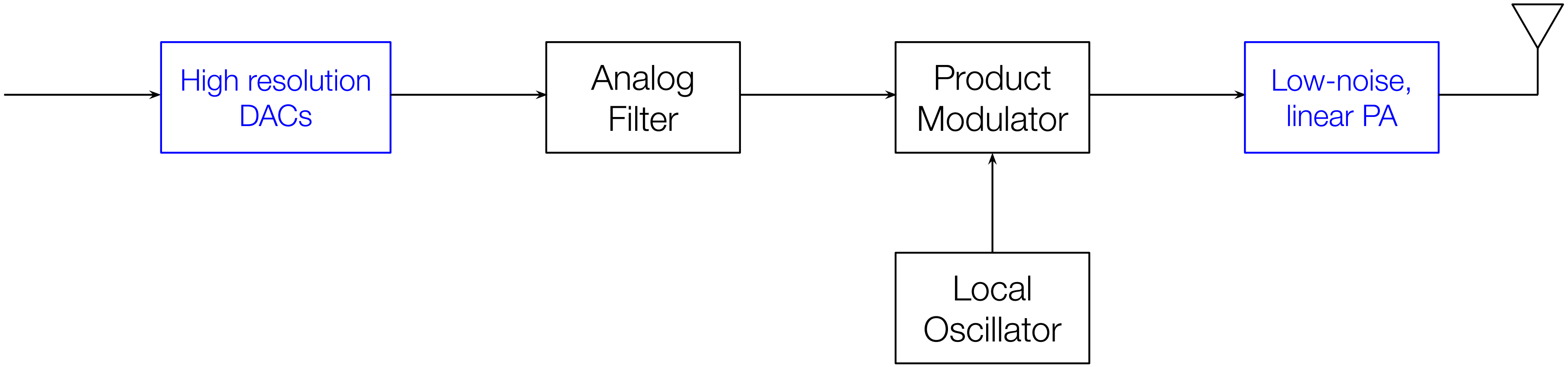
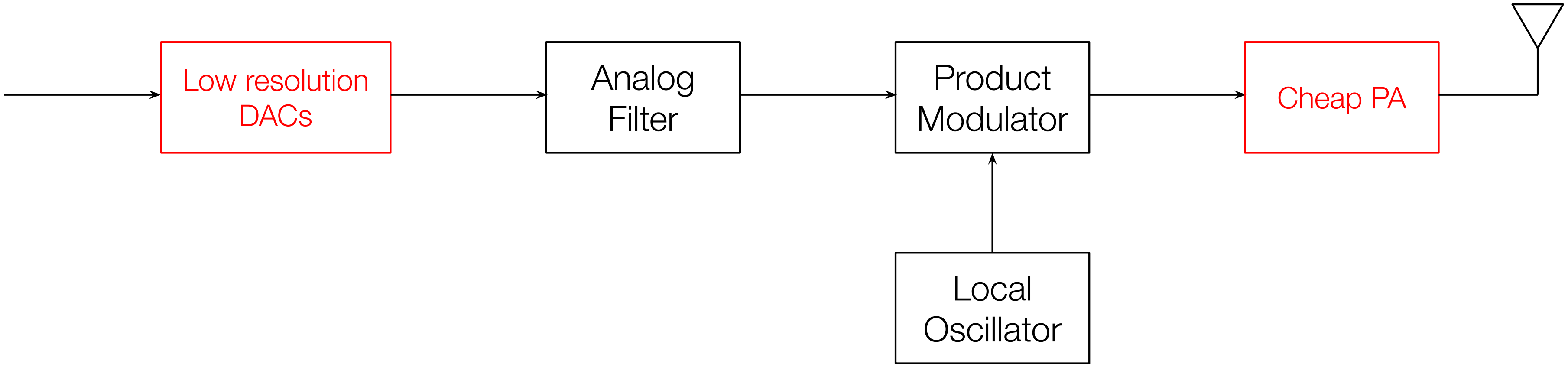
Motivation — The RF Chain in Massive MIMO Downlink



- conventional MIMO precoding assumes the BS to emit a free-space signal vector
- but it is costly to implement such high resolution digital-to-analog converters, particularly in the context of **massive** MIMO
- and you need a massive amount of power to operate with them in massive MIMO...

Motivation — The RF Chain in Massive MIMO Downlink



- cheap replacement: use low resolution DACs to allow the use of cheaper power amp.; this allows an economic build of the massive MIMO transmitter

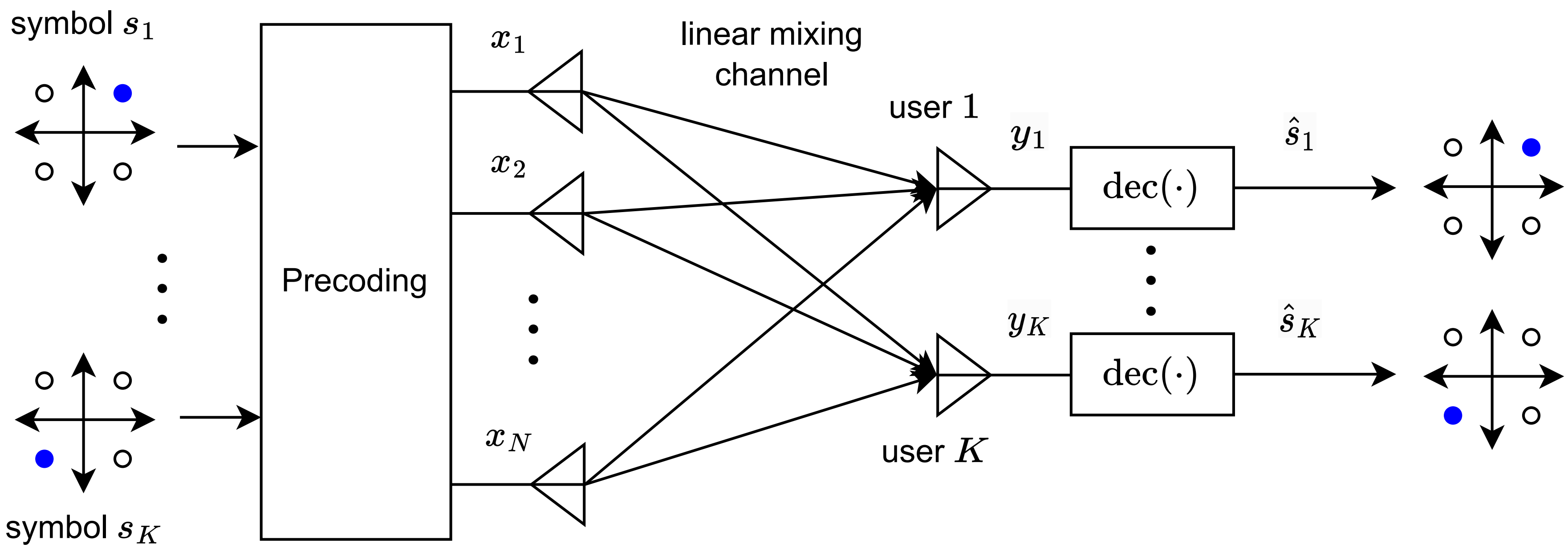
- this calls for researches in quantized MIMO precoding

