A High-Performance Triple Patterning Layout Decomposer with Balanced Density

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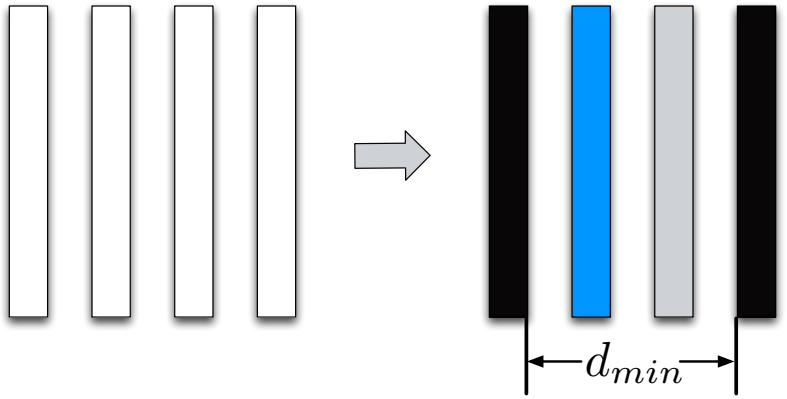
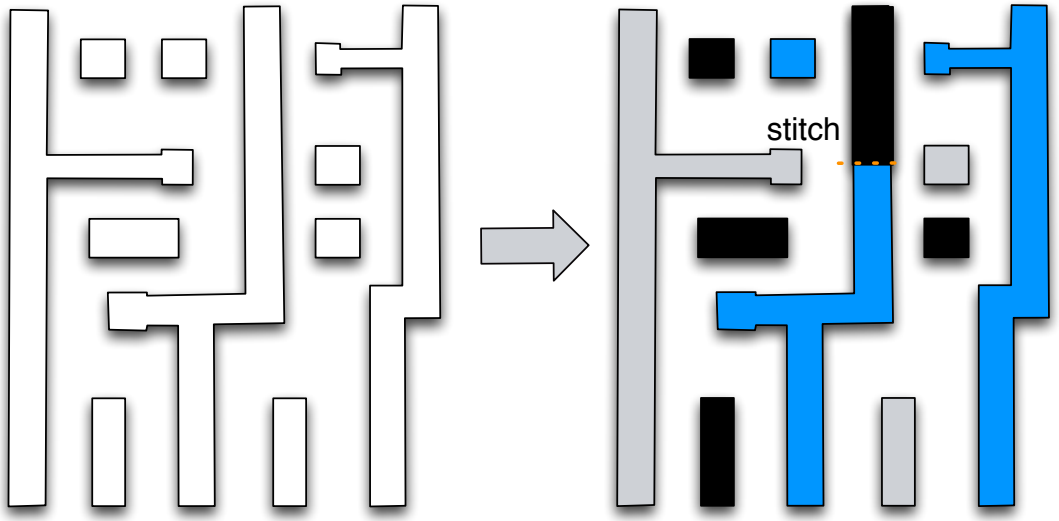
Triple Patterning Lithography (TPL)

ITRS roadmap

- 28nm single-patterning
- 20nm double-patterning
- 14nm triple-patterning / EUV
- 10nm quadruple-patterning / EUV





TPL Decomposition Works

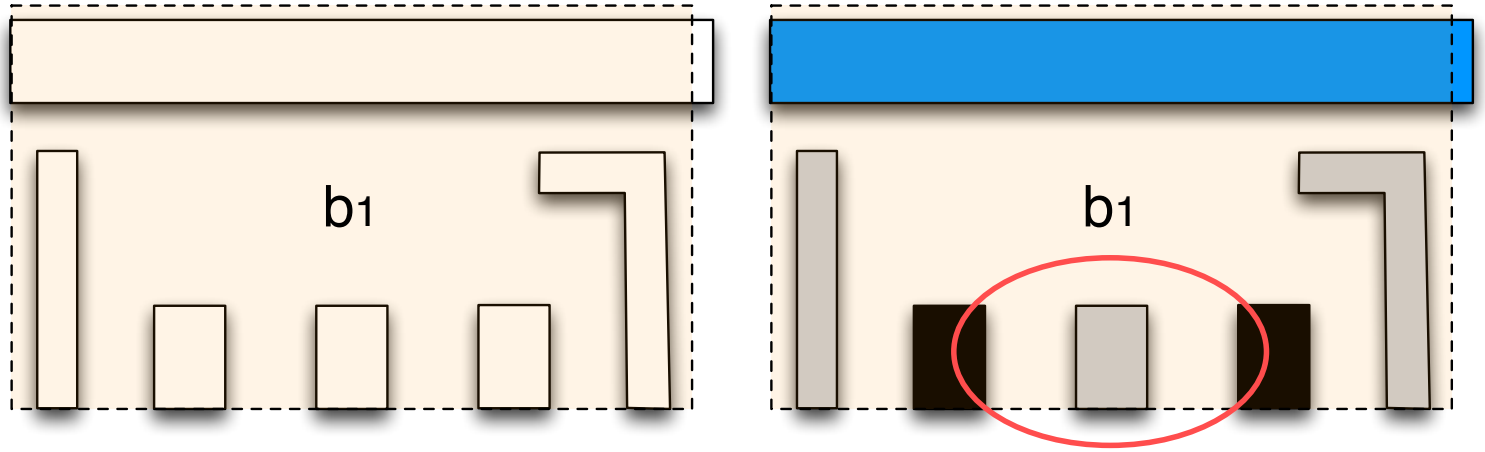
– **ILP or SAT**
[Cork+,SPIE'08][Yu+,ICCAD'11][Cork+,SPIE'13]

– **Graph Search for Row based Layout**
[Tian+, ICCAD'12][Tian+,SPIE'13]

– **Heuristic**
[Ghaida+,SPIE'11][Fang+,DAC'12][Chen,ISQED'13]
[Kuang+,DAC'13][Tang+,Patent'13]

– **Semidefinite Programming (SDP)** (trade-off)
[Yu+, ICCAD'11][Yu+,ICCAD'13]

- Global Balanced Density?



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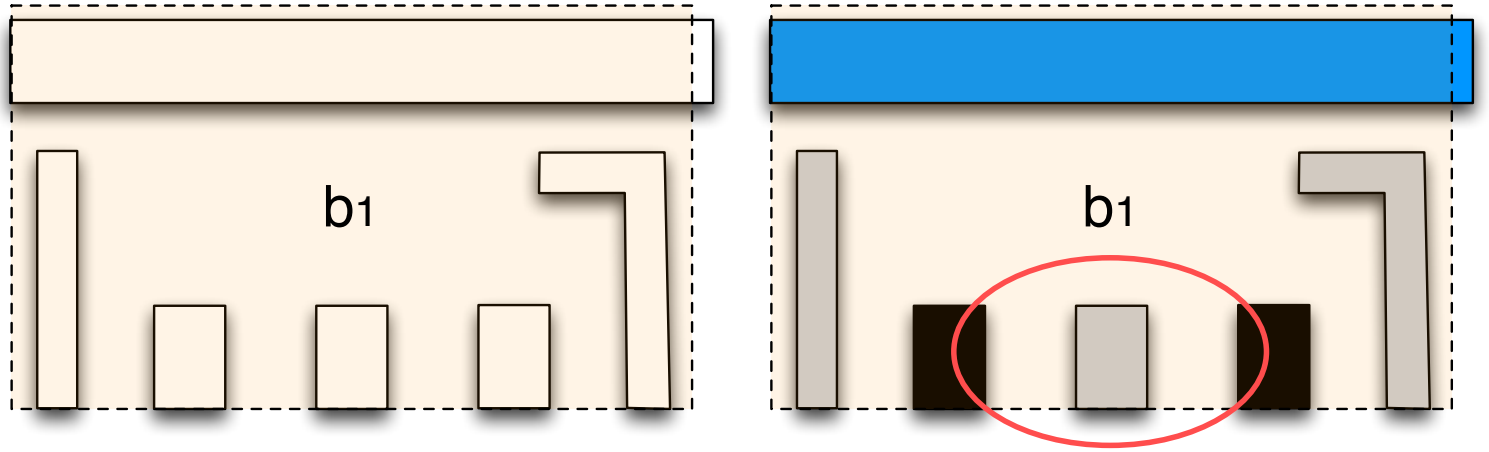
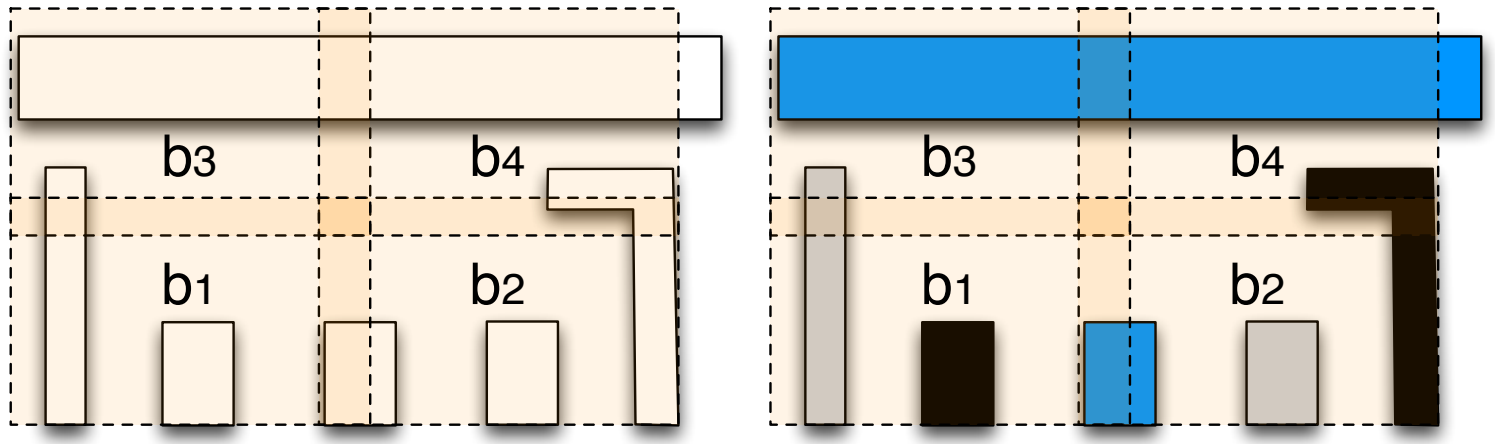
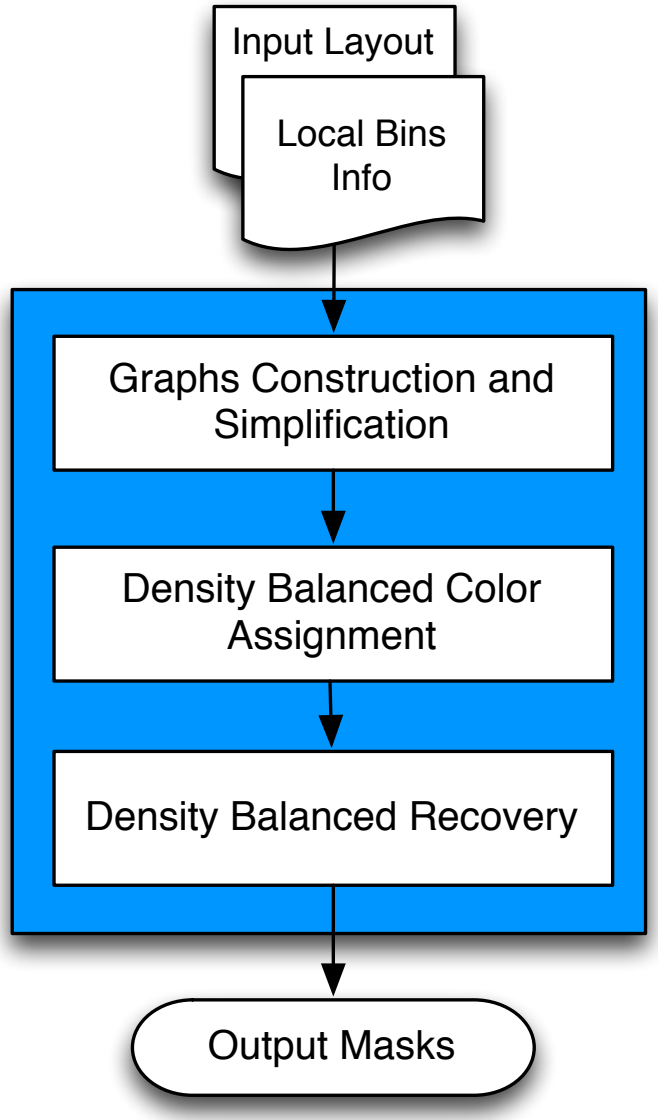
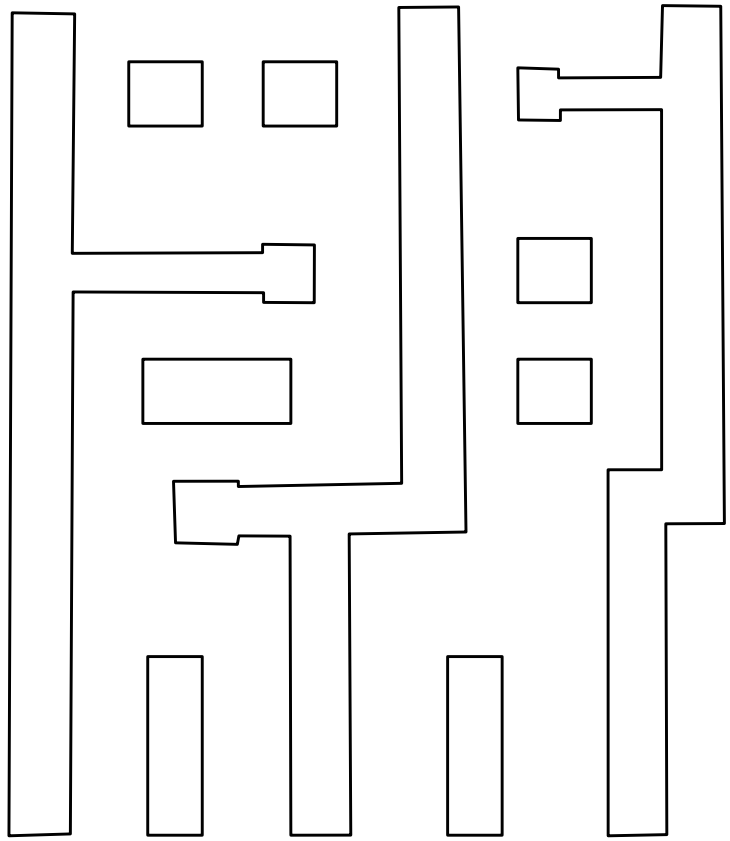


- Local Balanced Density!



