Network Flow-based Simultaneous Retiming and Slack Budgeting for Low Power Design

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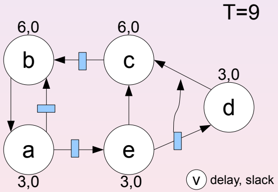
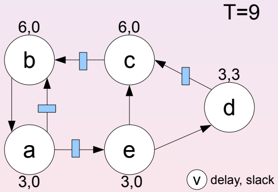
Outline

Retiming and Slack Budgeting

- Timing constraint and Low Power become significant requirement.
- Retiming: relocate flip-flops (FFs)
- Slack Budgeting: relax the timing constraints of components





Previous Works

–**Retiming**:

- [C.E.Leiserson et al. Algorithmica 1991]: first work
- [N. Maheshwari et al. TCAD 1998]: flow based Min-area retiming
- [H. Zhou, ASPDAC 2005]: incremental Min-period retiming
- [J. Wang & H. Zhou DAC 2008]: incremental Min-period retiming

–**Slack Budgeting**:

- [R. Nair et al. TCAD 1989]: ZSA, suboptimal heuristic
- [C. Chen et al. TCAD 2002]: Maximum-Independent-Set, NP-complete
- [S.Ghiasi et al. ICCAD 2004]: Flow based algorithm

Previous Works (Cont.)

–**Retiming + Slack Budgeting**:

- [Y. Hu et al. DAC 2006]: dual-Vdd, MILP
- [S. Liu et al. ASPDAC 2010]: heuristic; MIS based

–**In previous works**:

- A few works consider simultaneous Retiming and Slack Budgeting
- MILP method or heuristic method

–**In our works**:

- Network-Based Algorithm
- Speedup

Problem Formulation

Input:

- Directed graph $G = (V, E, d, w)$ as synchronous sequential circuit.
  - $i \in V$: combinational gate
  - $e_{ij} \in E$: signal passing from gate i to j
  - d_i: delay of gate i
  - w_{ij}: number of FF on edge e_{ij}
- period constraint T
- power-slack tradeoff for each slack level

Output: reallocation represented by r, so

- minimize power consumption
- under the period constraint

MILP Formulation

Condition for $\Phi(G) \leq T$

$$\begin{aligned} a_i &\geq d_i + s_i & \forall i \in V \\ a_i &\leq T & \forall i \in V \\ r_i - r_j &\leq w_{ij} & \forall (i,j) \in E \\ a_j &\geq a_i + d_i + s_i & \text{if } r_i - r_j = w_{ij} \end{aligned}$$

Suppose $R_i = r_i + a_i / T$
$\Rightarrow a_i = T \cdot R_i - T \cdot r_i$.

$$\begin{aligned} \min \quad & \sum_{i \in V} P(\bar{s}_i) & & (II) \\ \text{s.t.} \quad & \bar{R}_i - \bar{r}_i \geq \bar{s}_i & \forall i \in V & \quad (IIa) \\ & \bar{R}_i - \bar{r}_i \leq T & \forall i \in V & \quad (IIb) \\ & \bar{r}_j - \bar{r}_i \geq -T \cdot w_{ij} & \forall (i,j) \in E & \quad (IIc) \\ & 0 \leq \bar{R}_i, \bar{r}_i \leq \bar{N}_{ff} & \forall i \in V & \quad (IId) \\ & \bar{s}_i = \{\bar{s}_i^1, \cdots, \bar{s}_i^k\} & \forall i \in V & \quad (IIe) \\ & 0 \leq \bar{s}_i \leq T & \forall i \in V & \quad (IIf) \\ & \bar{R}_j - \bar{R}_i \geq t_{ij} & \forall (i,j) \in E & \quad (IIg) \\ & t_{ij} \geq \bar{s}_j - T \cdot w_{ij} & \forall (i,j) \in E & \quad (IIh) \end{aligned}$$

MILP Formulation (cont.)