  - linear method: apply direct quant. on existing/conventional precoders (e.g. ZF); computationally efficient but ineffective performance-wise **[MGN09, SFS17, DCJM$^+$19]**

  - non-linear method: re-design the signal vector using opt.; generally has better data accuracy but requires much more computation **[SSMF17, LMLS18]**

- **$\Sigma\Delta$ precoding: a linear method that offers reasonable performance! [SMLS19]**

Scenario: One-Bit Quantized Precoding



- we consider multiuser MISO downlink system where

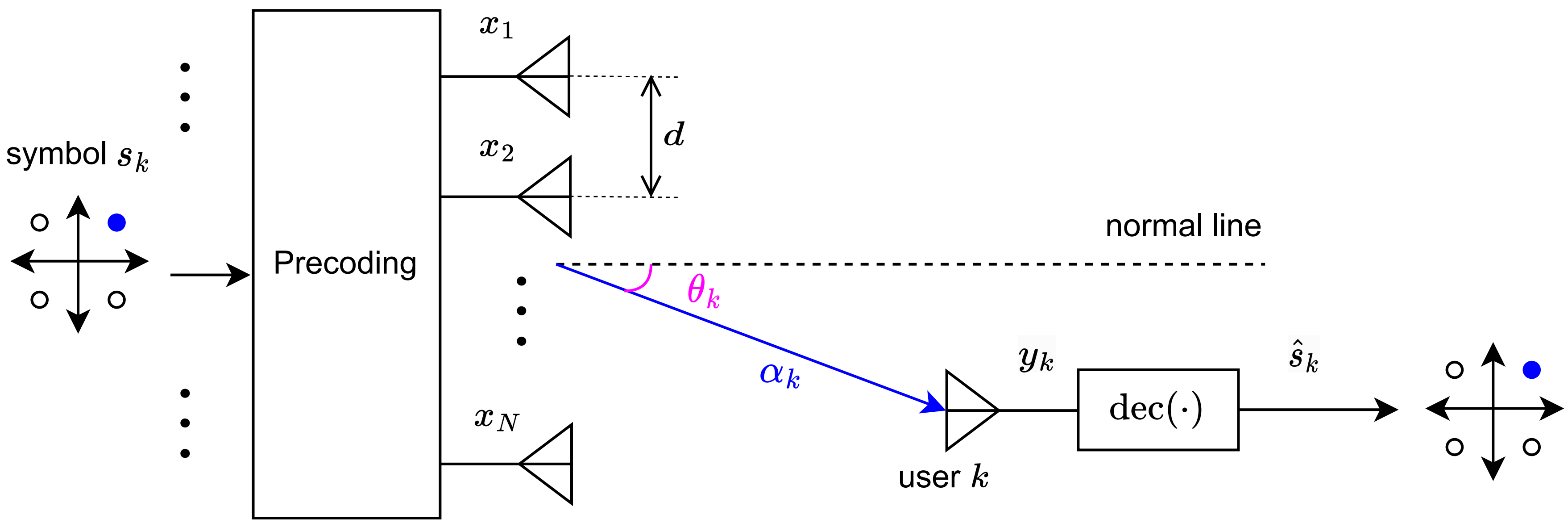
$$y_k = \boldsymbol{h}_k^\top \boldsymbol{x} + v_k, \qquad k = 1, \ldots, K,$$

  is the received symbol; $\boldsymbol{h}_k$ is the channel gain vector; v_k is AWGN with power σ^2

- problem: given $\boldsymbol{h}_k$ and a dedicated transmission symbol s_k at the base station, design a signal vector $\boldsymbol{x} \in \mathcal{X}^N = \{\pm 1 \pm \mathrm{j}\}^N$ such that the received symbol $y_k \approx c_k \cdot s_k$

- we will extend $\mathcal{X}$ to general case later on

Assumption: Uniform Linear Array



- we will assume a specific channel model — the uniform linear array (ULA):

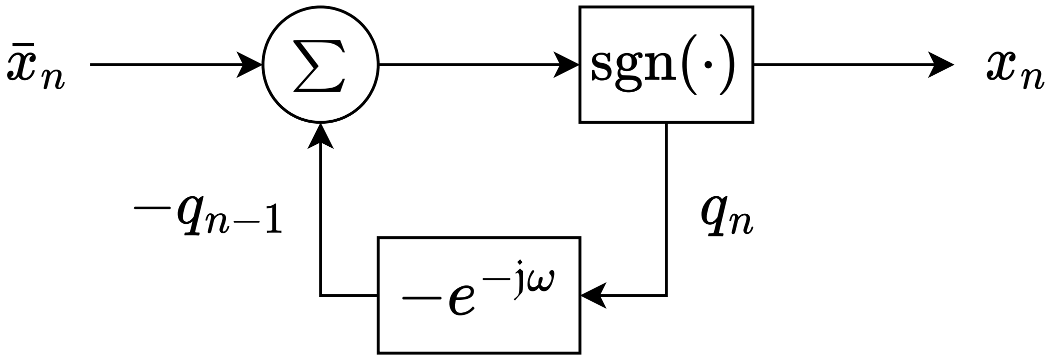
$$\boldsymbol{h}_k = \alpha_k \boldsymbol{a}_k, \qquad \boldsymbol{a}_k = (0, e^{-\mathrm{j}\omega_k}, \ldots, e^{-\mathrm{j}\omega_k(N-1)}), \qquad \omega_k = \tfrac{2\pi d}{\lambda} \sin(\theta_k)$$

where α_k is the channel gain, $\boldsymbol{a}_k$ is the steering vector, θ_k is the angle of departure, and ω_k is the spatial frequency; d is the antenna dist. and λ is the wavelength used

- observation: the rx signal model turns into a discrete time Fourier transform like form

$$y_k = \alpha_k \cdot \boldsymbol{a}_k^\top \boldsymbol{x} = \alpha_k \sum_{n=0}^{N-1} x_n e^{-\mathrm{j}\omega_k n}$$

