On a Moreau Envelope Wirelength Model for Analytical Global Placement

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Outline

- We first illustrate our motivation in the block that briefly describes different wirelength models.
- We derive the representations of proximal mapping of the non-smooth HPWL function.
- The water-filling algorithm is applied to solve the proximal mapping and Moreau envelope problem.

Wirelength Models and Approximations

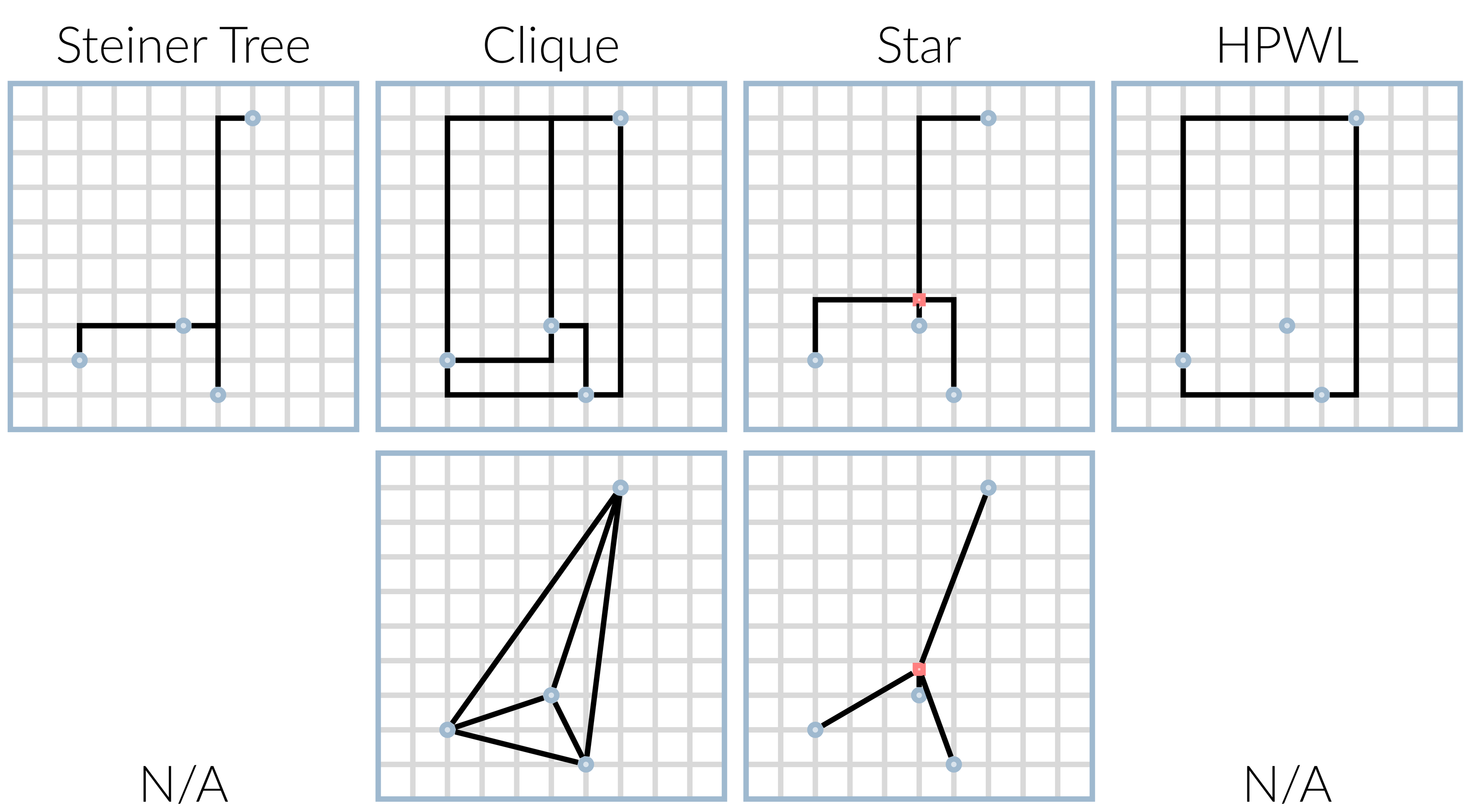


Table 1. Examples of different wirelength models. The first row shows models using ℓ_1 distance, while the second shows those using ℓ_2 distance.