Overall Flow





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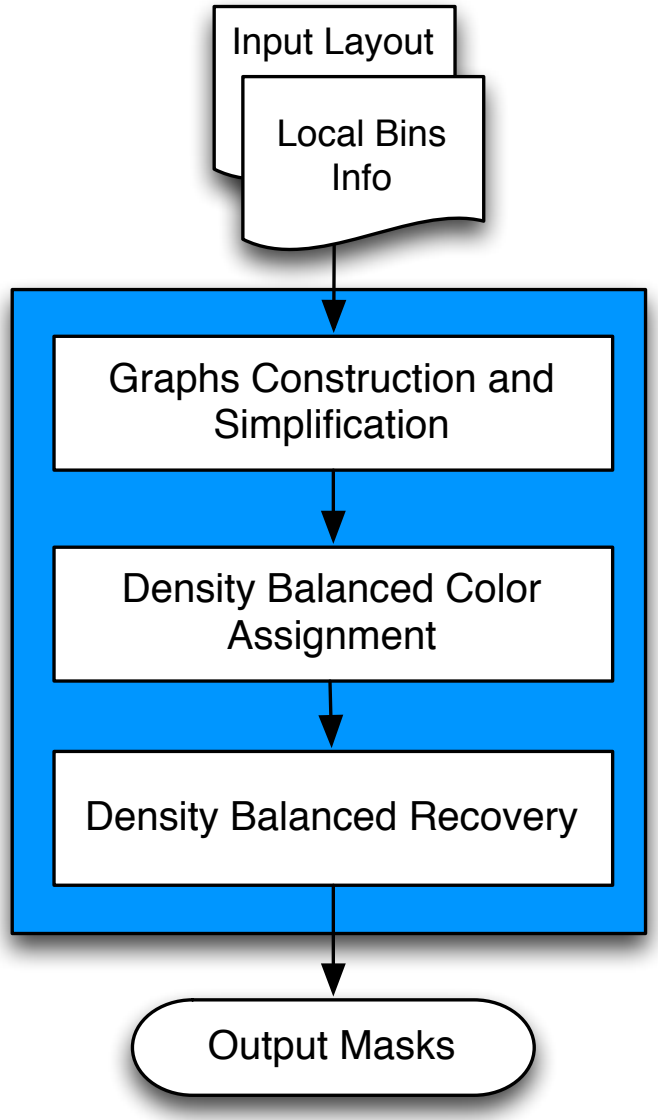

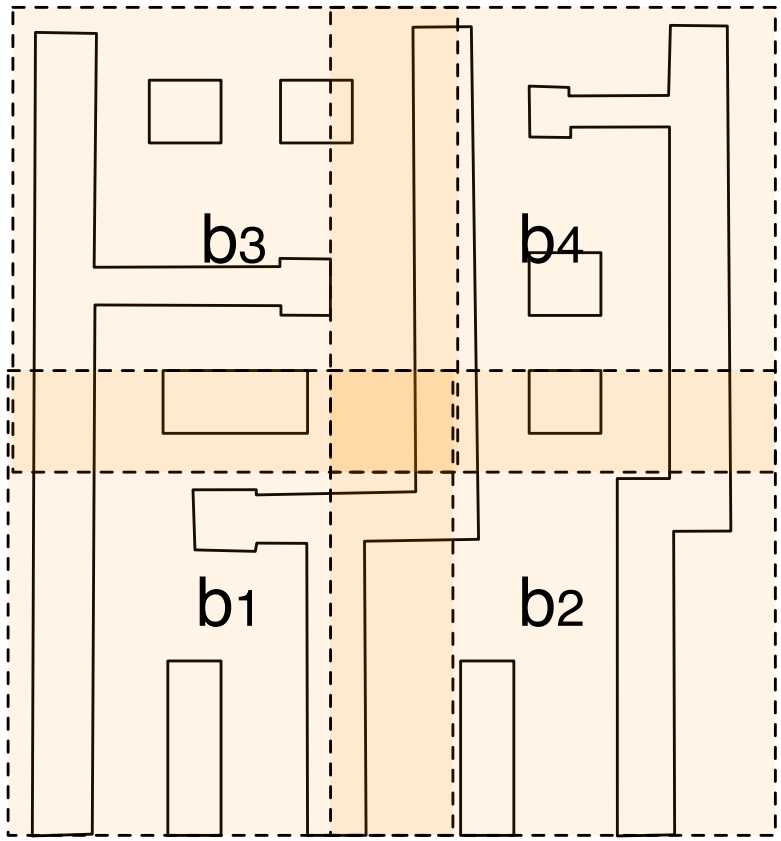

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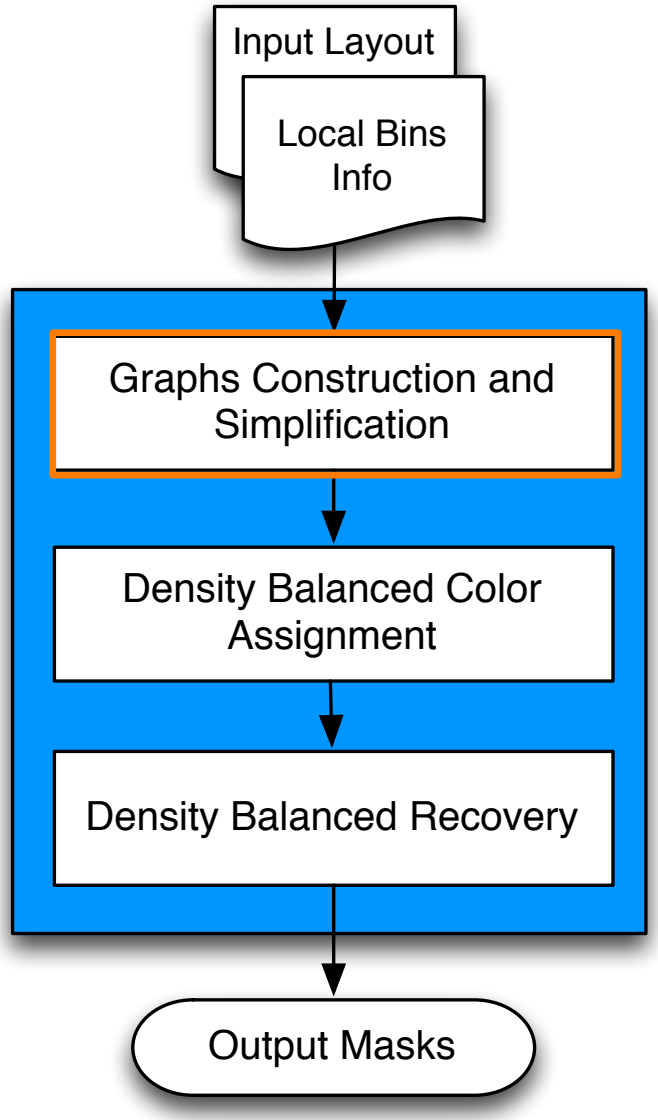

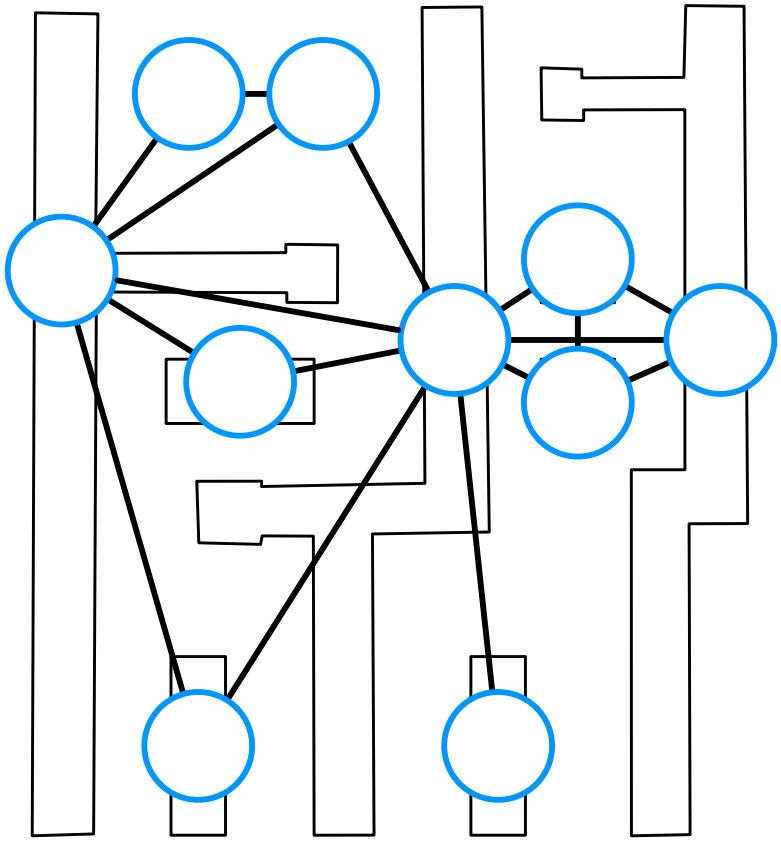

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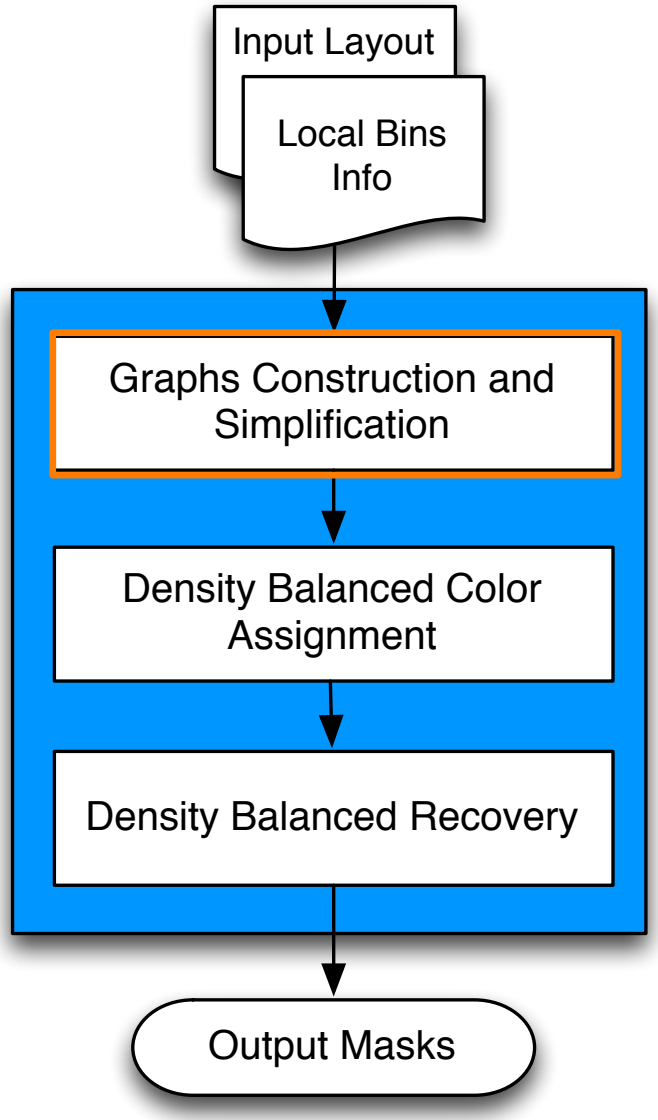