Temporal Sigma Delta Modulation



- to begin with, we take one step back to study a classical DAC: $\Sigma\Delta$ modulator [ASVDS96]
- principle: given a high resolution sequence $\bar{x}_n$, generate one-bit sequence x_n by

$$x_n = \mathrm{sgn}(\bar{x}_n - q_{n-1}) = \bar{x}_n - q_{n-1} + q_n$$

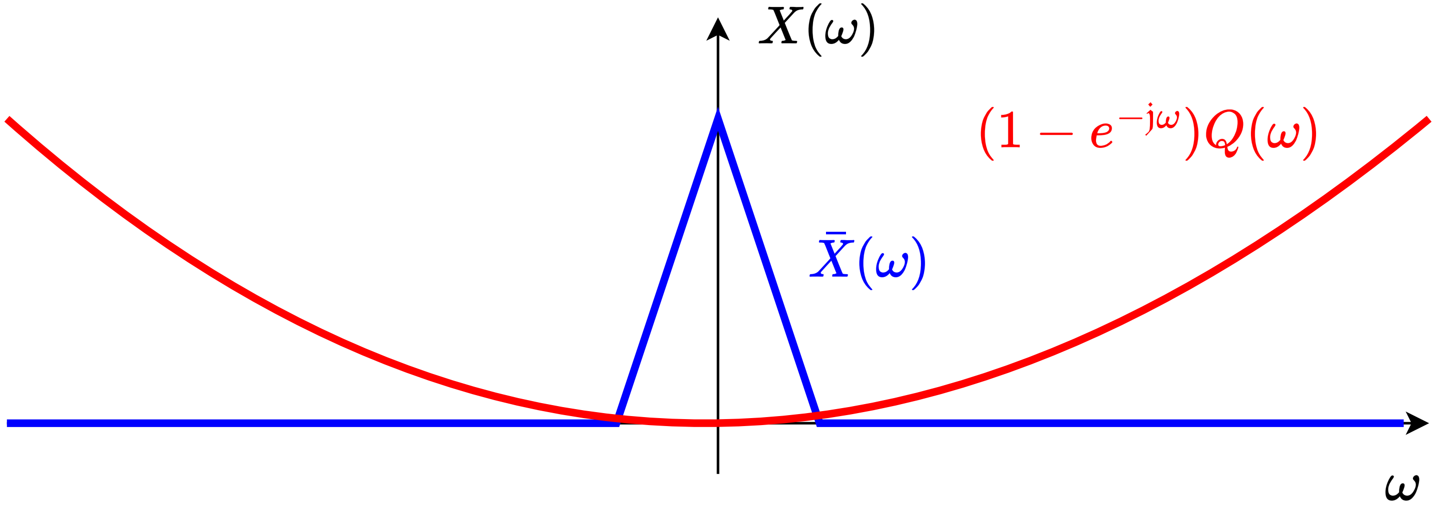
  where q_n is the quant. error incurred by the one-bit quantizer $\mathrm{sgn}(\cdot)$

- observation: the DTFT of x_n follows:

$$\underbrace{X(\omega)}_{\text{one-bit output}} = \underbrace{\bar{X}(\omega)}_{\text{full res. input}} + \underbrace{(1 - e^{-j\omega})}_{\text{HPF}} \underbrace{Q(\omega)}_{\text{quant. error}}$$

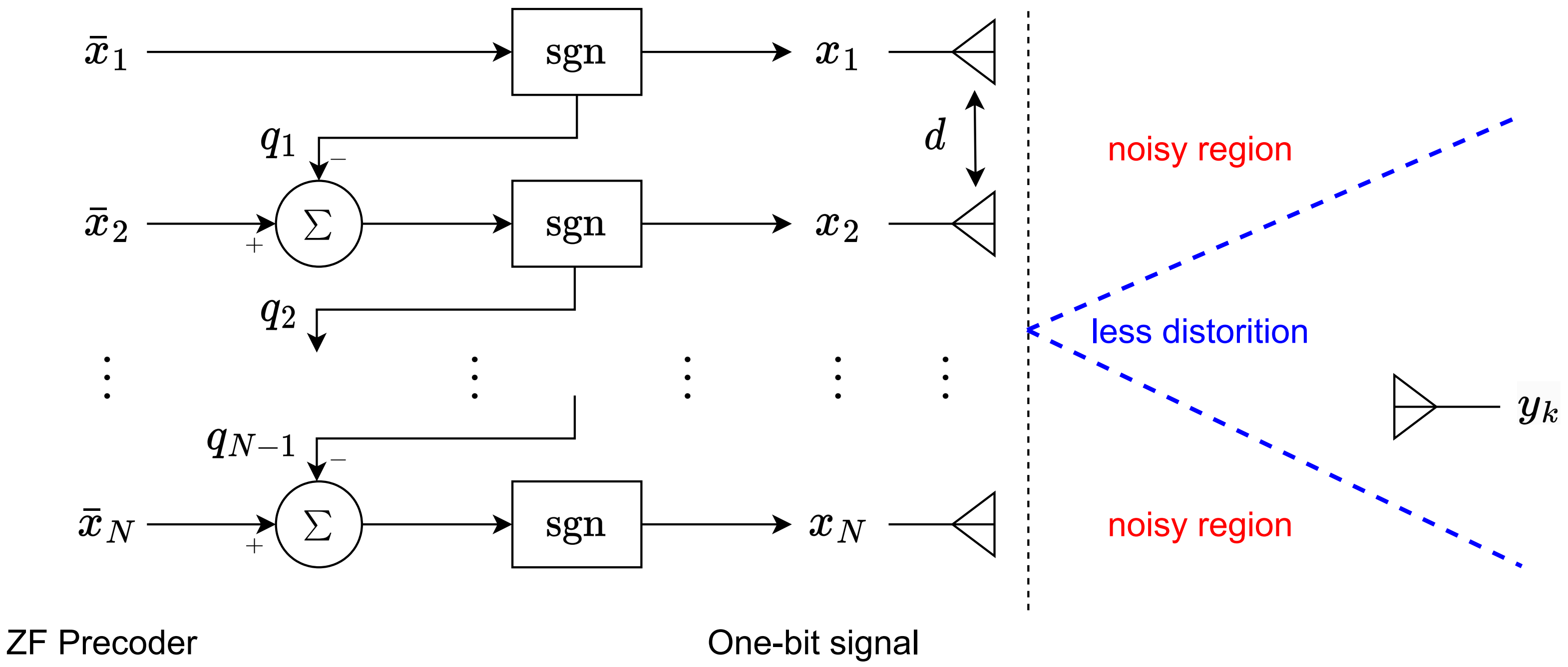
ΣΔ Principle: A Spectrum Illustration

$$\underbrace{X(\omega)}_{\text{one-bit output}} = \underbrace{\bar{X}(\omega)}_{\text{full res. input}} + \underbrace{(1 - e^{-j\omega})}_{\text{HPF}} \underbrace{Q(\omega)}_{\text{quant. error}}$$



