# A Unified Framework for Simultaneous Layout Decomposition and Mask Optimization

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#### Outline

Introduction

Algorithms

**Experimental Results** 

Conclusion



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# **VLSI Chip Design Flow**





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# Layout Decomposition (LD)

• Conflict: two features with the same color, while distance  $< d_{min}$ 



#### **Problem Formulation**

Input: layout and  $d_{min}$ Output: decomposed layout, minimizing conflict #



# Mask Optimization (MO)



- The quality of printed image may be poor due to the diffraction effect of the light.
- Optical Proximity Correction(OPC): Refine the mask to compensate the diffraction effect.
- Method for OPC:
  - rule-based [Park+,ISQED'2010];
  - model-based [Kuang+,DATE'2015][Su+,TCAD'2016];
  - inverse lithography technique [Gao+,DAC'2014].



# Mask Optimization (cont.)

- Edge Placement Error (EPE): Geometric displacement between the image contour and the edge of target image on the layout.
- EPE Violation: The perpendicular displacement is greater than an EPE threshold value.



#### **Problem Formulation**

Input: target layout Output: refined mask, minimizing EPE violation #.



# Two-Stage Flow for Layout Optimization

Two stages:

- Layout Decomposition (LD)
- Mask Optimization (MO)



#### Issues



Solution 1: #EPE Violation = 1



Solution 2: #EPE Violation = 3



### **Options?**

- **Exhaustive MO** for all LD solutions.
  - Running time overhead due to thousands of LD solutions.





# Options? (cont.)

- Heuristic selection among LD solutions.
  - Local region density [Yu+,ICCAD'13]: balance the pattern density on each mask.



- Spacing vector [Chen+, ISQED'13]: maximize minimum distance between patterns.





- Limited effectiveness.

#### **Motivation**

How about combining LD and MO together?



- It is an open problem.
- It is expected to be more effective and more efficient.



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#### **Preliminaries**

- Lithography model:
  - The aerial image is formed by a series of convolution operation between mask  ${\bf M}$  and lithography kernel  ${\bf h}.$

$$\mathbf{I} = f_{optical}(\mathbf{M}) = \sum_{k=1}^{K} w_k \cdot |\mathbf{M} \otimes \mathbf{h}_k|^2$$

- Photo-resist model
  - Set a threshold  $I_{th}$  to binarize aerial image.

$$\mathbf{Z}(x, y) = f_{resist}(\mathbf{I}) = \begin{cases} 1, & \text{if } \mathbf{I}(x, y) \ge I_{th}, \\ 0, & \text{otherwise.} \end{cases}$$



#### **Problem Formulation**

LDMO: Given a target image  $Z_t$ , find two masks  $M_1$  and  $M_2$  which can form printed image with high fidelity.

$$\begin{split} \min_{\mathbf{M}_1,\mathbf{M}_2} & F = \|\mathbf{Z}_t - \mathbf{Z}\|_2^2\\ \text{s.t.} & \mathbf{M}_1(x,y) \in \{0,1\}, \ \forall x,y,\\ & \mathbf{M}_2(x,y) \in \{0,1\}, \ \forall x,y,\\ & \mathbf{I}_1 = \sum_{k=1}^K w_k \cdot |\mathbf{M}_1 \otimes \mathbf{h}_k|^2,\\ & \mathbf{I}_2 = \sum_{k=1}^K w_k \cdot |\mathbf{M}_2 \otimes \mathbf{h}_k|^2,\\ & \mathbf{Z} = f_{resist}(\mathbf{I}_1) \lor f_{resist}(\mathbf{I}_2). \end{split}$$



#### **Overall Flow**





#### **Overall Flow**





### **Grid Construction**

- Extract target pattern.
- Add bounding box.
- Construct grid.
- Merge grid.