Moreau Envelope

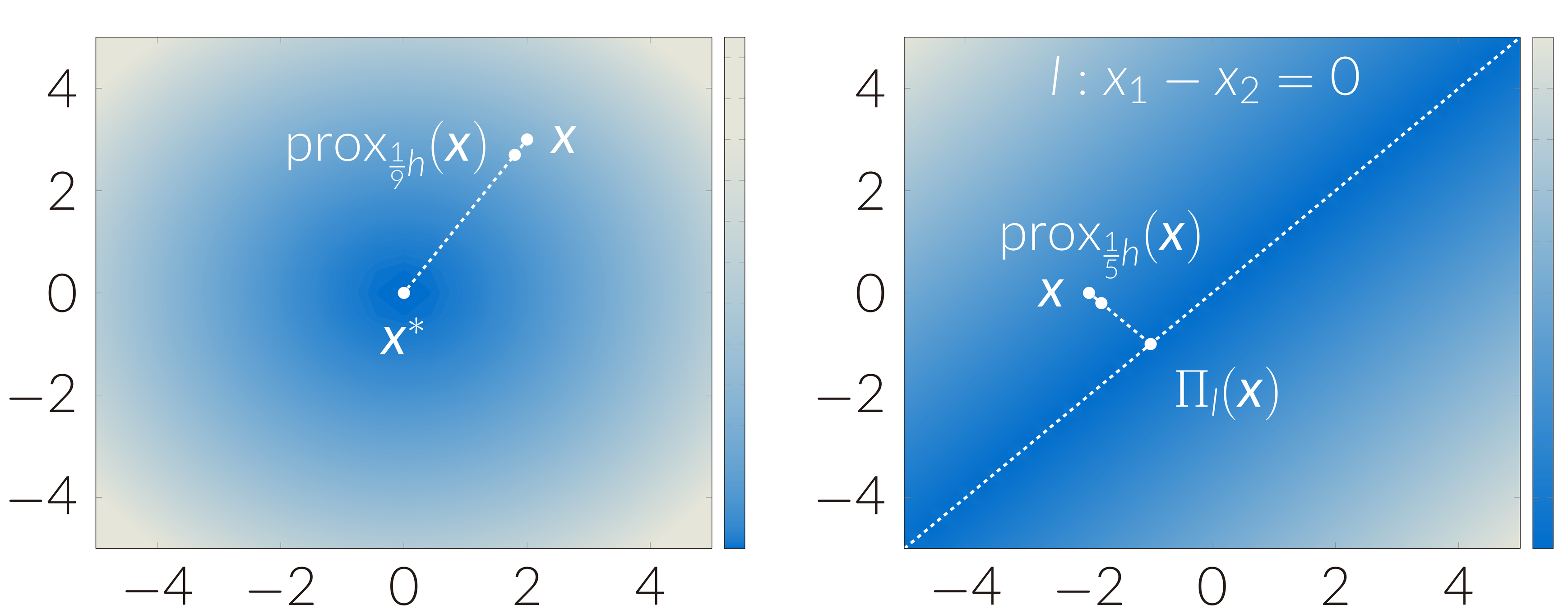
For any $t > 0$, the **Moreau envelope** function h^t and the proximal mapping prox is defined by

$$h^t(\boldsymbol{x}) = \min_{\boldsymbol{u}\in\mathbb{R}^n}\left\{h(\boldsymbol{u}) + \frac{1}{2t}\|\boldsymbol{u}-\boldsymbol{x}\|_2^2\right\},$$

$$\operatorname{prox}_{th}(\boldsymbol{x}) = \arg\min_{\boldsymbol{u}\in\mathbb{R}^n}\left\{h(\boldsymbol{u}) + \frac{1}{2t}\|\boldsymbol{u}-\boldsymbol{x}\|_2^2\right\}.$$

- **Convergence.** $h^t(\boldsymbol{x})$ approximates $h(\boldsymbol{x})$: $\lim_{t\to 0^+} h^t(\boldsymbol{x}) = h(\boldsymbol{x})$.
- **Differentiability.** $h^t(\boldsymbol{x})$ is differentiable: $\nabla_{\boldsymbol{x}} h^t(\boldsymbol{x}) = \frac{1}{t}(\boldsymbol{x} - \operatorname{prox}_{th}(\boldsymbol{x}))$.