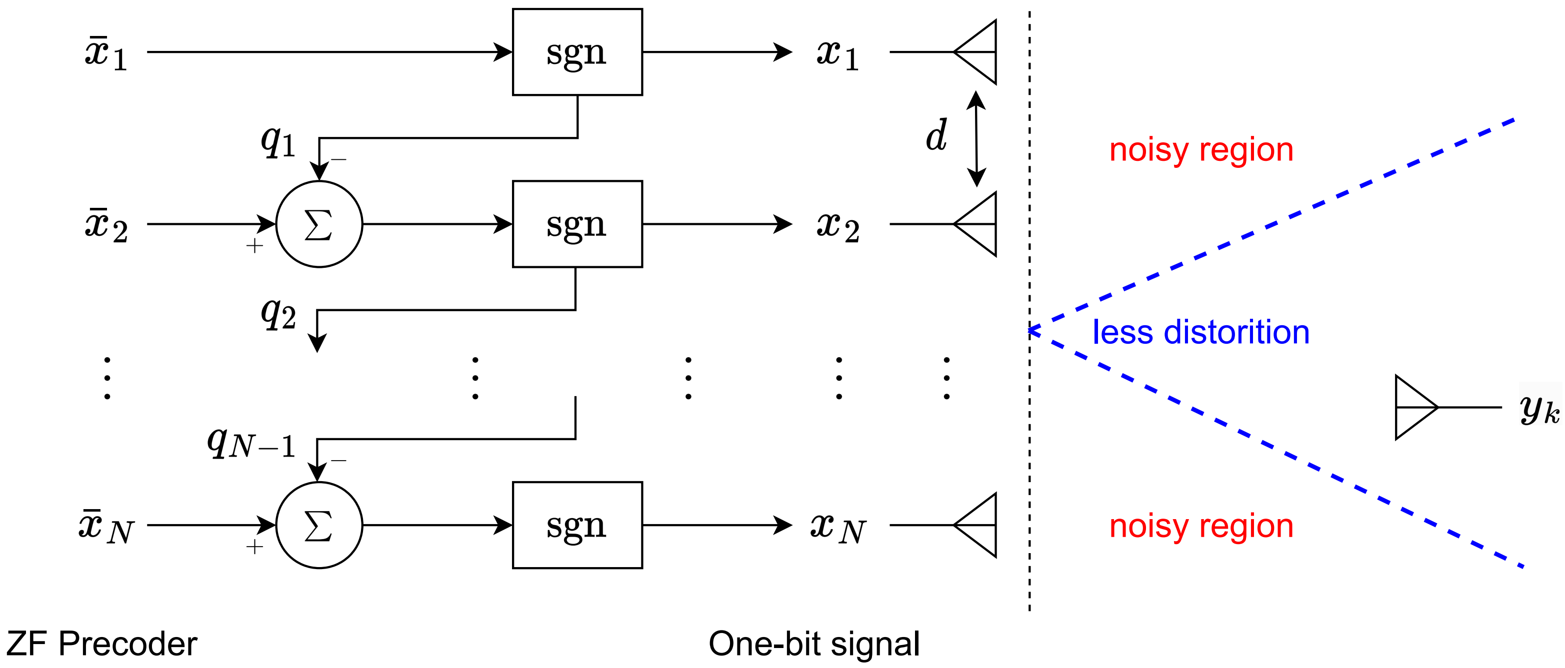
- assumptions: i) $\bar{X}(\omega)$ is low-pass and ii) $Q(\omega)$ is bounded and flat
- observation: quant. noise is shaped toward the high-pass region
- implication: apply low-pass filter to recover the full res. $\bar{x}_n$ from the one-bit signal x_n

Spatial $\Sigma\Delta$ Modulator in MIMO Precoding [SMLS19]



- putting $\Sigma\Delta$ to MIMO precoding we observe the following duality
  - signal at the time index n = tx. signal at the n-th antenna element
  - error feedback in temp. $\Sigma\Delta$ = passing quant. error to the next antenna element
  - LPF in temp. $\Sigma\Delta$ = restrict users to lie in low angular region

Spatial ΣΔ Modulator in MIMO Precoding [SMLS19]



- received signal model (when $\alpha_k = 1$):

$$
\begin{aligned}
y_k &= \textstyle\sum_{n=0}^{N-1} (\bar{x}_n + q_n - q_{n-1}) e^{-j\omega_k n} \\
&= \left[\textstyle\sum_{n=0}^{N-1} \bar{x}_n e^{-j\omega_k n}\right] + \left[\textstyle\sum_{n=0}^{N-1} (q_n - q_{n-1}) e^{-j\omega_k n}\right] \\
&\approx \bar{X}(\omega) + (1 - e^{-j\omega_k}) Q(\omega) \qquad \text{(holds when } N \text{ is large)}
\end{aligned}
$$

- recall $\omega_k = \frac{2\pi d}{\lambda} \sin(\theta_k)$, this means the red term zeros out when $\theta_k = 0^\circ$

Some Technical Remarks

- recall that we made two assumptions on slide 7, namely, i) $\bar{X}(\omega)$ is low-pass and ii) $Q(\omega)$ is bounded and flat

- simply put, i) is done by restricting $|\theta_k|$ in a small angular region that is close to 0°, so that $\omega_k = \frac{2\pi d}{\lambda}\sin(\theta_k)$ will also be small