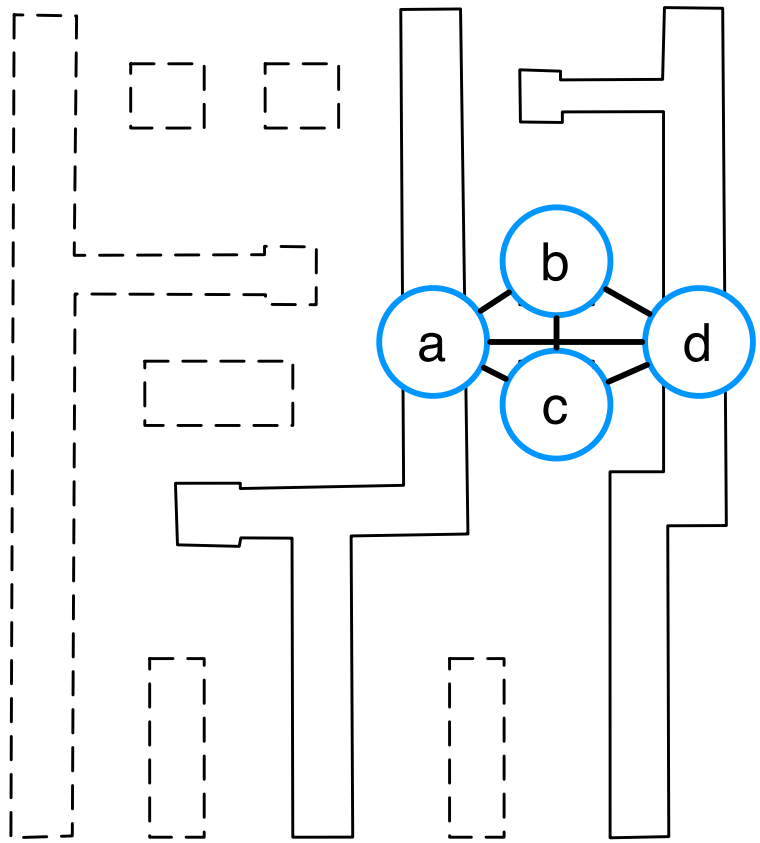

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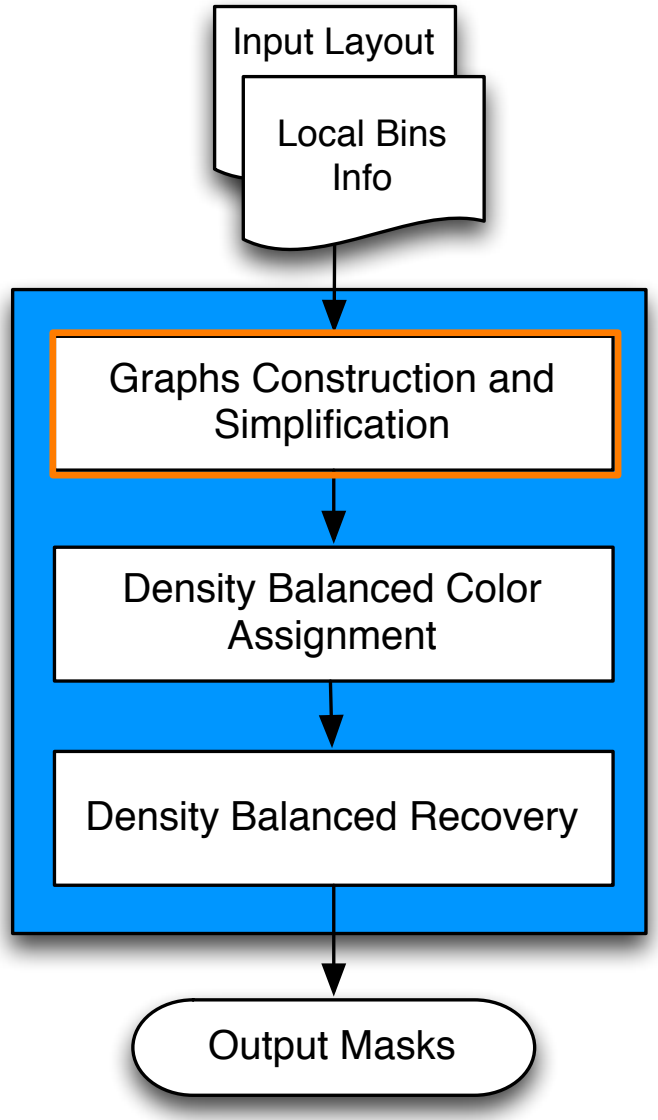

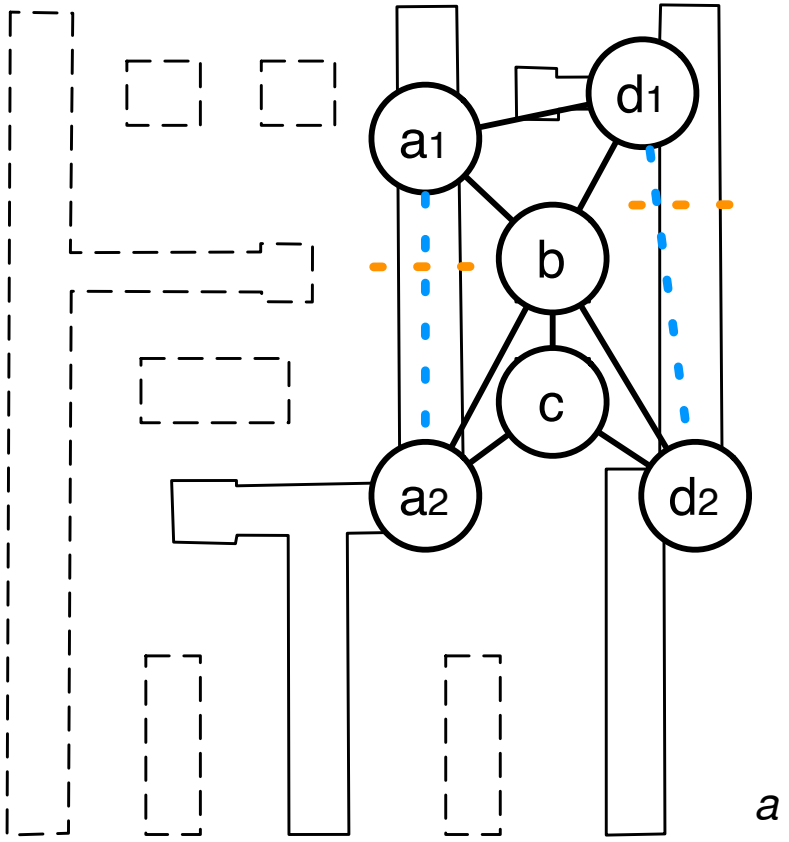

[a]Stitch candidate generation [Kuang+,DAC'13]

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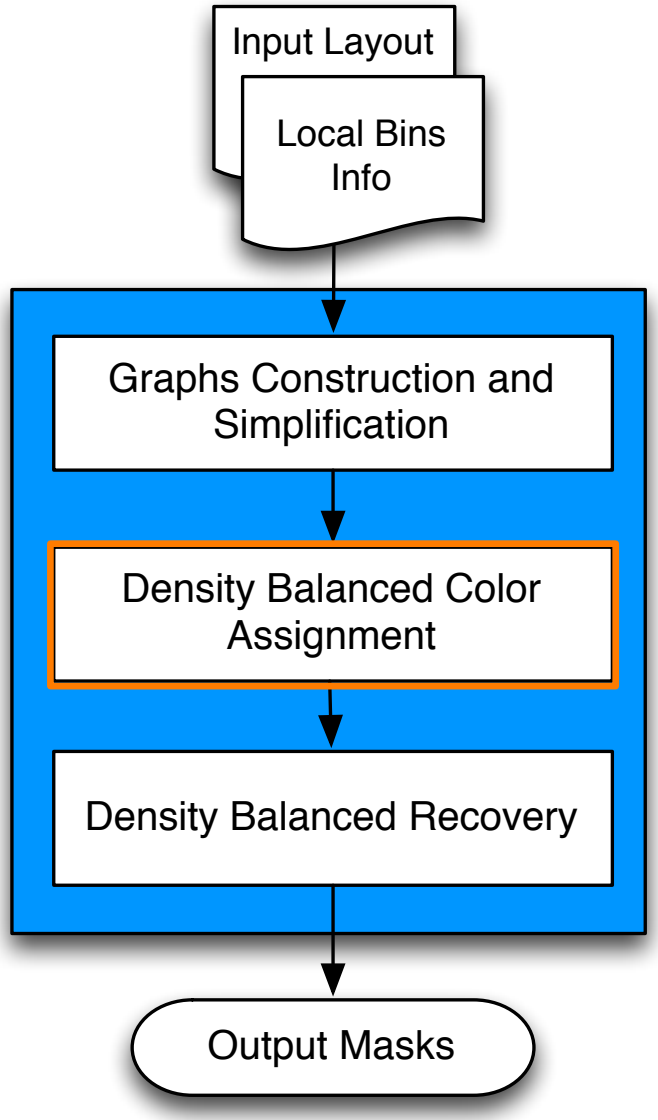

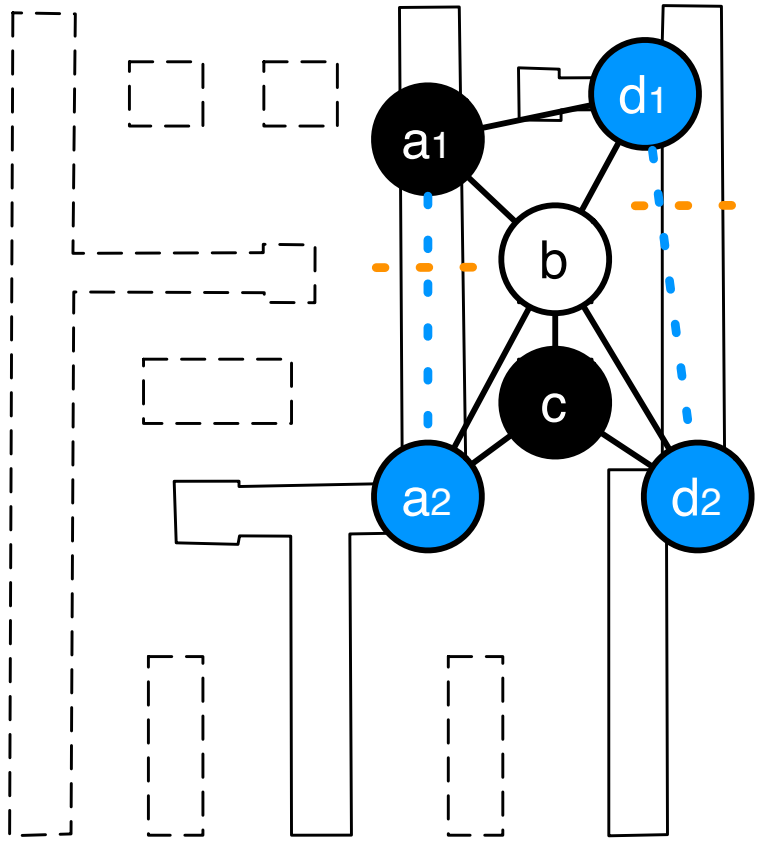

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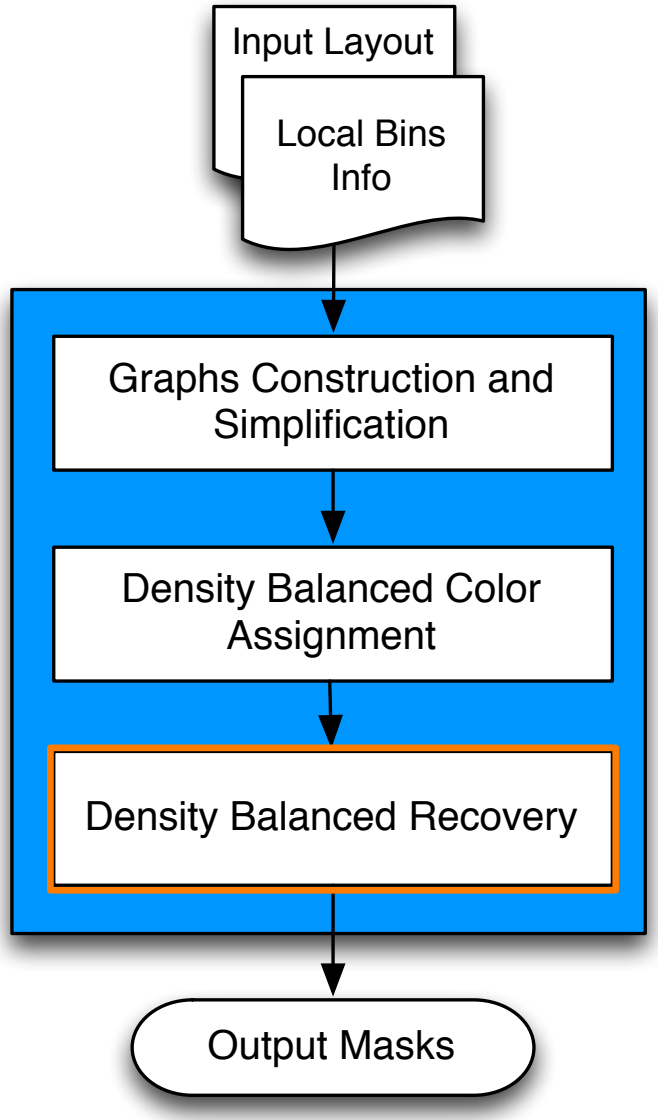