- Solved by ILP Solver, but unacceptable runtime
- Need more effective method
- Without two constraints, convex cost dual network algorithm [R. K. Ahuja et al. 2003]
- Removes constraint (IIh), add penalty function $P(tij)$:
- Generate new problem (III)

$$
\begin{array}{llrr}
\min & \sum_{i \in V} P(\bar{s}_i) & & (II) \\
\text{s.t.} & \bar{R}_i - \bar{r}_i \geq \bar{s}_i & \forall i \in V & (IIa) \\
& \bar{R}_i - \bar{r}_i \leq T & \forall i \in V & (IIb) \\
& \bar{r}_j - \bar{r}_i \geq -T \cdot w_{ij} & \forall (i,j) \in E & (IIc) \\
& 0 \leq \bar{R}_i, \bar{r}_i \leq \bar{N}_{ff} & \forall i \in V & (IId) \\
& \bar{s}_i = \{\bar{s}_i^1, \cdots, \bar{s}_i^k\} & \forall i \in V & (IIe) \\
& 0 \leq \bar{s}_i \leq T & \forall i \in V & (IIf) \\
& \bar{R}_j - \bar{R}_i \geq t_{ij} & \forall (i,j) \in E & (IIg) \\
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$$

MILP Formulation (cont.)

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- Removes constraint (IIh), add penalty function $P(tij)$:
- Generate new problem (III)

$$\min \sum_{i \in V} P(\bar{s}_i) \quad (II)$$

$$\begin{aligned}
\text{s.t.}\quad & \bar{R}_i - \bar{r}_i \geq \bar{s}_i && \forall i \in V && (IIa)\\
& \bar{R}_i - \bar{r}_i \leq T && \forall i \in V && (IIb)\\
& \bar{r}_j - \bar{r}_i \geq -T \cdot w_{ij} && \forall (i,j) \in E && (IIc)\\
& 0 \leq \bar{R}_i, \bar{r}_i \leq \bar{N}_{ff} && \forall i \in V && (IId)\\
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& t_{ij} \geq \bar{s}_j - T \cdot w_{ij} && \forall (i,j) \in E && (IIh)
\end{aligned}$$

MILP Formulation (cont.)

$$\min \sum_{i \in V} P(\bar{s}_i) + \sum_{(i,j) \in E} P(t_{ij}) \quad (III)$$

$$\text{s.t. } (IIa) - (IIg)$$

$$t_{ij} \geq -T \cdot w_{ij}, \quad \forall (i,j) \in E$$

$$\min \sum_{i \in V} P(\bar{s}_i) \quad (II)$$

$$\begin{aligned}
\text{s.t. } & \bar{R}_i - \bar{r}_i \geq \bar{s}_i & \forall i \in V & \quad (IIa)\\
& \bar{R}_i - \bar{r}_i \leq T & \forall i \in V & \quad (IIb)\\
& \bar{r}_j - \bar{r}_i \geq -T \cdot w_{ij} & \forall (i,j) \in E & \quad (IIc)\\
& 0 \leq \bar{R}_i, \bar{r}_i \leq \bar{N}_{ff} & \forall i \in V & \quad (IId)\\
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& t_{ij} \geq \bar{s}_j - T \cdot w_{ij} & \forall (i,j) \in E & \quad (IIh)
\end{aligned}$$

- Given solutions of (III), heuristic generate solution of (II):
- $\bar{s}_j = \min(t_{ij} + T \cdot w_{ij}, \bar{s}_j), \forall i \in FI(j)$

Remove Redundant Constraint

- Denote s_i^* where $P(\bar{s}_i)$ is minimum
- Define $Q(\bar{s}_i)$:

$$Q(\bar{s}_i) = \begin{cases} P(\bar{s}_i^*) & \text{if} \quad \bar{s}_i \le s_i^* \\ P(\bar{s}_i) & \text{if} \quad \bar{s}_i > s_i^* \end{cases}$$