Intuition: Replace $h(\boldsymbol{x})$ with the net HPWL $W_e(\boldsymbol{x}) = \max_i x_i - \min_i x_i$ for an approximation of HPWL, as long as $\operatorname{prox}_{tW_e}(\boldsymbol{x})$ is cheap to compute.



Gradient Property

Consider the horizontal part of W_e, the gradient of Moreau envelope function W_e^t is $\boldsymbol{g} = \nabla W_e^t(\boldsymbol{x})$ where

$$g_i = \begin{cases} \frac{1}{t}(x_i - \tau_2), & \text{if } x_i > \tau_2; \\ 0, & \text{if } \tau_1 \le x_i \le \tau_2; \\ \frac{1}{t}(x_i - \tau_1), & \text{otherwise} \end{cases} \quad (1)$$

is defined for any $i = 1, \cdots, n$, such that

$$\sum_{i=1}^{n}(x_i - \tau_2)^+ = \sum_{i=1}^{n}(\tau_1 - x_i)^+ = t, \quad (2)$$

if the solution τ_1, τ_2 to (2) satisfy $\tau_1 \le \tau_2$, otherwise $\boldsymbol{g} = \nabla W_e^t(\boldsymbol{x})$ is determined by the average coordinate: $g_i = \frac{1}{t}x_i - \frac{1}{tn}\sum_{i=1}^{n} x_i$ for any index $i = 1, \cdots, n$.