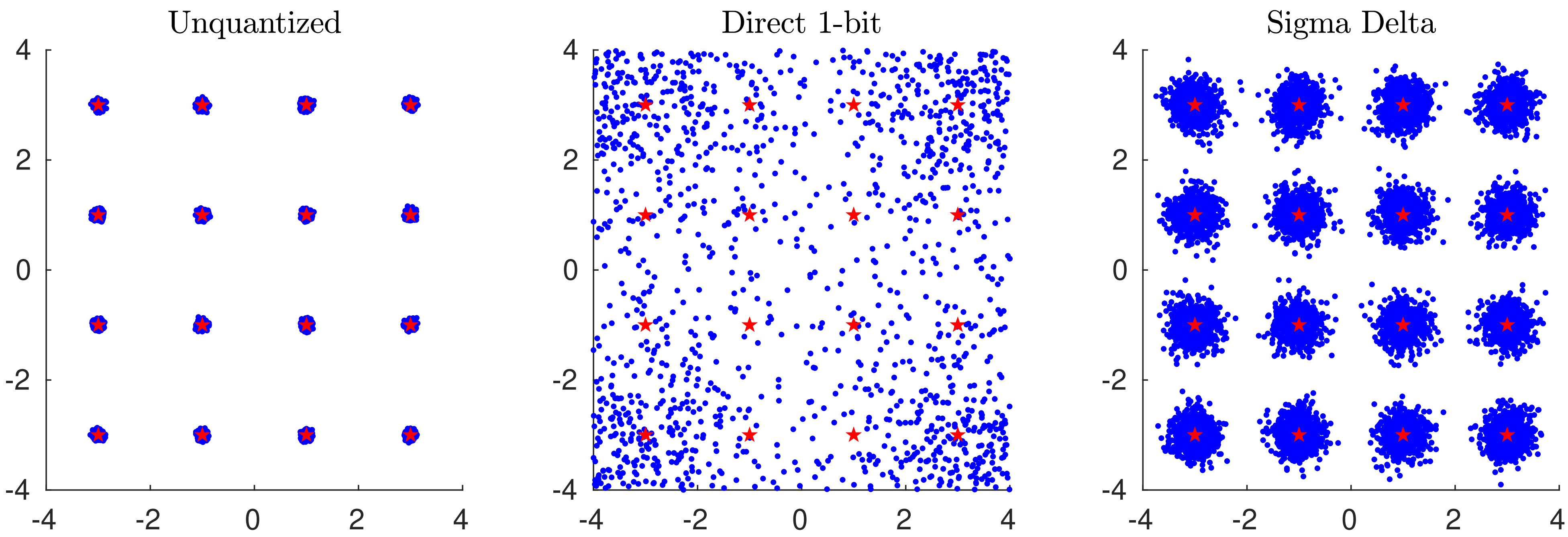
- as for ii), we use —

  – **no-overload condition**: avoid $q_n \to \infty$ by limiting $|\bar{x}_n| \leq 1$ (which is easily done by normalization); as a result we have $|q_n| \leq 1$, i.e. $Q(\omega)$ is bounded

  – **assumption**: under the above condition, we further assume q_n is uniformly i.i.d. over $[-1, 1]$ and is independent of $\bar{x}_n$, i.e. $Q(\omega)$ is flat

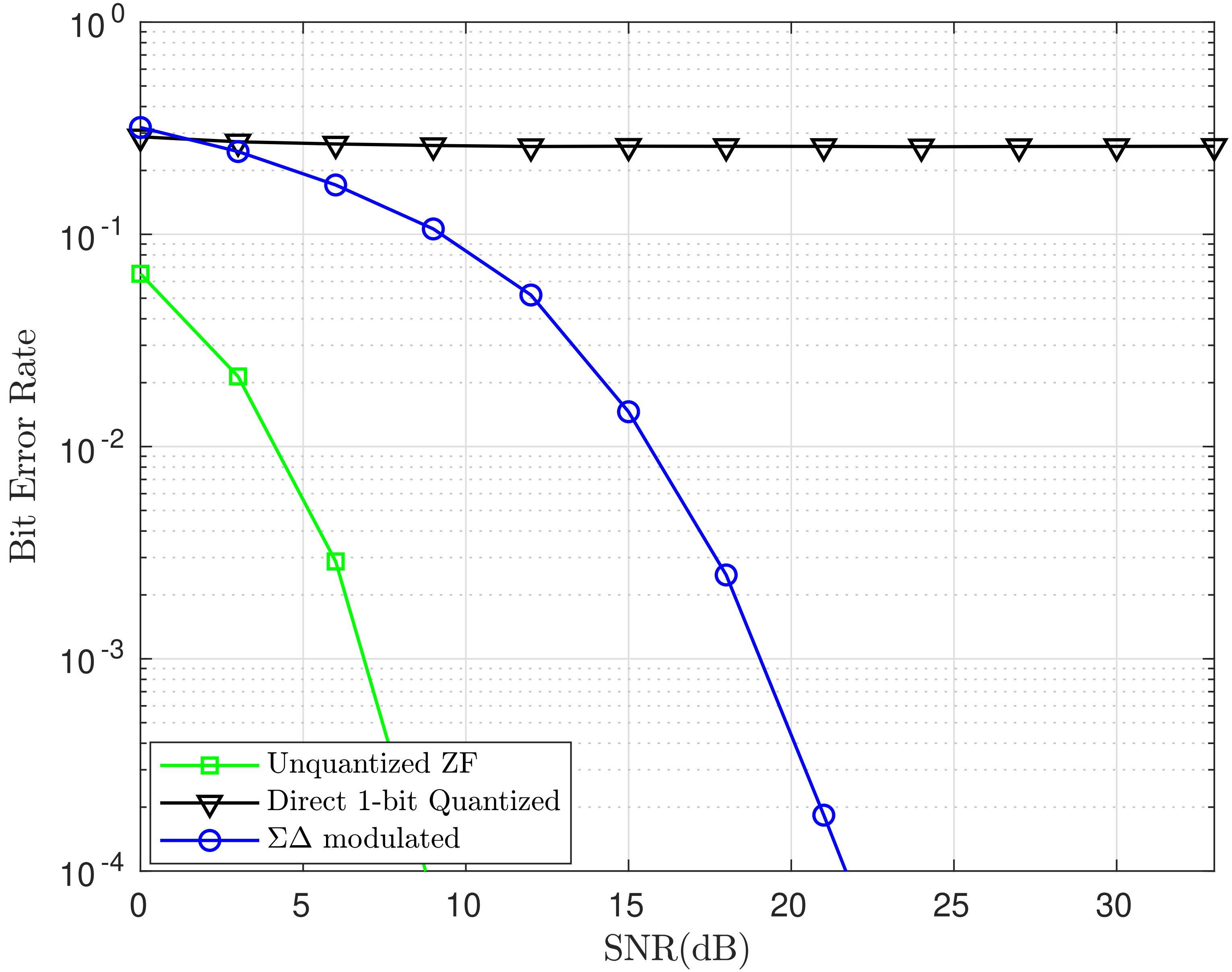
- technically speaking the assumption is wrong because q_n is dependent on $\bar{x}_n$; one way to overcome this violation is to use dithering

Simulation: Scatter Plot



- we demonstrate the effectiveness of $\Sigma\Delta$ precoding by observing the scatter plot
- red stars $\star$ are the 16-QAM constellation points; blue dots $\bullet$ are the normalized received symbols prior to being put forth to the detector y_k/α_k
- settings: $N = 512$ Tx antenna; $K = 12$ users with $\theta_k \in [-30^\circ, 30^\circ]$; the antenna spacing is set as $d = \lambda/8$; the background SNR is fixed to 20dB

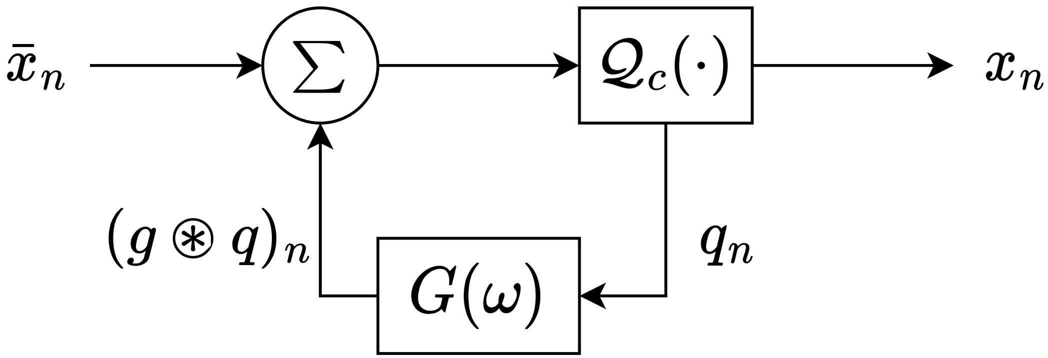
Simulation: Bit Error Rate Performance



Inspirations Taken

- what we already know:
  - spatial $\Sigma\Delta$ modulation is a suitable candidate for 1-bit massive MIMO
  - if the BS adopts an uniform linear array, $\Sigma\Delta$ mod. pushes the quan. noise to the end-fire so that users near the broadside are less affected
- what we still don't know:
  - can we alter the noise-shaping effect to suit our need?
  - any systematical framework for the design of $\Sigma\Delta$ mod, even for multi-bit quantizer?
- we set out to continue our study on how to bake a design framework for $\Sigma\Delta$ precoder design (which turns out to be Chebyshev-like!)