- Consider new problem (III'), which replaces (IIa) and (IIb) by $\bar{R}_i - \bar{r}_i = \bar{s}_i$

$$\begin{aligned} \min \quad & \sum_{i \in V} Q(\bar{s}_i) + \sum_{(i,j) \in E} P(t_{ij}) && (III') \\ \text{s.t.} \quad & (IIc) - (IIg) \\ & \bar{R}_i - \bar{r}_i = \bar{s}_i && \forall i \in V \\ & t_{ij} \ge -T \cdot w_{ij} && \forall (i,j) \in E \end{aligned}$$

$$\begin{aligned} \min \quad & \sum_{i \in V} P(\bar{s}_i) + \sum_{(i,j) \in E} P(t_{ij}) && (III) \\ \text{s.t.} \quad & (IIa) - (IIg) \\ & t_{ij} \ge -T \cdot w_{ij}, \quad \forall (i,j) \in E \end{aligned}$$

Theorem 1

For every optimal solution $(\bar{R}, \bar{r}, \bar{s})$ of problem (III), there is an optimal solution $(\bar{R}, \bar{r}, \hat{s})$ of problem (III'), and the converse also holds.

Theorem 2

The constraint (IIb) in problem (III) can be removed.

Remove Redundant Constraint

- Denote s_i^* where $P(\bar{s}_i)$ is minimum
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Theorem 1

For every optimal solution $(\bar{R}, \bar{r}, \bar{s})$ of problem (III), there is an optimal solution $(\bar{R}, \bar{r}, \hat{s})$ of problem (III'), and the converse also holds.

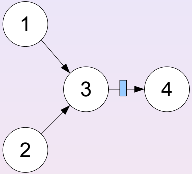
Theorem 2

The constraint (IIb) in problem (III) can be removed.

Primal Network Flow Problem

Solve problem (III) by Convex Cost Dual Flow[a]:
Step 1: Transformation to Primal Network Flow Problem

- Split vertex i into two vertex $\bar{r}_i$ and $\bar{R}_i$
- $(\bar{r}_i, \bar{R}_i) \in \bar{E}_1$, $(\bar{R}_i, \bar{R}_j) \in \bar{E}_2$, $(\bar{r}_i, \bar{r}_j) \in \bar{E}_3$
- Further simplify problem as follow:

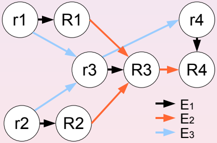


$$\min \sum_{(i,j)\in\bar{E}} P(s_{ij}) \qquad (IV)$$

$$\text{s.t.} \quad \mu_j - \mu_i \geq s_{ij} \qquad \forall (i,j) \in \bar{E} \qquad (IVa)$$

$$0 \leq \mu_i \leq \bar{N}_{ff} \qquad \forall i \in \bar{V} \qquad (IVb)$$

$$l_{ij} \leq s_{ij} \leq u_{ij} \qquad \forall (i,j) \in \bar{E} \qquad (IVc)$$



[a] Refer to [R. K. Ahuja et al. 2003] for detail about Convex Cost Dual Flow.

Primal Network Flow Problem (cont.)

Step 1: Transformation to Primal Network Flow Problem (cont.)

$$\min \sum_{(i,j)\in \bar{E}} P(s_{ij}) \qquad (IV)$$

$$\text{s.t.} \quad \mu_j - \mu_i \geq s_{ij} \qquad \forall (i,j) \in \bar{E} \qquad (IVa)$$

$$0 \leq \mu_i \leq N_{ff} \qquad \forall i \in \bar{V} \qquad (IVb)$$

$$l_{ij} \leq s_{ij} \leq u_{ij} \qquad \forall (i,j) \in \bar{E} \qquad (IVc)$$

Remove constraints by $P(s_{ij})$ and $B(\mu_i)$

$$\bar{P}(s_{ij}) = \begin{cases} P(u_{ij}) + M(s_{ij} - u_{ij}) & \bar{s}_{ij} > u_{ij} \\ P(s_{ij}) & 0 \leq \bar{s}_i \leq T \\ P(l_{ij}) - M(s_{ij} - l_{ij}) & \bar{s}_{ij} < l_{ij} \end{cases}$$

$$B(\mu_i) = \begin{cases} M \cdot (\mu_i - \bar{N}_{ff}) & \text{if} \quad \mu_i > \bar{N}_{ff} \\ 0 & \text{if} \quad 0 \leq \bar{\mu}_i \leq \bar{N}_{ff} \\ -M \cdot \mu_i & \text{if} \quad \mu_i < 0 \end{cases}$$