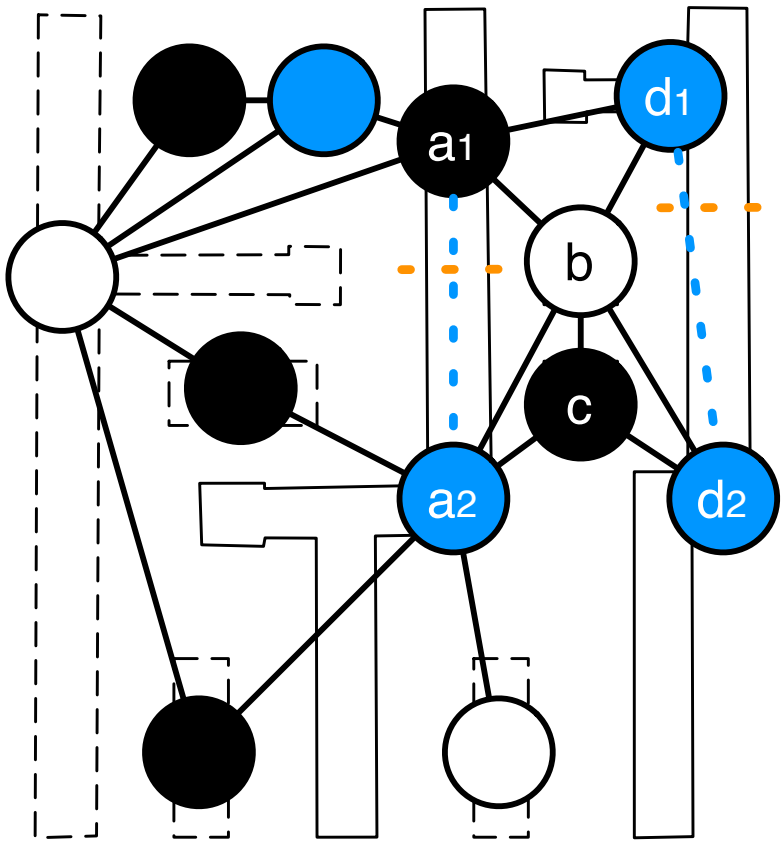

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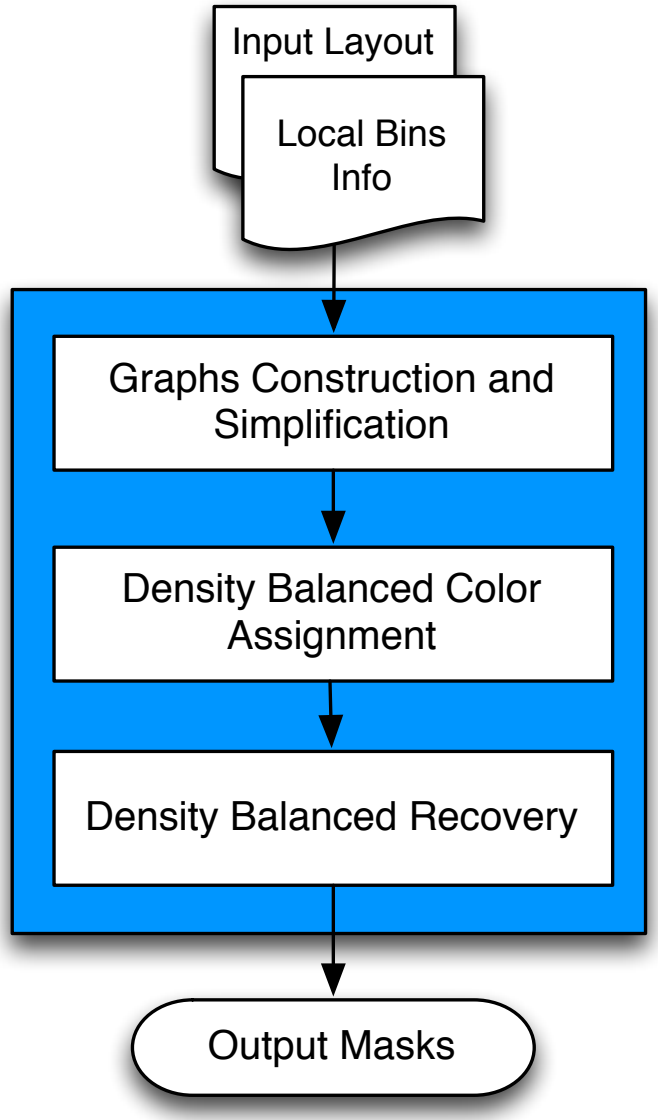
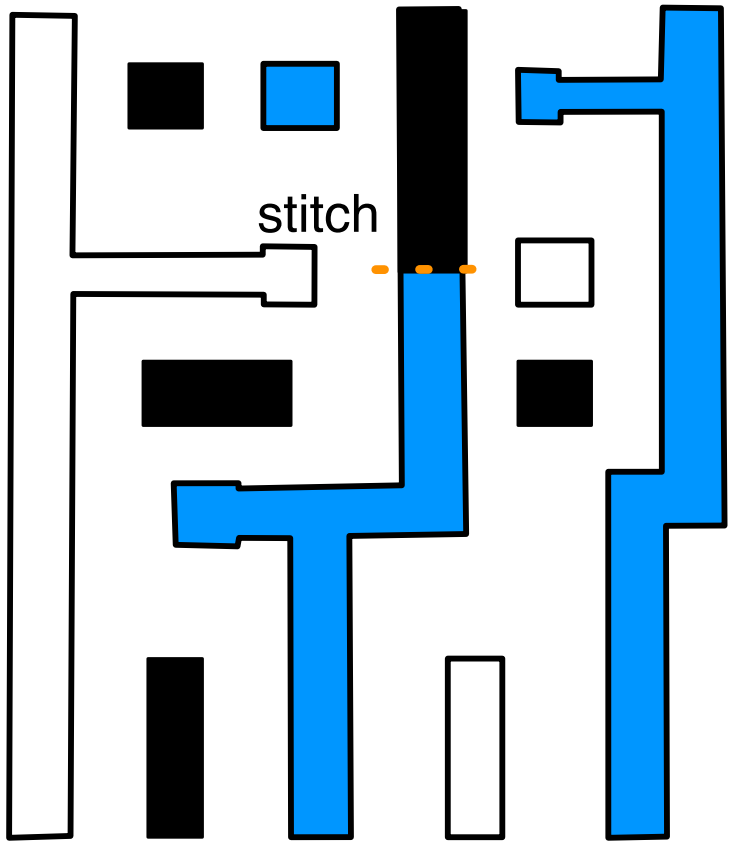
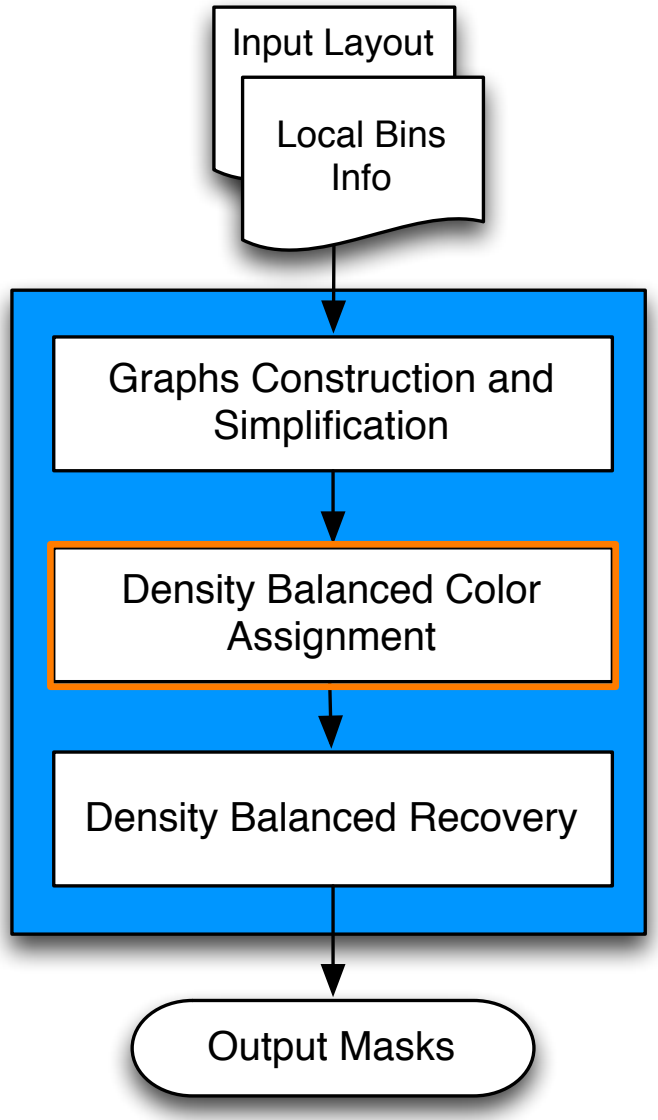





Input Layout
Local Bins Info
Graphs Construction and Simplification
Density Balanced Color Assignment
Density Balanced Recovery
Output Masks

Problem Formulation

Local density uniformity in bin b_k

$$DU_k = d_{k1} \cdot d_{k2} + d_{k1} \cdot d_{k3} + d_{k2} \cdot d_{k3}$$

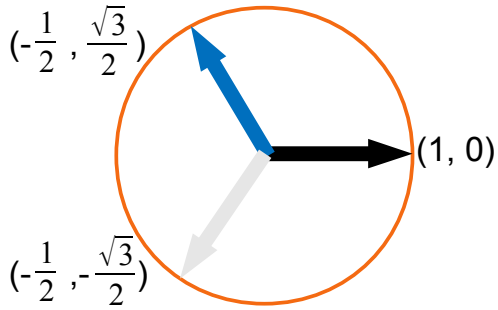
Lemma: Maximizing DU_k can achieve better density balance.

Density Balanced Color Assignment

Input	Graph model
Output	Color Assignment to the graph
Objective	min conflict#, stitch#, and max DU_k

Color representation

- Three unit vectors [Yu+,ICCAD'11]
- same color: $\vec{v}_i \cdot \vec{v}_j = 1$
- different color: $\vec{v}_i \cdot \vec{v}_j = -1/2$



Our Vector Programming

$$\min \sum_{e_{ij} \in CE} (\vec{v}_i \cdot \vec{v}_j) - \alpha \sum_{e_{ij} \in SE} (\vec{v}_i \cdot \vec{v}_j) - \beta \cdot \sum_{b_k \in B} DU_k$$

$$\text{s.t.} \quad \vec{v}_i \in \{(1,0), (-\frac{1}{2}, \frac{\sqrt{3}}{2}), (-\frac{1}{2}, -\frac{\sqrt{3}}{2})\}$$

$$DU_k = -\sum_{i,j \in V} den_{ki} \cdot den_{kj} \cdot (\vec{v}_i \cdot \vec{v}_j) \quad \forall b_k \in B$$

den_{ki}: density of feature r_i in bin b_k

Relax to Semidefinite Programming (SDP)

$$\begin{aligned} \text{SDP: } \min \quad & A \bullet X \\ & X_{ii} = 1, \quad \forall i \in V \\ & X_{ij} \geq -\frac{1}{2}, \quad \forall e_{ij} \in CE \\ & X \succeq 0 \end{aligned}$$

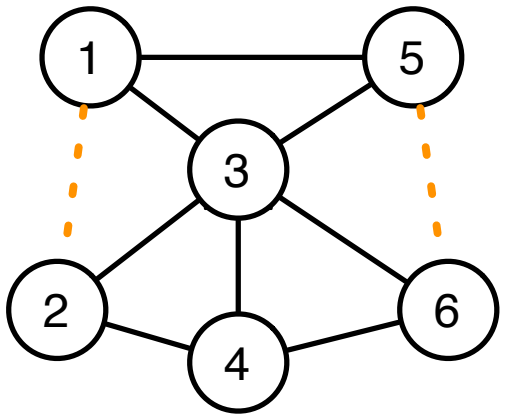
$$A_{ij} = \begin{cases} 1 + \beta \cdot \sum_k den_{ki} \cdot den_{kj}, & \forall b_k \in B, e_{ij} \in CE \\ -\alpha + \beta \cdot \sum_k den_{ki} \cdot den_{kj}, & \forall b_k \in B, e_{ij} \in SE \\ \beta \cdot \sum_k den_{ki} \cdot den_{kj}, & \text{otherwise} \end{cases}$$

Output matrix X:

- If X_{ij} close to 1, i, j same color
- If X_{ij} close to -0.5, i, j different colors

Mapping: From SDP to Color Assignment

$$X = \begin{pmatrix} 1.0 & 0.43 & -0.5 & 0.21 & -0.5 & 0.15 \\ & 1.0 & -0.5 & -0.5 & 0.15 & 0.95 \\ & & 1.0 & -0.5 & -0.5 & -0.5 \\ & & & 1.0 & 0.21 & -0.5 \\ & \ldots & & & 1.0 & 0.43 \\ & & & & & 1.0 \end{pmatrix}$$



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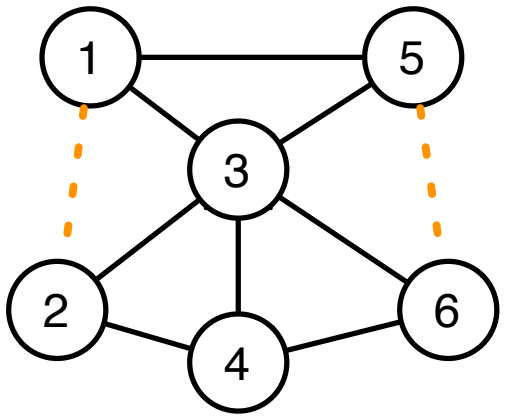
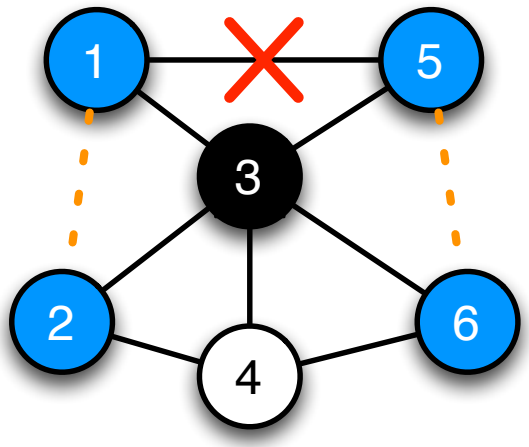
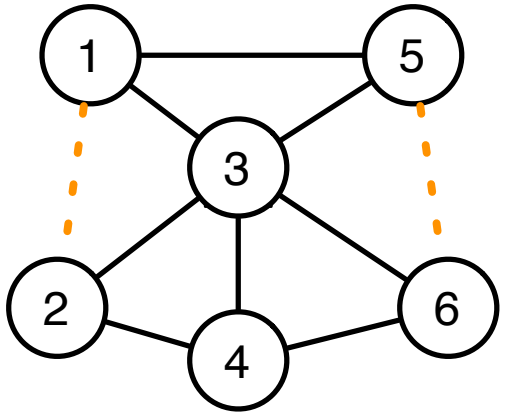


- Greedy may lose optimality
- 0.95? 0.43?