A General ΣΔ Modulator



- $\Sigma\Delta$ modulator can be generalized in two senses:
  - the quantizer $\mathcal{Q}_c$ is a multi-level one, e.g. let $M = 4$ levels of quan., then the output domain is $x_n \in \mathcal{X} = \{\pm 1 \pm 1\mathrm{j}, \pm 1 \pm 3\mathrm{j}, \pm 3 \pm 1\mathrm{j}, \pm 3 \pm 3\mathrm{j}\}$
  - the noise-shaping filter $G(\omega)$ can be altered by choosing the filter tap coefficient vector $\boldsymbol{g} \in \mathbb{C}^D$, whereas D is the filter order

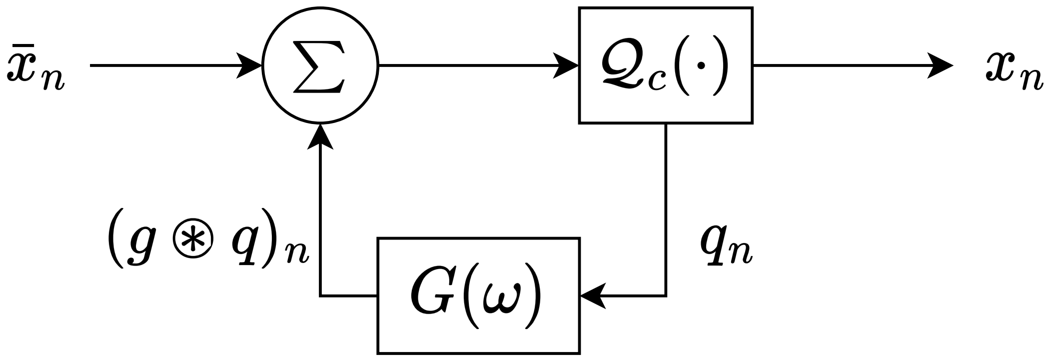
- end-to-end relation: $x_n = \bar{x}_n + \sum_{i=0}^{D} g_i q_{n-i}$, where we defined $g_0 = 1$.

- noise shaping effect: consider the DTFT

$$X(\omega) = \bar{X}(\omega) + (1 + G(\omega))Q(\omega)$$

is governed by the choice of $\{g_i\}_{i=1}^{D}$; i.e. we shape the quan. noise, according to our desire, using an order D FIR filter

A General ΣΔ Modulator: No-Overload Condition



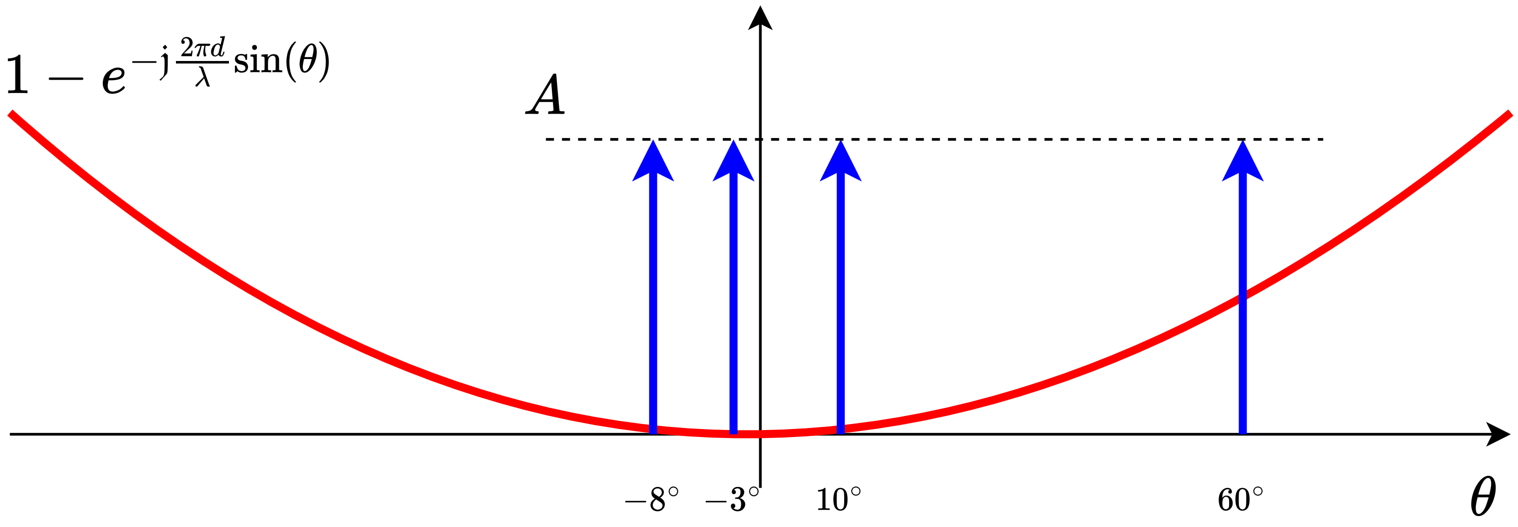
- no-overload condition: (partly inspired by **[SS91]**)

$$A + \sum_{i=1}^{D} |\Re(g_i)| + |\Im(g_i)| \le M$$

  - $A = \max_n\{|\Re(\bar{x}_n)|, |\Im(\bar{x}_n)|\}$ is the maximum input signal amplitude
  - M is the number of quantization levels of $\mathcal{Q}_c$