- Get Primal Network Flow Problem:

$$\min \sum_{(i,j)\in \bar{E}} \bar{P}(s_{ij}) + \sum_{i\in \bar{V}} B(\mu_i) \qquad (V)$$

$$\text{s.t.} \quad \mu_j - \mu_i \geq s_{ij} \qquad \forall (i,j) \in \bar{E}$$

Primal Network Flow Problem (cont.)

Step 1: Transformation to Primal Network Flow Problem (cont.)

$$\min \sum_{(i,j)\in \bar{E}} P(s_{ij}) \qquad (IV)$$

$$\text{s.t. } \mu_j - \mu_i \geq s_{ij} \qquad \forall (i,j) \in \bar{E} \qquad (IVa)$$

$$0 \leq \mu_i \leq N_{ff} \qquad \forall i \in \bar{V} \qquad (IVb)$$

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Remove constraints by $P(s_{ij})$ and $B(\mu_i)$

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$$B(\mu_i) = \begin{cases} M \cdot (\mu_i - \bar{N}_{ff}) & \text{if} \quad \mu_i > \bar{N}_{ff} \\ 0 & \text{if} \quad 0 \leq \bar{\mu}_i \leq \bar{N}_{ff} \\ -M \cdot \mu_i & \text{if} \quad \mu_i < 0 \end{cases}$$

- Get Primal Network Flow Problem:

$$\min \sum_{(i,j)\in \bar{E}} \bar{P}(s_{ij}) + \sum_{i\in \bar{V}} B(\mu_i) \qquad (V)$$

$$\text{s.t. } \mu_j - \mu_i \geq s_{ij} \qquad \forall (i,j) \in \bar{E}$$

Lagrangian Relaxation

Step 2: Lagrangian Relaxation

- Lagrangian relaxation to eliminate constraints
- Lagrangian sub-problem:
-

$$L(\vec{x}) \quad = \quad \sum_{e(i,j)\in\bar{E}} \bar{P}(s_{ij}) + \sum_{i\in\bar{V}} B_i(\mu_i) - \sum_{e(i,j)\in\bar{E}} (\mu_j - \mu_i - s_{ij})x_{ij}$$

- Introduce start node v_0
- Final Lagrangian subproblem:

$$L(\vec{x}) = \min \sum_{e(i,j)\in E} [P_{ij}(s_{ij}) + x_{ij}s_{ij}] \tag{1}$$

$$\text{s.t.} \sum_{j:e(i,j)\in E} x_{ij} - \sum_{j:e(j,i)\in E} x_{ji} = 0 \quad \forall i \in V$$

$$x_{ij} \geq 0 \qquad\qquad \forall (i,j) \in E_1 \cup E_2 \cup E_3$$

Convex Cost-scaling Approach

Step 3: Convex Cost-scaling Approach

- Define function $H_{ij}(x_{ij}) = \min_{s_{ij}}\{P_{ij}(s_{ij}) + x_{ij}s_{ij}\}$:
- $H_{ij}(x_{ij})$ is concave, so $C_{ij}(x_{ij}) = -H_{ij}(x_{ij})$ is convex
- Final problem is a min-cost flow problem:

$$
\begin{aligned}
L(\vec{x}) = \min & \sum_{e(i,j)\in E} C_{ij}(x_{ij}) && (VI) \\
\text{s.t.} & \sum_{j:e(i,j)\in E} x_{ij} - \sum_{j:e(j,i)\in E} x_{ji} = 0 \quad \forall i \in V \\
& 0 \le x_{ij} \le M \qquad \forall (i,j) \in E_1 \cup E_2 \cup E_3 \\
& -M \le x_{ij} \le M \qquad \forall (i,j) \in E_4
\end{aligned}
$$

- For optimal flow x^*, construct residual network $G(x^*)$
- In $G(x^*)$, solve shortest path distance $d(i)$
- Apply $\mu(i) = d(i)$ and $s_{ij} = \mu(i) - \mu(j)$
- Final solve the problem!!