#### **Overall Flow**





#### Formulation Relaxation

▶ Relaxation on binary constraints with *sigmoid* function.

$$\mathbf{M}_{1}(x, y) \in \{0, 1\} \to \mathbf{M}_{1}(x, y) = \operatorname{sig}(\mathbf{P}_{1}(x, y)) = \frac{1}{1 + \exp[-\theta_{M}\mathbf{P}_{1}(x, y)]}$$
$$\mathbf{Z}_{1}(x, y) = f_{resist}(\mathbf{I}_{1}) \to \mathbf{Z}_{1}(x, y) = \operatorname{sig}(\mathbf{I}_{1}(x, y)) = \frac{1}{1 + \exp[-\theta_{Z}(\mathbf{I}_{1}(x, y) - I_{th})]}$$

Relaxation on Z.

$$\mathbf{Z} = f_{resist}(\mathbf{I}_1) \lor f_{resist}(\mathbf{I}_2) \to \mathbf{Z}(x, y) = \min\{\mathbf{Z}_1(x, y) + \mathbf{Z}_2(x, y), 1\}$$



### Gradient-Based Optimization

Algorithm 1 Gradient-Based Mask Update

- 1: function MaskUpdate( $\mathbf{P}_1, \, \mathbf{P}_2$ )
- 2: Initialize stepsize *t*;
- 3: Compute the relaxed masks  $M_1, M_2$ ;
- 4: Compute  $\mathbf{Z}$  according to current  $\mathbf{P}_1$  and  $\mathbf{P}_2$ ;
- 5: Compute the gradient  $\nabla_{\mathbf{P}_1} F$ ,  $\nabla_{\mathbf{P}_2} F$
- 6:  $\mathbf{P}_1 \leftarrow \mathbf{P}_1 t \times \nabla_{\mathbf{P}_1} F$ ;
- 7:  $\mathbf{P}_2 \leftarrow \mathbf{P}_2 t \times \nabla_{\mathbf{P}_2} F$ ;
- 8: return  $\mathbf{P}_1, \mathbf{P}_2, \nabla_{\mathbf{P}_1} F, \nabla_{\mathbf{P}_2} F$ ;
- 9: end function



#### **Overall Flow**





### **Violation Graph**



$$w_{ij} = \begin{cases} 1, & \text{if } v_i \text{ and } v_j \text{ have conflict,} \\ \beta, & \text{if } v_i \text{ and } v_j \text{ have large #EPEV,} \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbf{W} = \begin{bmatrix} 0 & 0 & 0 & 0 & \beta \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \beta & 0 & 0 & 1 & 0 \end{bmatrix}$$



#### Semidefinite Programming

• Use  $\mathbf{x} = [x_1, x_2, \cdots, x_n]^{\mathsf{T}}$  to denote the grid assignment solution.

Max-Cut:

$$\max_{x_i} \sum_{(i,j)\in E} w_{ij}(1-x_ix_j)$$
  
s.t.  $x_i \in \{-1,1\}, \quad \forall v_i \in V$ 

Relax to Semidefinite Programming:

$$\begin{array}{l} \min_{\mathbf{X}} \ \mathbf{W} \bullet \mathbf{X} \\ \text{s.t.} \ \operatorname{diag}(\mathbf{X}) = \mathbf{e} \\ \mathbf{X} \succeq \mathbf{0} \end{array}$$



### Semidefinite Programming (cont.)

Randomized rounding [Goemans+, JACM'1995]

- Obtain  $\mathbf{X}^*$  by solving SDP.
- Cholesky decomposition with X\*.

 $\mathbf{X}^* = \mathbf{U}^\intercal \mathbf{U}$ 

- Get x<sub>i</sub> as follows. **u**<sub>i</sub> is the *i*-th column of **U** and **r** is random unit vector.

$$x_i = \operatorname{sgn}(\mathbf{u}_i^{\mathsf{T}}\mathbf{r}) = \begin{cases} 1, & \text{if } \mathbf{u}_i^{\mathsf{T}}\mathbf{r} \ge 0, \\ -1, & \text{otherwise.} \end{cases}$$



### Pruning

- Obtain multiple solutions by randomized rounding.
- Efficient pruning.



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### #EPE Violation Convergence Curve





### Comparison – EPE Violation Num





# Comparison – Runtime



#### Distribution of #EPE violations





Examples of Printed Image

```
(a) [ICCAD'13] + [DAC'14];
(b) [ISQED'13] + [DAC'14];
(c) Ours.
```







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- Two collaborative flows are designed:
  - A gradient-based numerical optimization
  - A set of discrete optimization.
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#### Future Exploration

- More advanced lithography process, e.g., triple patterning lithography.
- More optimization targets, such as process variation band.



# Thank You

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