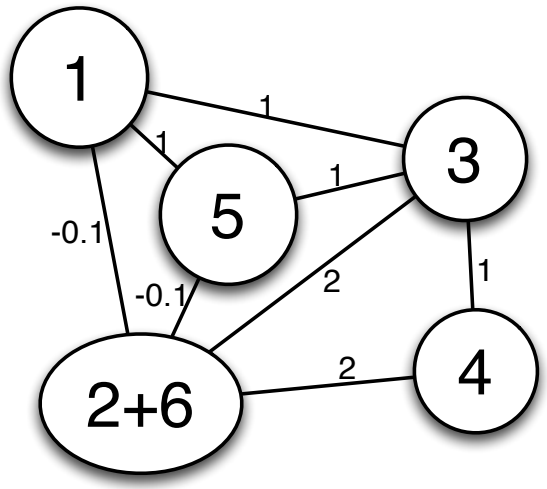


Mapping: 3-Way Max-Cut

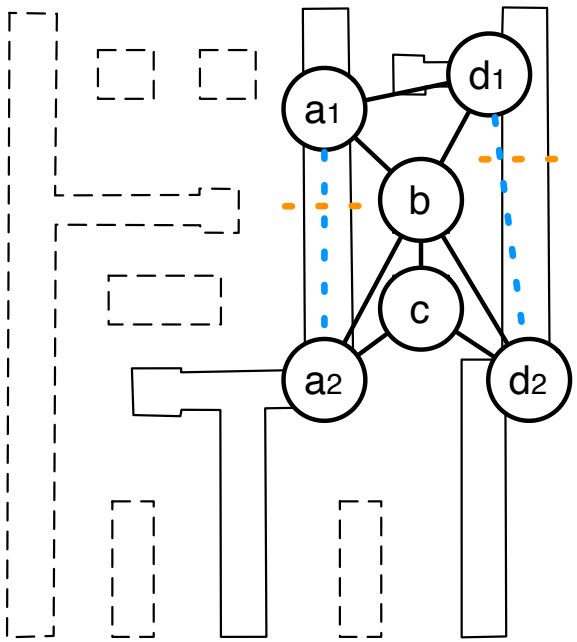
$$X = \begin{pmatrix} 1.0 & 0.43 & -0.5 & 0.21 & -0.5 & 0.15 \\ & 1.0 & -0.5 & -0.5 & 0.15 & 0.95 \\ & & 1.0 & -0.5 & -0.5 & -0.5 \\ & & & 1.0 & 0.21 & -0.5 \\ & \ldots & & & 1.0 & 0.43 \\ & & & & & 1.0 \end{pmatrix}$$

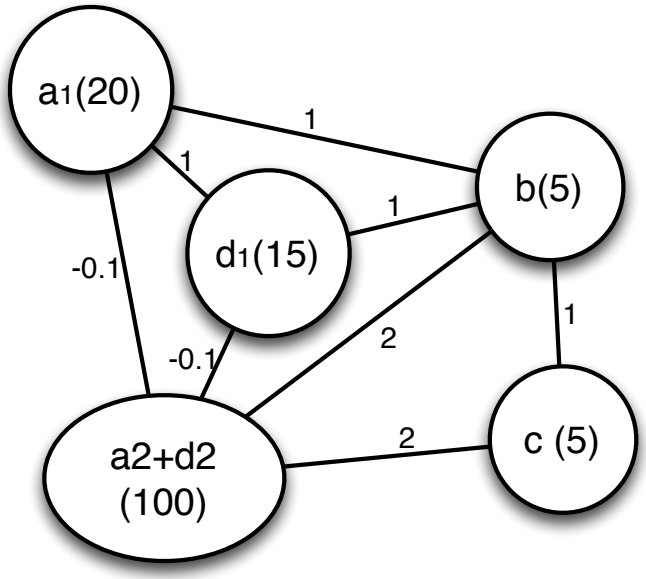


- Smaller merged graph
- 3-Way Max-Cut
- FM Heuristic v.s. Search



Mapping: Extend to Density Balance





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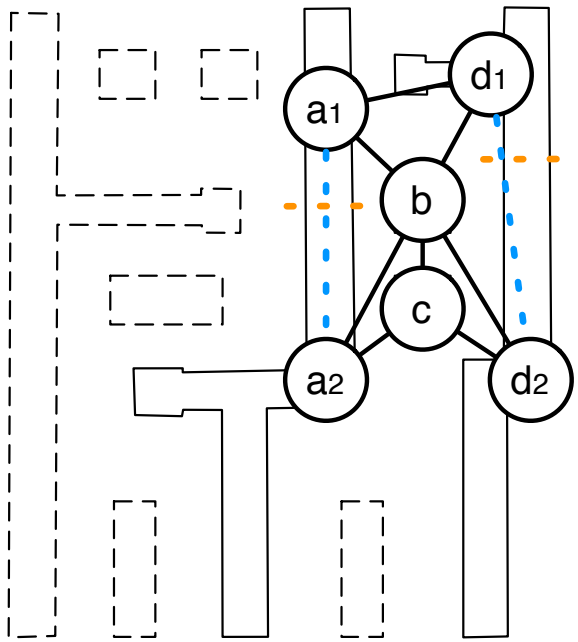
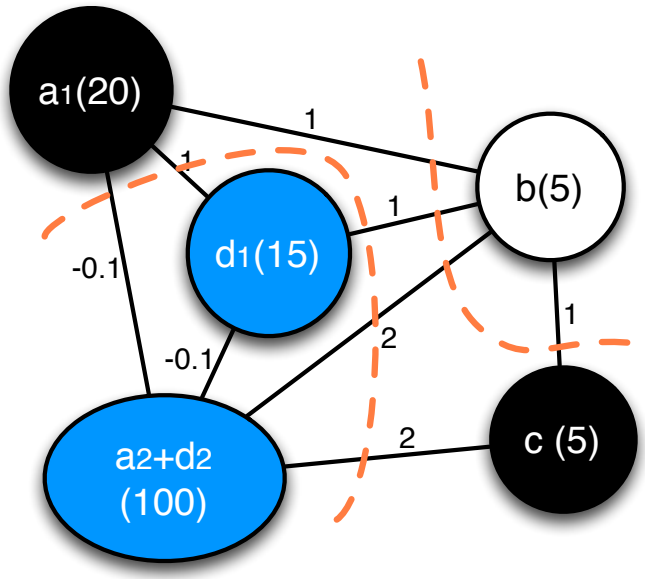
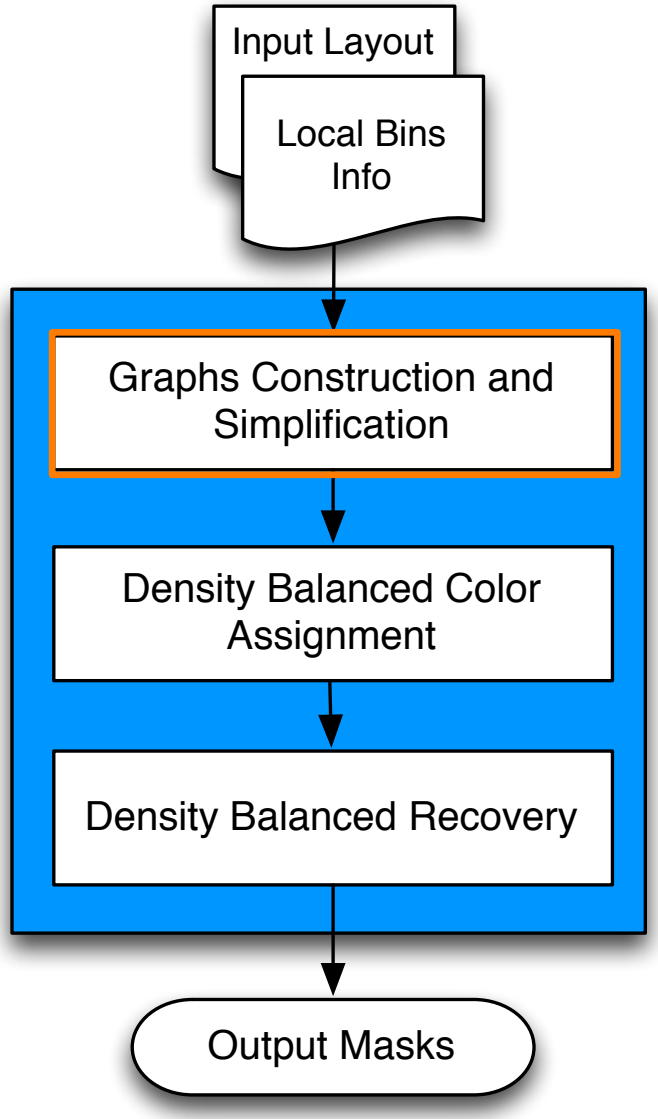


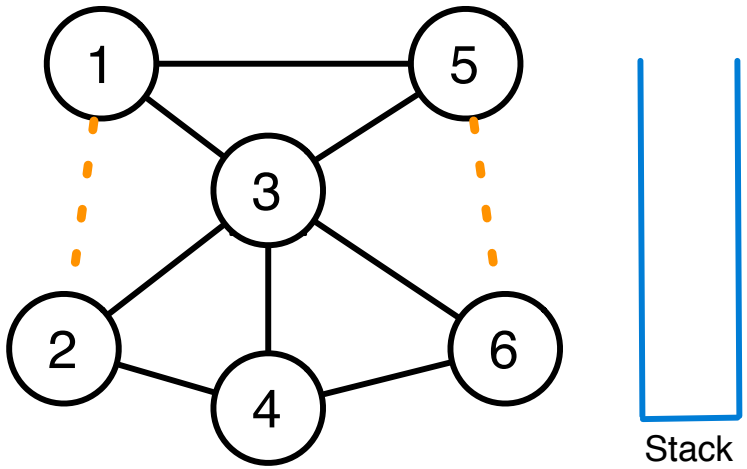




- Implement techniques
  [Yu+,ICCAD'11]
  [Fang+,DAC'12]
  [Kuang+,DAC'13]
- 3 new techniques
- Integrate density balance

Fast Color Assignment Trial

- Iteratively remove vertex with:
  - Conflict degree < 3
  - Stitch degree < 2
- Linear runtime
- Keep conflict # optimality



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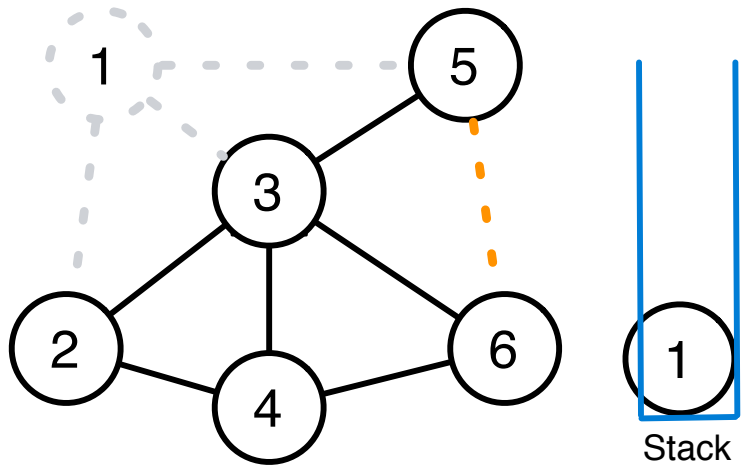

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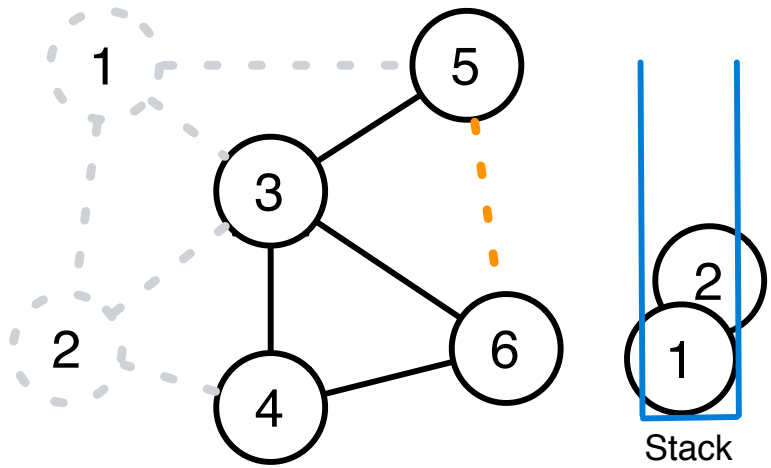

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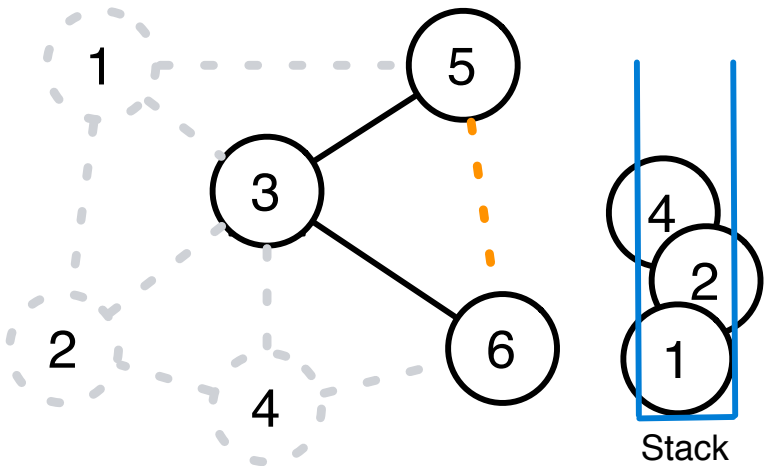

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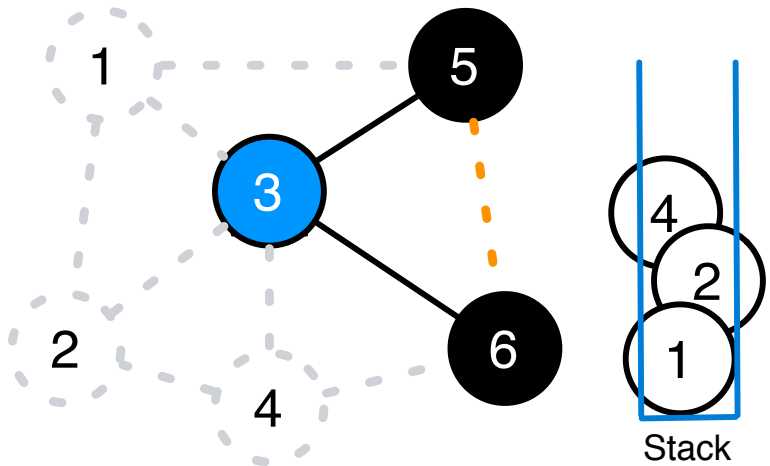