Water-Filling for Gradient Computation

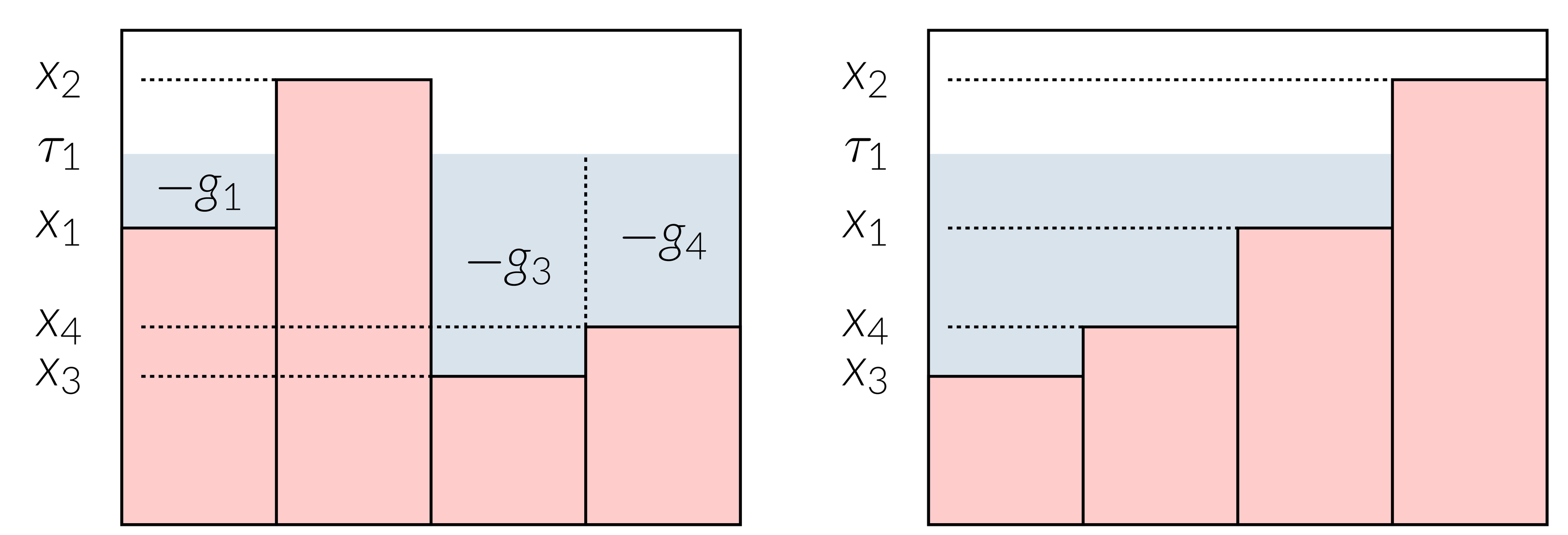
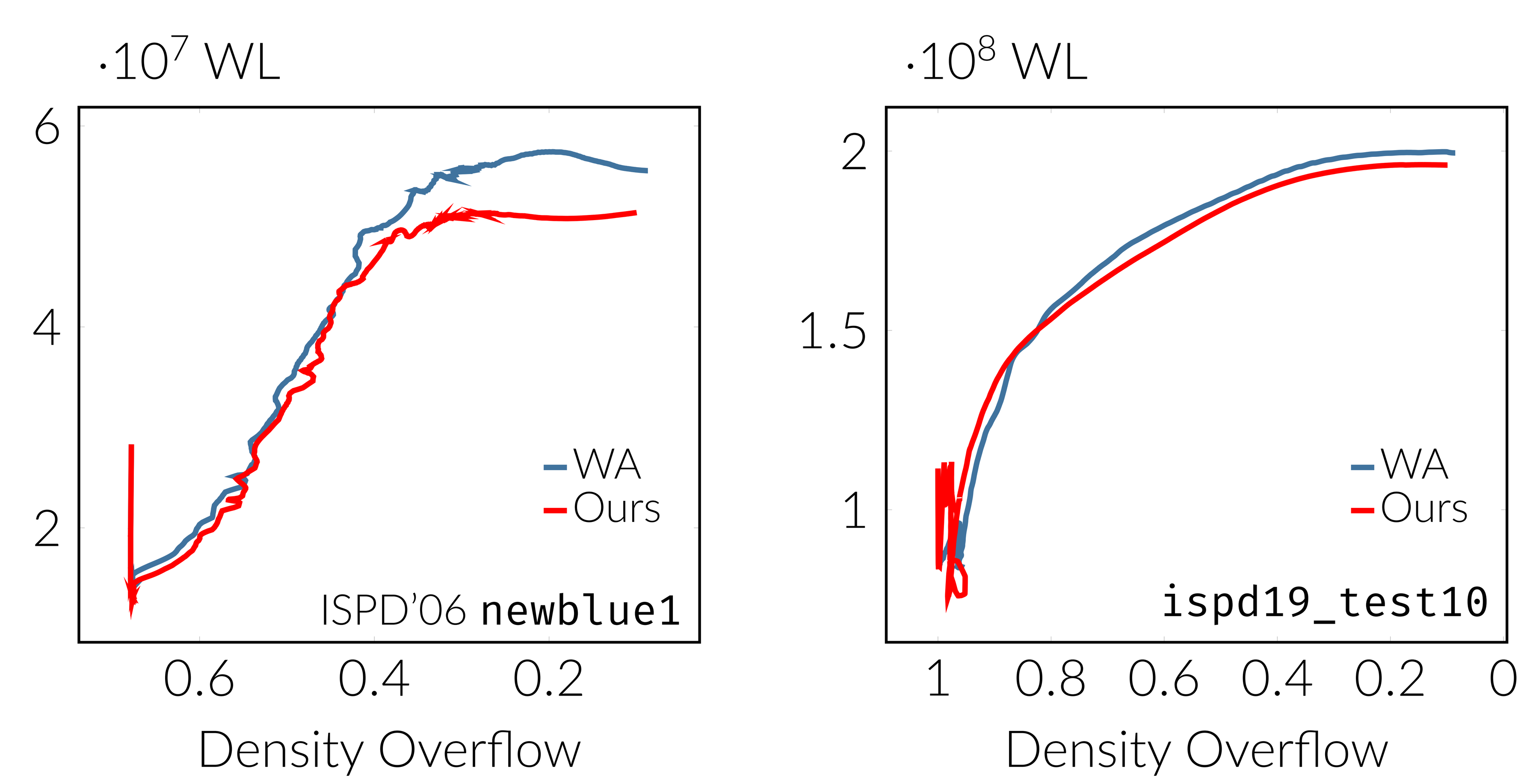


Figure 1. The illustration of water-filling to solve τ_1 in Equation (2).

The water-filling algorithm is applied to solve equations (2) for τ_1 (similar for τ_2). Then we can calculate the gradient $\nabla W_e^t(\boldsymbol{x})$ by (1) accordingly.

Experimental Results



Conclusions

- We propose a novel HPWL-based differentiable wirelength model.
- The derivation of proximal mapping will enlighten more promising research on numerical optimization of global placement.

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