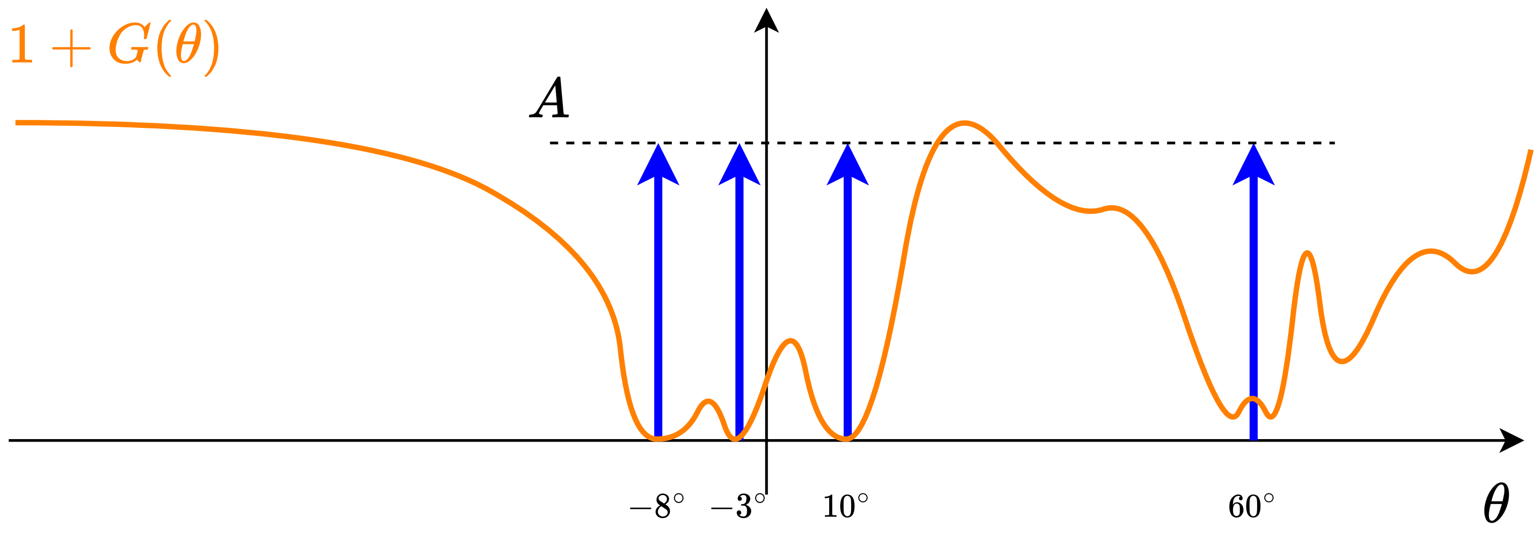
- example (first-order ΣΔ mod.): when $G(\omega) = -e^{-\mathrm{j}\omega}$, we have $D = 1$, and $g_1 = -1$; this implies $A \le M - 1$ must hold in order to satisfy the no-overload criterion

Design Strategy: Illustration of the Idea



- consider the $K = 4$ users locating in $[-8^\circ, -3^\circ, 10^\circ, 60^\circ]$ respectively
- first order $\Sigma\Delta$ mod. will work fine in suppressing quant. noise for the users at the low angle region; but the one locating at 60° suffers from a huge quant. noise
- **question:** can we customise the error filter to target our users?

Design Strategy: Illustration of the Idea



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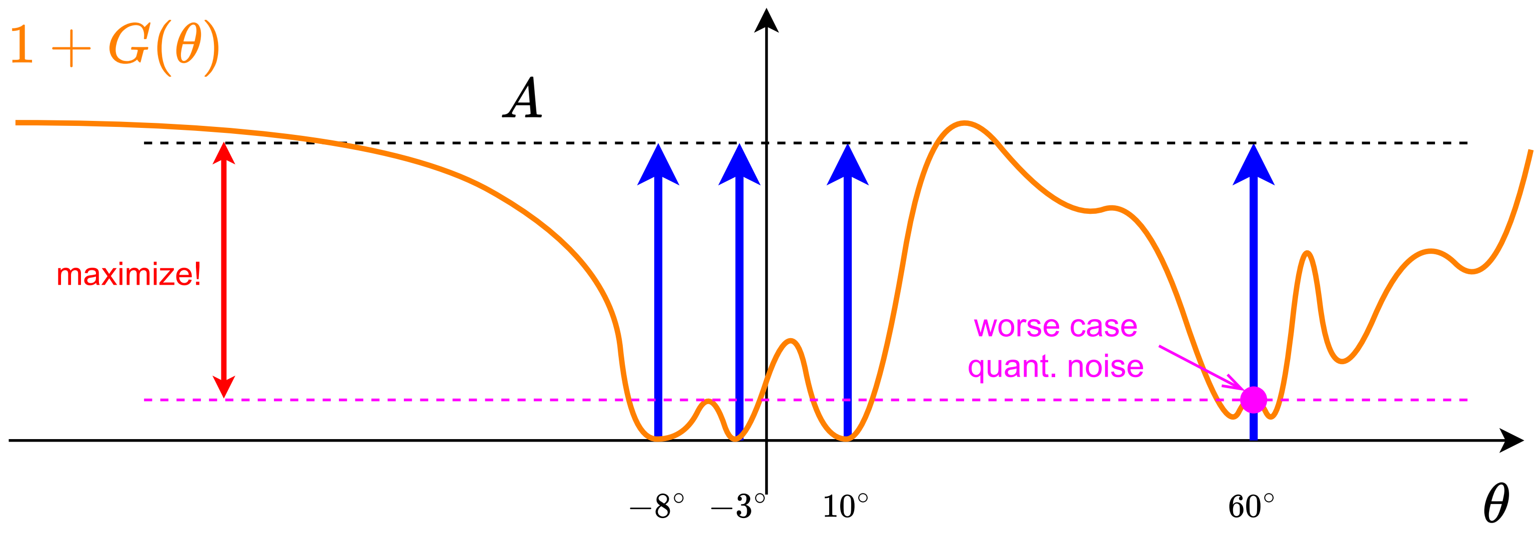
- first order $\Sigma\Delta$ mod. will work fine in suppressing quant. noise for the users at the low angle region; but the one locating at 60° suffers from a huge quant. noise
- question: can we customise the error filter to target our users?
- **idea:** design $G(\theta)$ by picking the error filter's coefficient $\boldsymbol{g} \in \mathbb{C}^D$ such that the signal-to-quantization-plus-noise ratio (SQNR) is maximized