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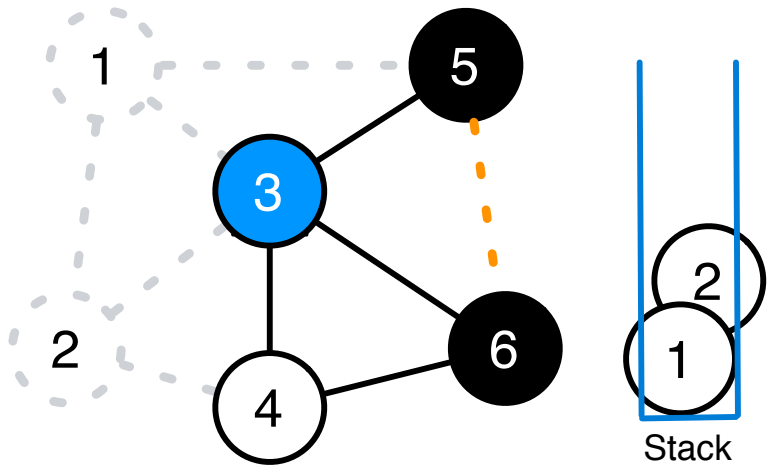

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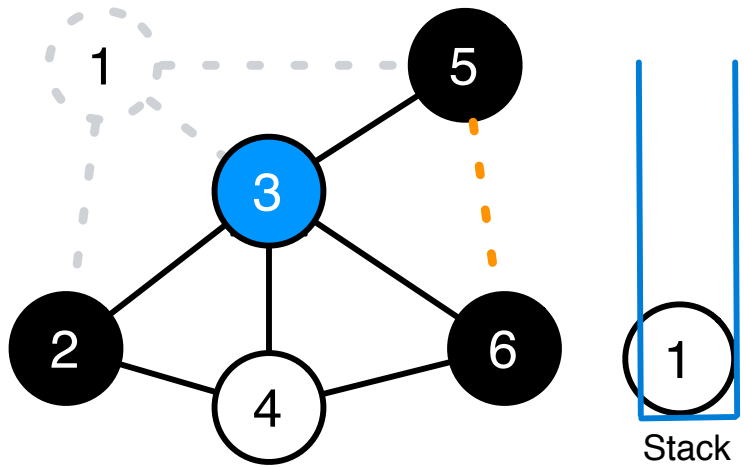

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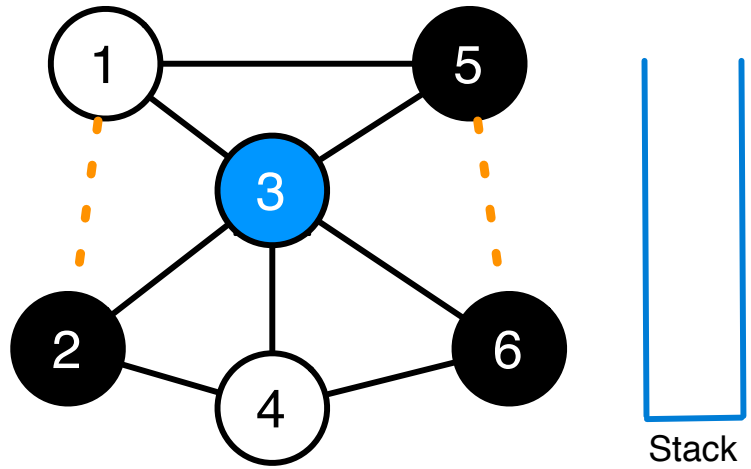

Cut Vertex Stitch Forbiddance

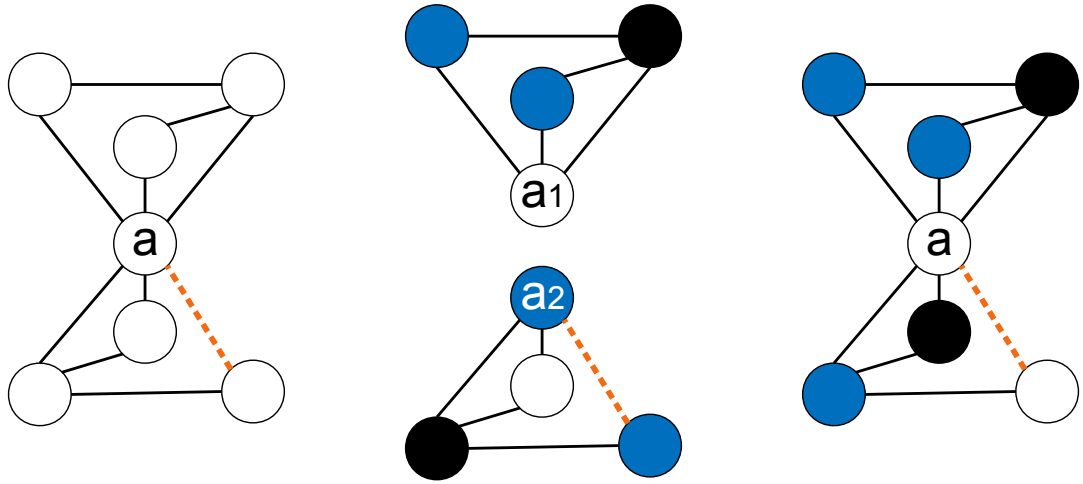

Figure : Cut Computation [Fang+,DAC'12]

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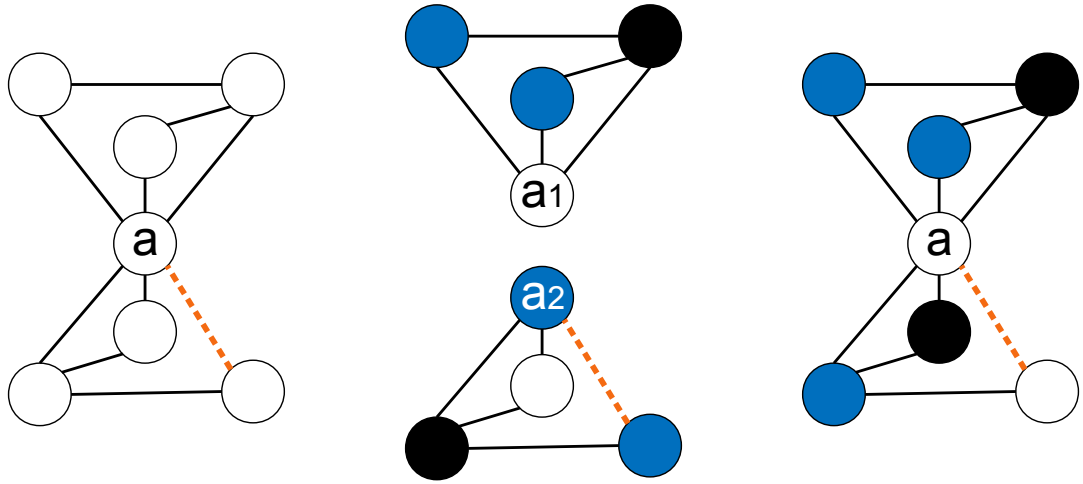

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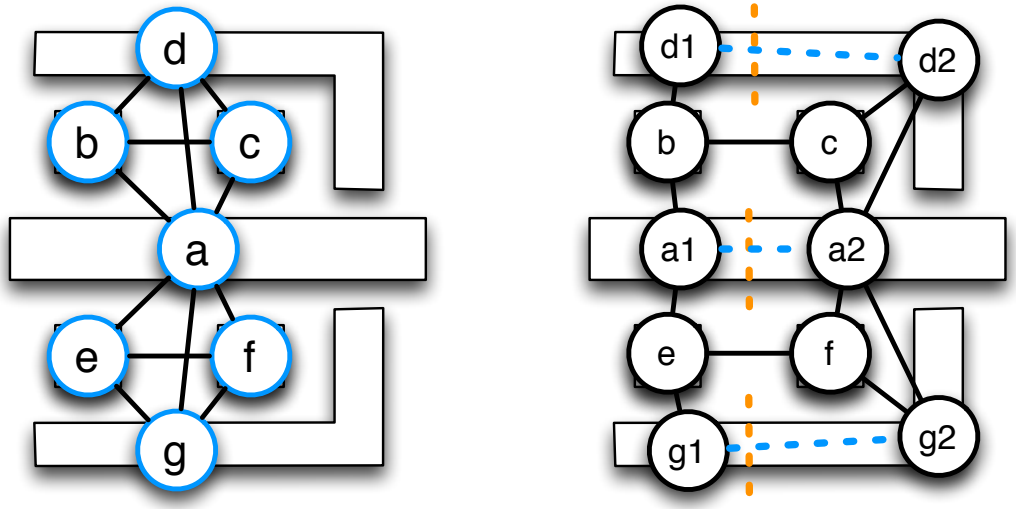

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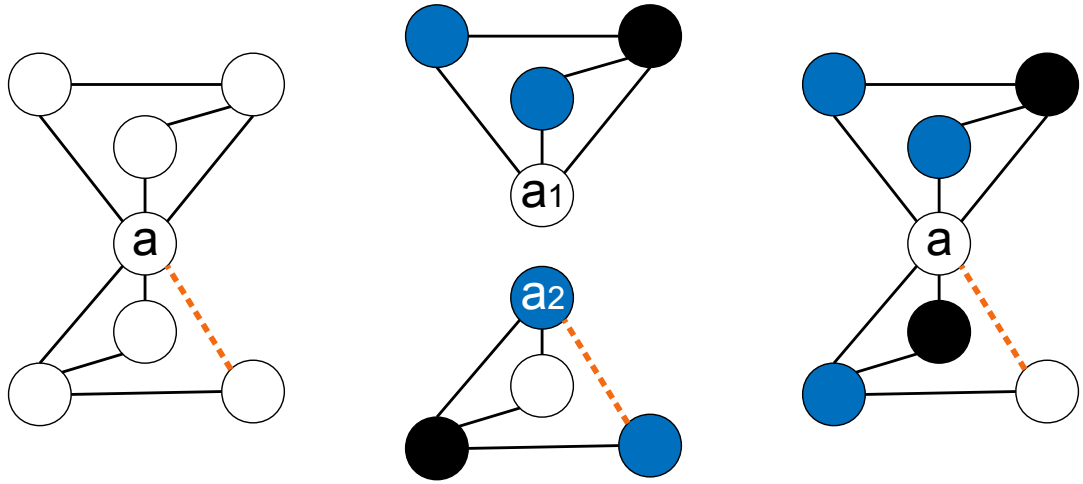

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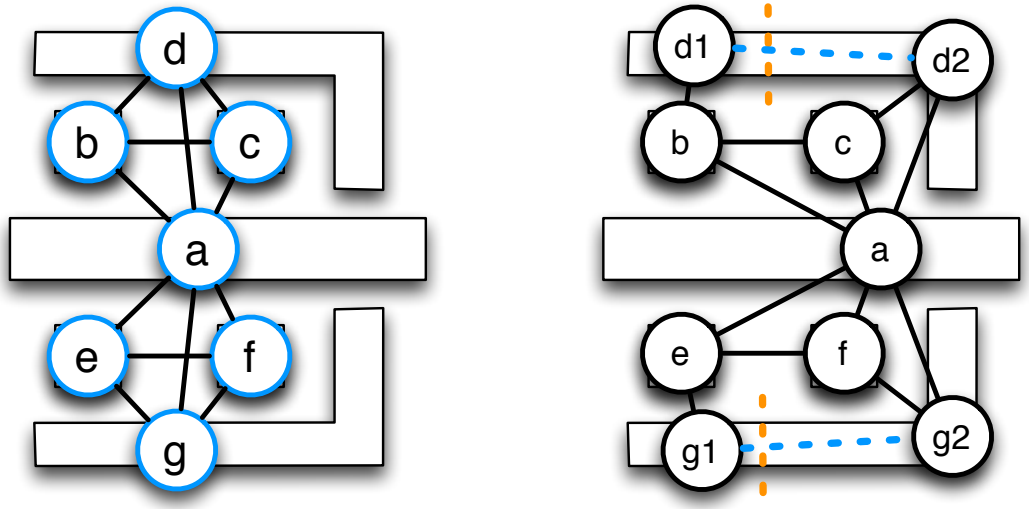