Experiments Setup

- Implemented in C++
- 3.0GHz CPU and 6GB Memory
- 19 cases from the ISCAS89

Case Name	Gate #	Edges #	Max Output	Max Inputs	Tmin
s27.test	11	19	4	2	20
s208.1.test	105	182	19	4	28
s298.test	120	250	13	6	24
s382.test	159	312	21	6	44
s386.test	160	354	36	7	64
s344.test	161	280	12	11	46
s349.test	162	284	12	11	46
s444.test	182	358	22	6	46
s526.test	194	451	13	6	42
s526n.test	195	451	13	6	42
s510.test	212	431	28	7	42
s420.1.test	219	384	31	4	50
s832.test	288	788	107	19	98
s820.test	290	776	106	19	92
s641.test	380	563	35	24	238
s713.test	394	614	35	23	262
s838.1.test	447	788	55	4	80
s1238.test	509	1055	192	14	110
s1488.test	654	1406	56	19	166

Experimental Results

Results for Power Consumption and Total Slacks

Benchmark	T	Power Consumption			Total Slacks		
		Optimal ILP	[18][1]	ours	Optimal ILP	[18]	ours
s27.test	20	800	824	850	40	40	30
s208.1.test	28	3542	9118	4772	1770	290	1988
s298.test	24	6498	8888	8010	1330	660	1240
s382.test	44	6456	9038	9958	3011	2071	1895
s386.test	64	8836	12870	9564	2484	807	2324
s344.test	46	9876	11848	9894	1855	1064	1760
s349.test	46	9938	12472	9894	1852	912	1780
s444.test	46	8938	14032	11884	2962	1025	1939
s526.test	42	7602	14106	11498	3626	1307	2356
s526n.test	42	7752	11734	11548	3616	2089	2366
s510.test	42	13976	17492	14846	2237	937	2040
s420.1.test	50	4574	17920	9224	5906	1050	4466
s832.test	98	13652	14518	16274	5175	4525	4171
s820.test	92	13552	17694	16448	5261	3493	4103
s641.test	238	13334	20408	14424	7925	6067	7604
s713.test	262	13018	21228	14322	8522	6363	8112
s838.1.test	80	6004	18898	17556	14048	9016	9912
s1238.test	110	6096	10444	8208	16764	14635	15792
s1488.test	166	21292	23799	27836	15313	14791	13024
Avg	-	9249.3	14070	11947.9	5457.7	3744.3	4573.8
Diff	-	1	+52%	+29%	1	-31%	-16%

[1] S.Liu et al., "Simultaneous slack budgeting and retiming for synchronous circuits optimization", ASPDAC 2010

Experimental Results

Results for Runtime:

Benchmark	T	Runtime(s)		
		Optimal ILP	[18]	ours
s27.test	20	0.02	0.0	0.0
s208.1.test	28	0.39	0.44	0.06
s298.test	24	0.78	0.69	0.07
s382.test	44	$>$1000	10.56	0.12
s386.test	64	4.58	1.03	0.1
s344.test	46	0.82	2.53	0.09
s349.test	46	0.79	4.49	0.11
s444.test	46	$>$1000	12.04	0.12
s526.test	42	42.57	1.67	0.17
s526n.test	42	30.32	4.72	0.17
s510.test	42	$>$1000	1.62	0.17
s420.1.test	50	1.29	16.91	0.14
s832.test	98	71.96	151.26	0.24
s820.test	92	68.98	13.18	0.25
s641.test	238	2.24	92.97	0.26
s713.test	262	2.27	121.1	0.27
s838.1.test	80	1.48	256.9	0.4
s1238.test	110	0.23	448.6	0.34
s1488.test	166	$>$1000	670.7	0.53
Avg	-	-	95.3	0.19
Diff	-	-	1	0.002

Thank You !