Proposed Design



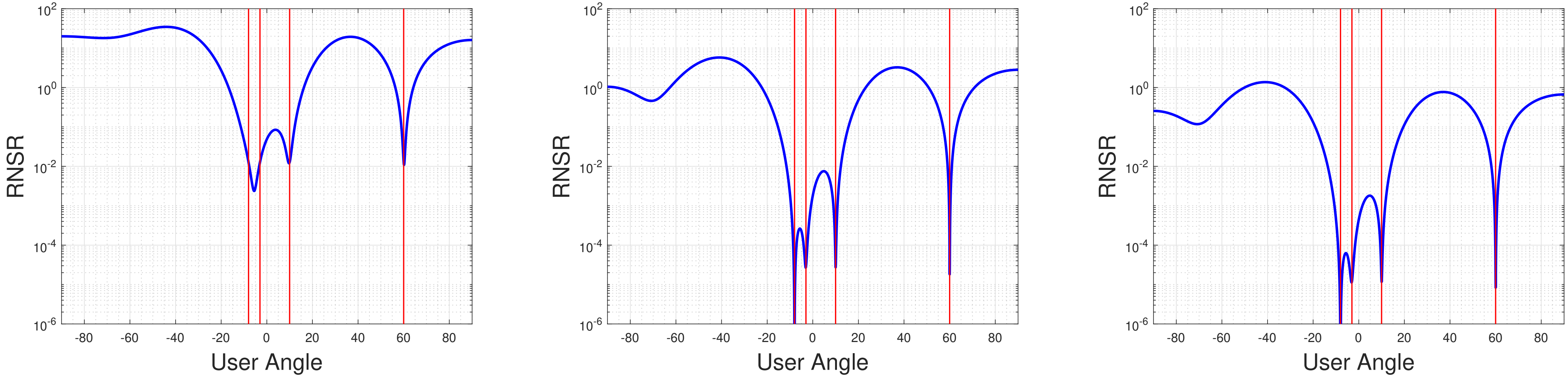
- the SQNR can be characterized as $\mathsf{SQNR}(\theta) = \frac{A^2}{c_1|1+G(\theta)|^2+c_0}$, where c_0, c_1 are const.
- we maximize $\mathsf{SQNR}(\theta)$ with $\boldsymbol{g}$ and A jointly

$$(\boldsymbol{g}^\star, A^\star) = \underset{\boldsymbol{g}\in\mathbb{C}^D, A\in\mathbb{R}_+}{\text{arg max}} \quad \min_{\theta\in\text{user's angle}} \quad \mathsf{SQNR}(\theta)$$

$$\text{subject to} \qquad \text{no overloading}$$

- best part: given the time constraint I can only ask for your trust that the problem can be transformed into a convex programme

Simulation: Resulting Noise-Shaping Response



$M = 3$ $\qquad$ $M = 4$ $\qquad$ $M = 5$

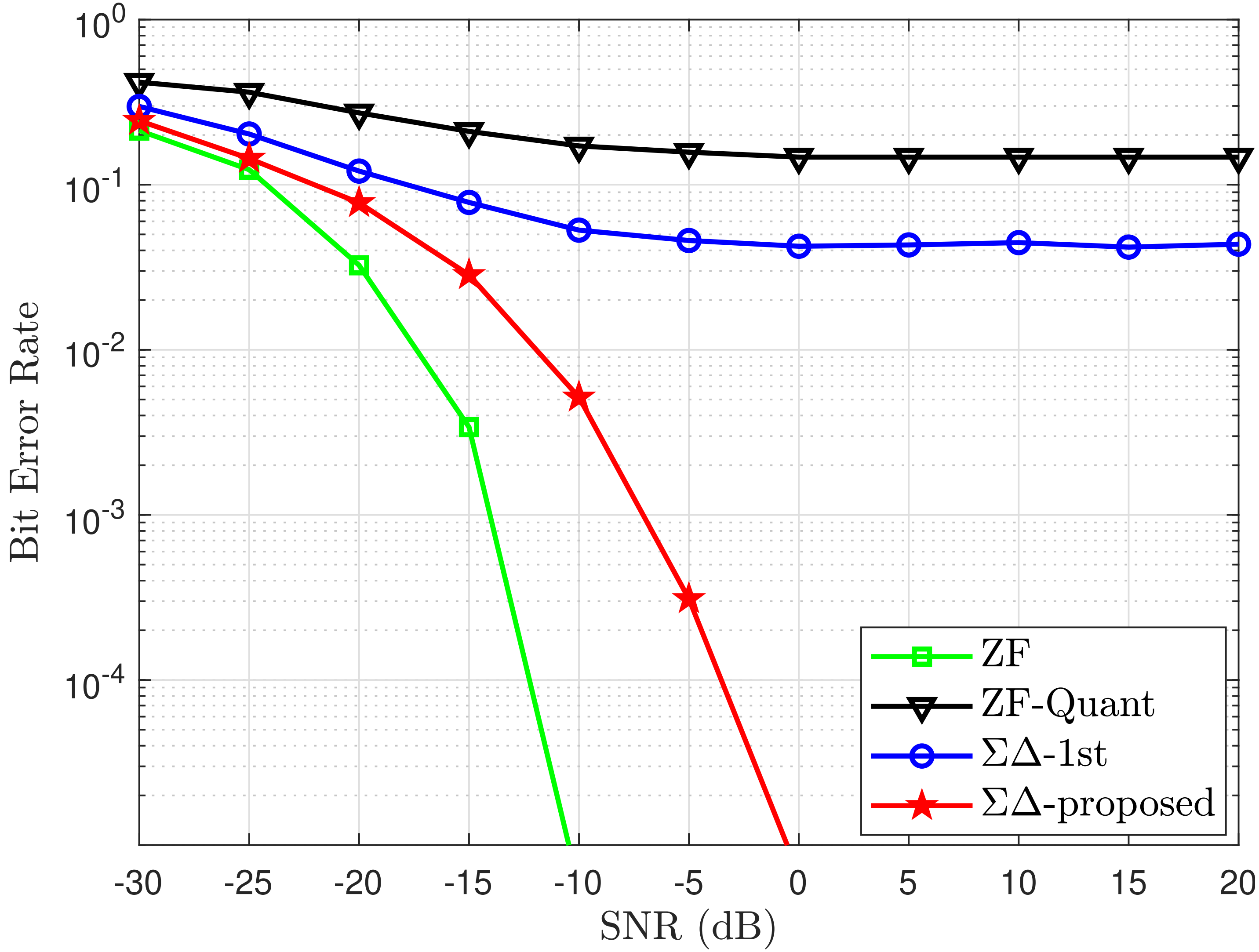
- problem size $N = 512, K = 4$, filter order $D = 8$, $d = \lambda/4$, all channel gains α_k's are fixed at unit gain; users are located in $[-8°, -3°, 10°, 60°]$

- we observe that the relative noise shaping response

$$\text{RNSR} = \frac{|1 + G(\theta)|^2}{A^2}$$

we see that more level of quantization gives better RNSR

Simulation: Bit Error Rate Performance



problem size $(N, K) = (1024, 6)$, filter order $D = 24$, $M = 4$ quant. levels, antenna spacing $d = \lambda/2$, channel gains are randomly generated, $\theta \in [-70^\circ, 70^\circ]$

Conclusions

- spatial $\Sigma\Delta$ modulation can be fitted into massive MIMO precoding efficiently and effectively
- the design of $\Sigma\Delta$ modulator in massive MIMO precoding can be turned into a filter design problem — which is convex
- simulation results showcase the advantage of our proposed design

That's all. Thank you!

Key References

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