Cut Vertex Stitch Forbiddance

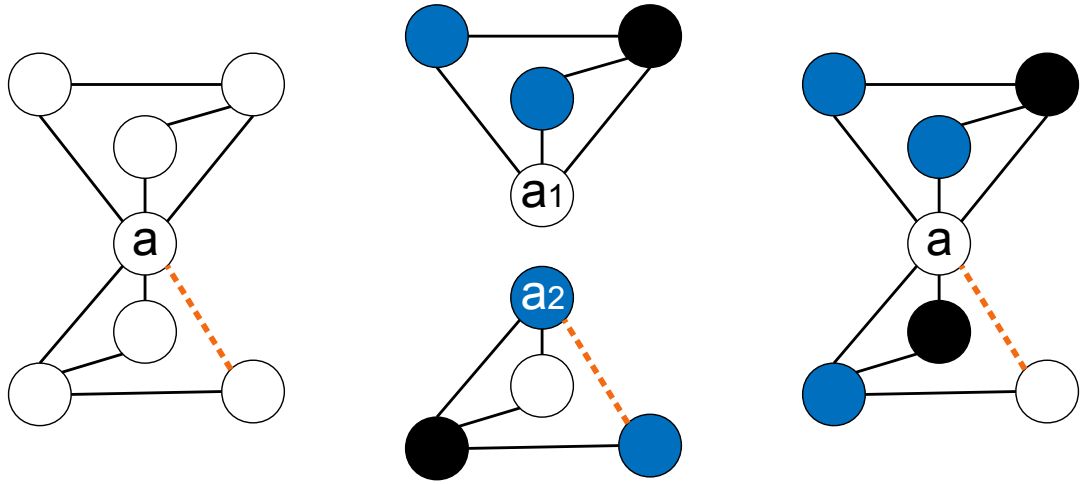
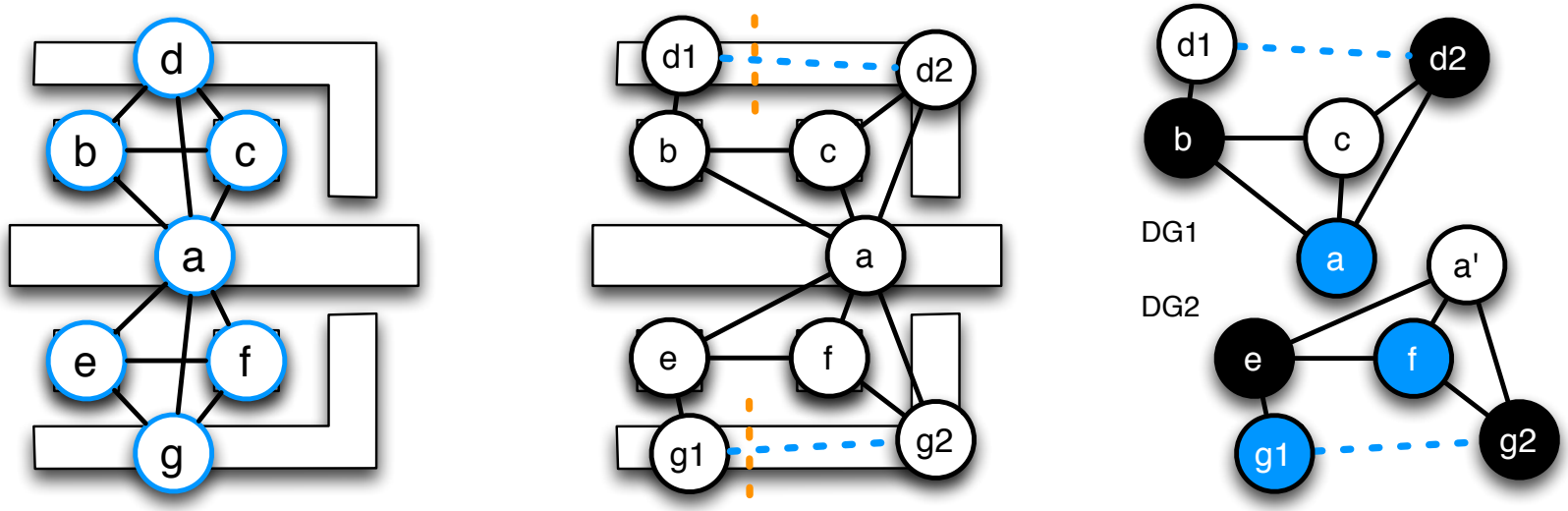
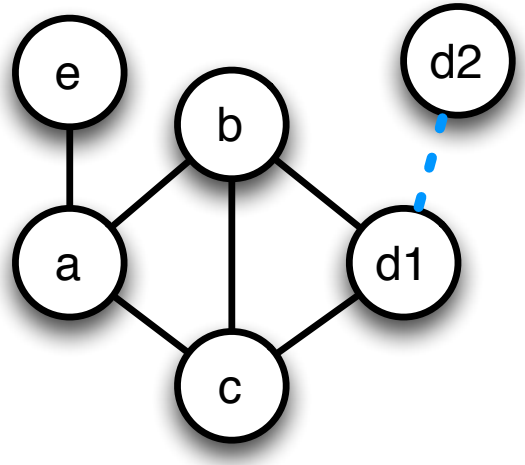


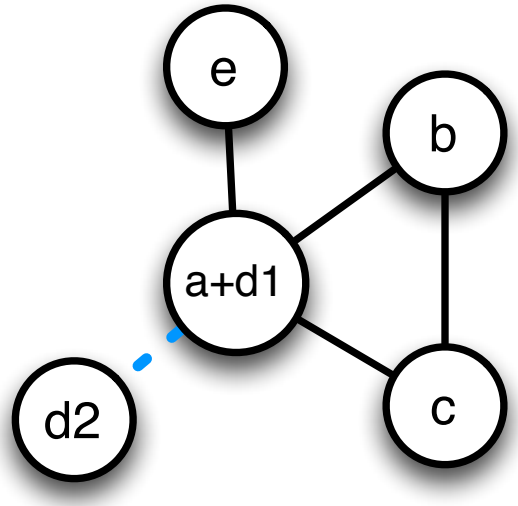
Figure : Cut Computation [Fang+,DAC'12]



Vertex Clustering



(a)

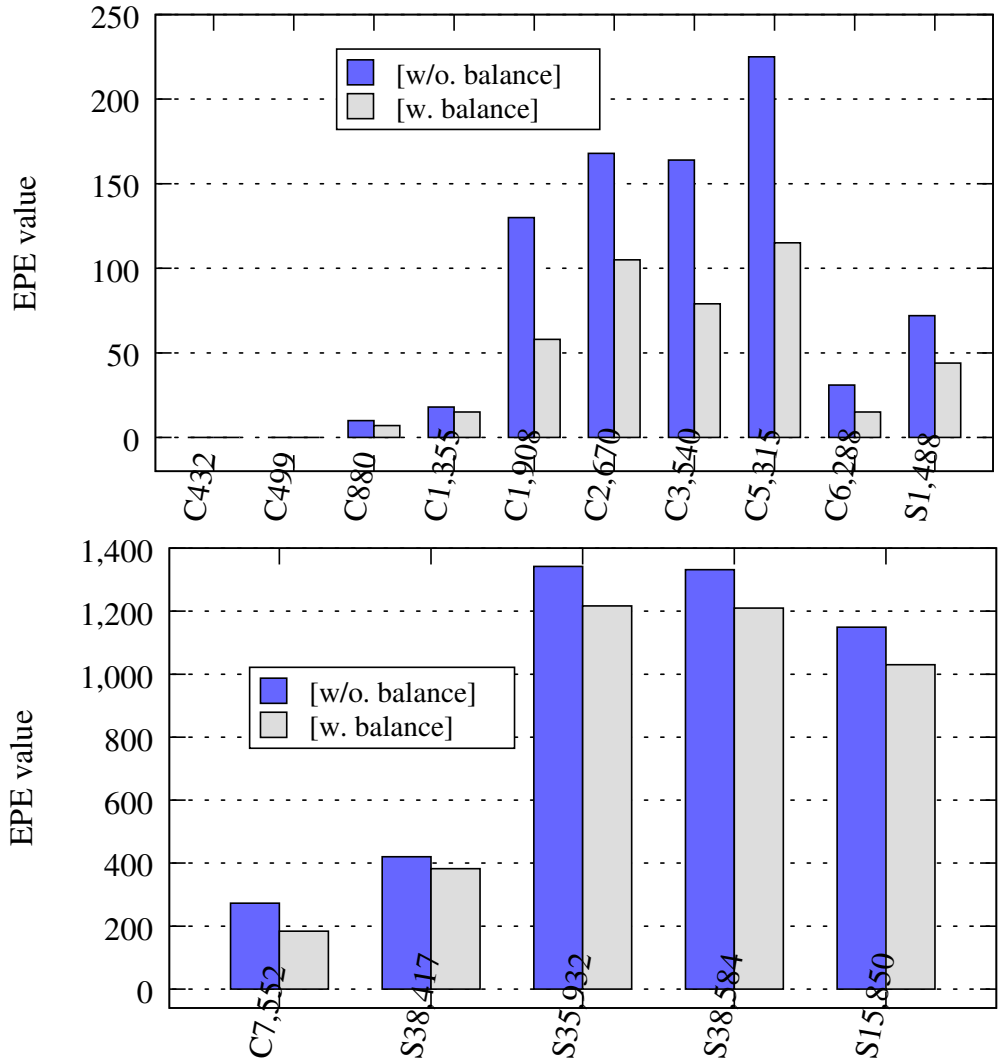


(b)

Experimental Results– Setup

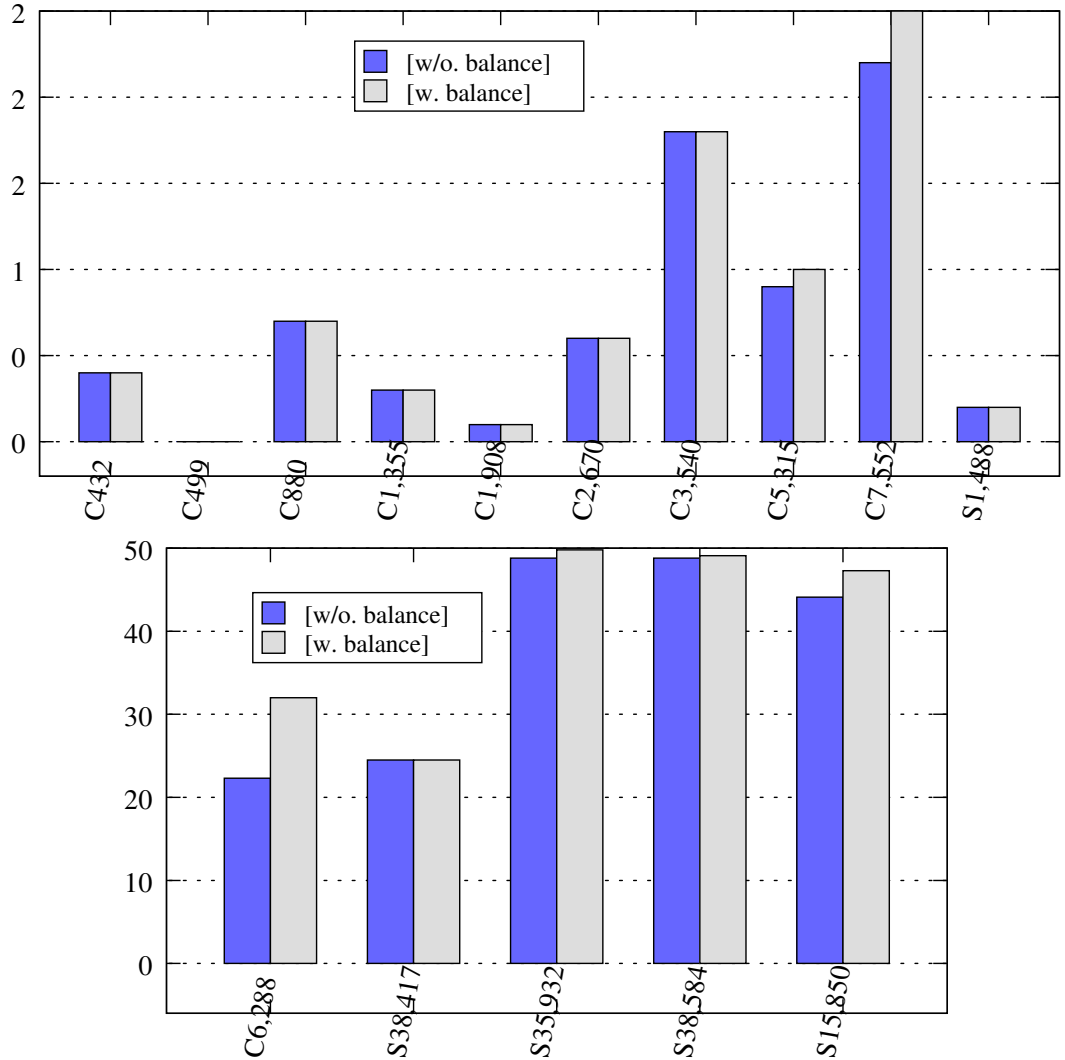
- Using C++ on 3.0GHz Linux machine
- **CSDP** as SDP solver
- Benchmarks
  - ISCAS benchmarks from [Yu+,ICCAD'11]
  - Two OpenSPARC T1 benchmarks mul_top and exu_ecc
  - Six dense benchmarks c9_total – s5_total

With or Without Density Balance?



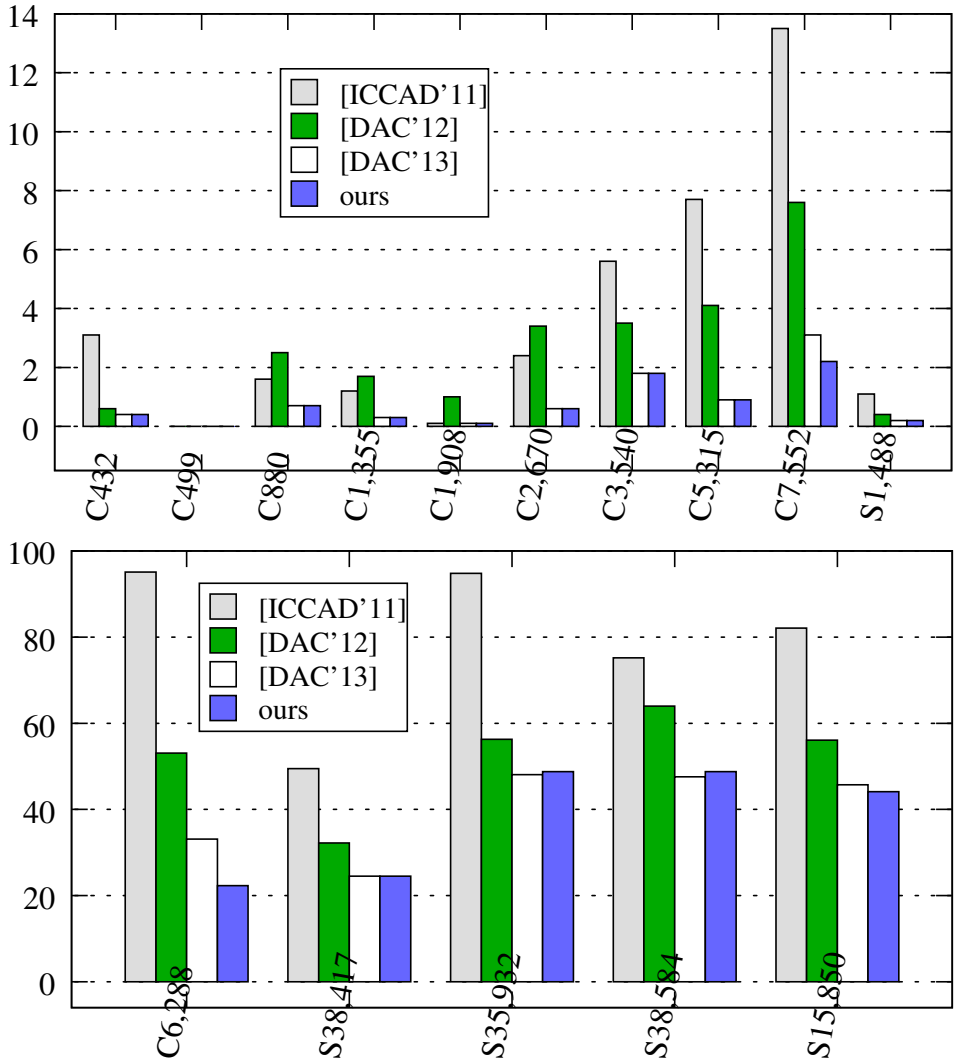
- Considering balance, -14% EPE#
- EPE: Edge placement error

With or Without Density Balance? (cont.)



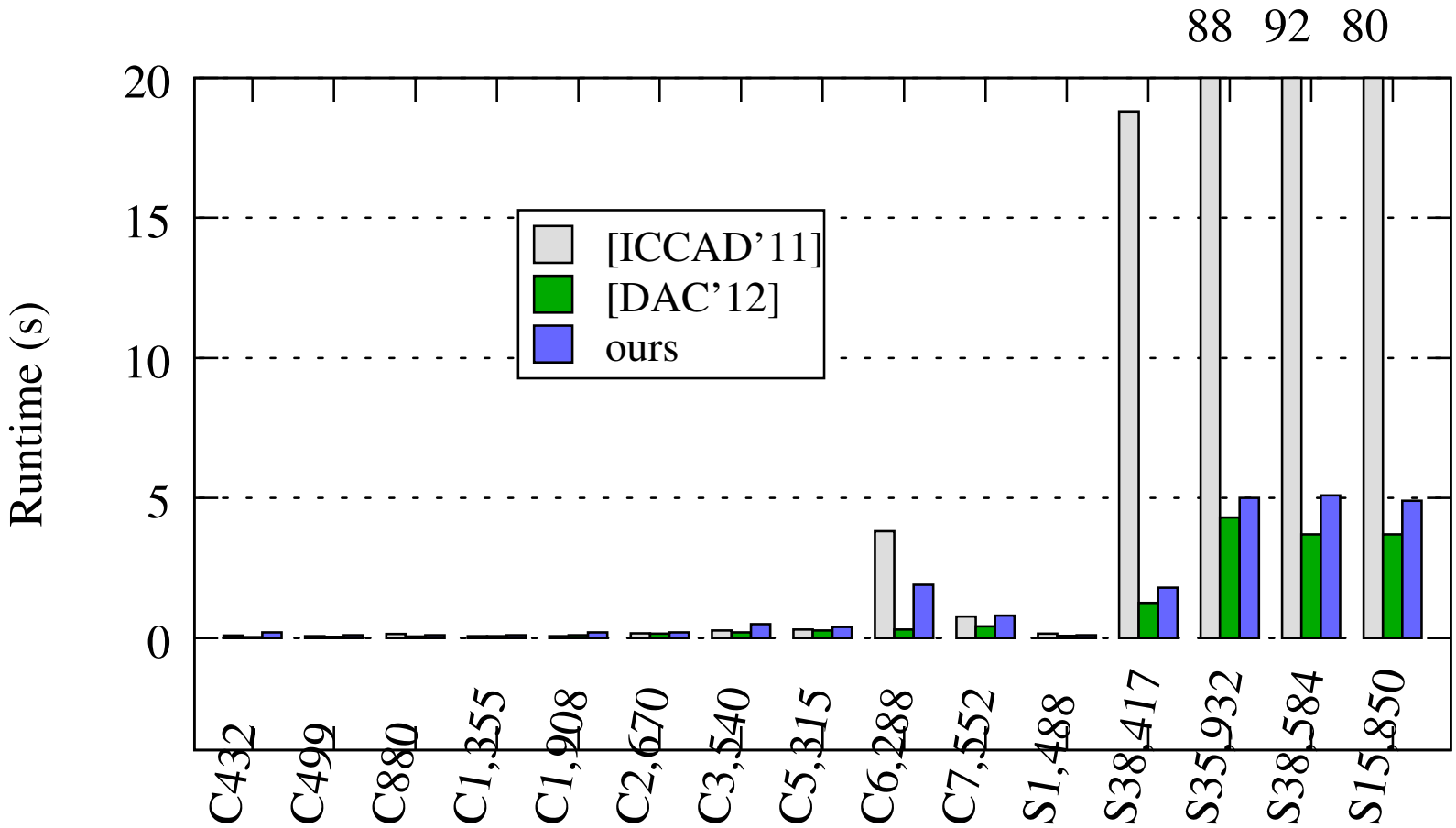
- Considering balance, +4% cost penalty
- Cost = conflict# + 0.1 stitch#

V.S. Other Decomposers ([Yu+,ICCAD'11] cases)



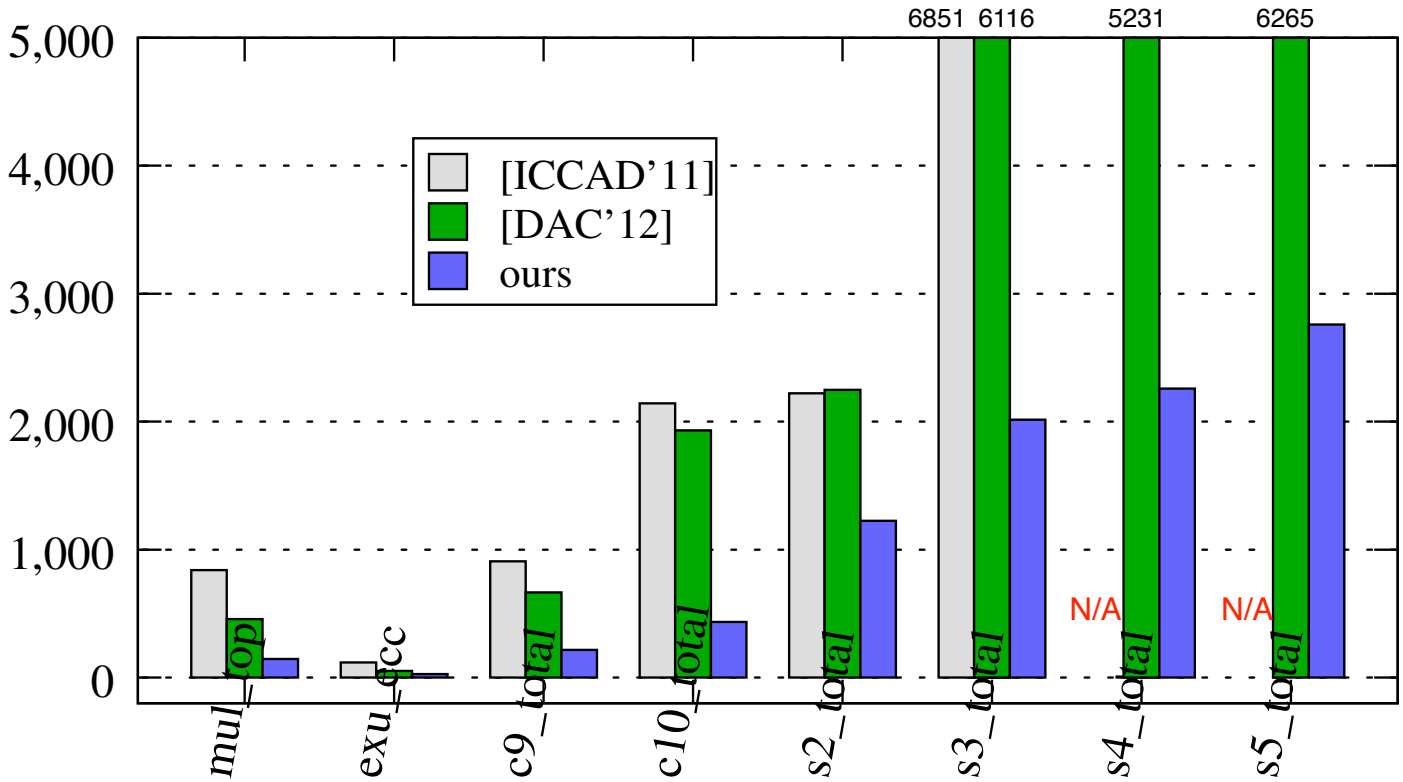
- Cost: -55% (v.s. ICCAD'11), -25% (v.s. DAC'12), -5% (v.s. DAC'13)

V.S. Other Decomposers ([Yu+,ICCAD'11] cases)



V.S. Other Decomposers (other cases)

Cost comparison

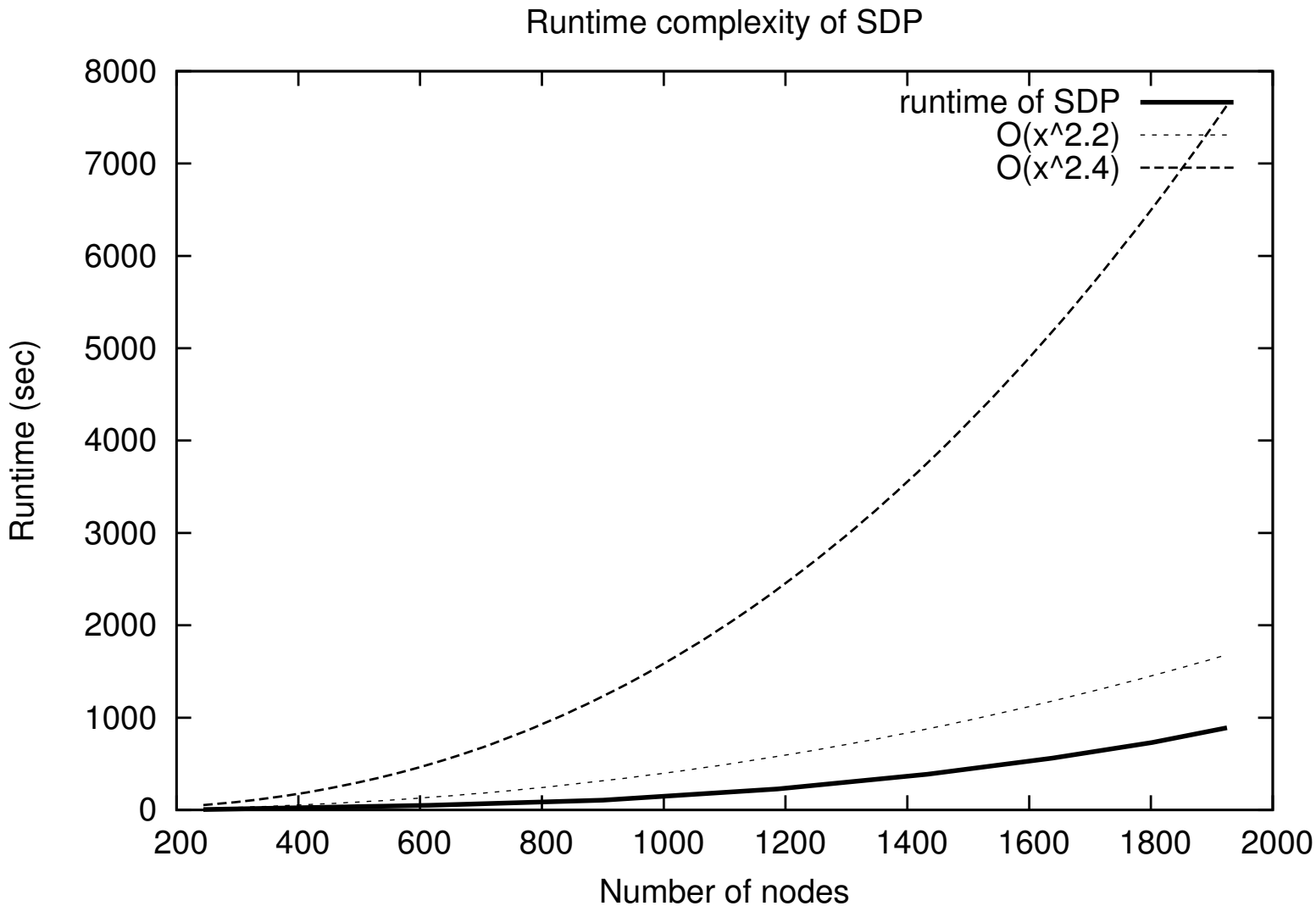


- -60% (v.s. ICCAD'11), -60% (v.s. DAC'12)

Runtime comparison

- ICCAD'11 : DAC'12 : ours = 3600s : 6s : 134s

Scalability of SDP



Conclusions

- Integrate density balance
  - SDP formulation
  - Mapping
  - Graph simplification
- High performance
  - Mapping
  - Graph model
- Faster
  - A set of graph simplification

Thank You !

Acknowledgement

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