How Good Are Slicing Floorplans? *

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Abstract

Given a set of modules with flexibility in shape, we show that there exists a slicing floorplan F such that $area(F) \leq \min\{(1+\frac{1}{\lfloor\sqrt{r}\rfloor}), \frac{5}{4}, (1+\alpha)\}A_{total}$ where A_{total} is the total area of all the modules, A_{max} is the maximum module area, $\alpha = \sqrt{\frac{2A_{max}}{rA_{total}}}$ and $r \geq 2$ is the shape flexibility of each module. Our result shows that slicing floorplans can provably pack modules tightly when the modules have flexibility in shape.

Keywords : slicing floorplan, module, aspect ratio, shape flexibility

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1 Introduction

Floorplan design plays an important role in the design of VLSI circuits in today's deep submicron technology. A slicing floorplan is a floorplan which can be obtained by recursively dividing a rectangle into two parts with either a vertical line or a horizontal line. Since slicing floorplans have very simple solution representations (e.g. slicing tree [4], Polish expression [6] etc.), it is easier to design efficient strategies to search for optimal slicing floorplans. As a result, slicing floorplans are used in many existing floorplanning systems [4, 3, 6, 5]. The only possible disadvantage of slicing floorplans is that even the optimal one may not pack the modules tightly and hence results in large chip area. Although, there are empirical evidences showing that slicing floorplans are quite good in packing modules tightly, it is important to have assurance of their performance by mathematical analysis.

Let R be a rectangle. We use $height(R)$, $width(R)$ and $area(R)$ to denote the height, the width and the area of R respectively. The *aspect ratio* of R is the ratio $height(R)/width(R)$. A *soft rectangle* is one which can have different shapes as long as the area remains the same. The *shape flexibility* of a soft rectangle specifies the range of its aspect ratio. A soft rectangle of area A is said to have a shape flexibility r if and only if R can be represented by any rectangle of area A as long as:

$$\frac{1}{r} \leq \frac{height(R)}{width(R)} \leq r \tag{1}$$

In our floorplan design problem, we are given n soft rectangles of area A_i for $i = 1, 2, \ldots, n$ and a shape flexibility r, we want to obtain an upper bound on the area of the optimal slicing floorplan. This is done by constructing a slicing floorplan F of these rectangles such that every rectangle satisfies the aspect ratio constraint in (1) and the area of F is as small as possible. We use A_{total} to denote $\sum_{i=1}^{n} A_i$ and use A_{max} to denote $\max_{1 \leq i \leq n}\{A_i\}$. Our objective is to minimize the dead space in F, $\Delta(F)$, which is defined as $\Delta(F) = area(F) - A_{total}$

In this paper, we show an upper bound for the area of the optimal slicing floorplan. We prove that if the rectangles have a shape flexibility of $r \geq 2$, there exists a slicing floorplan F of these rectangles such that $area(F) \leq \min\{(1+\frac{1}{\lfloor\sqrt{r}\rfloor}),\ \frac{5}{4},\ (1+\alpha)\}A_{total}$ where $\alpha = \sqrt{\frac{2A_{max}}{rA_{total}}}$, and the shape of the constructed floorplan resembles a square closely. The first term favors large r, e.g. when $r = 9$, $(1 + \frac{1}{\lfloor\sqrt{r}\rfloor}) = \frac{4}{3}$. The second term gives a better bound than the first one when r is small. The third term takes into account the relative sizes of the areas and it gives a good bound when all the areas are small comparing with the total area, e.g. when $r = 2$ and $A_{max} = \frac{A_{total}}{100}$, the percentage of dead space in the optimal slicing floorplan is at most 9%.

We will prove the main result in section 2 and section 3 gives some concluding remarks.

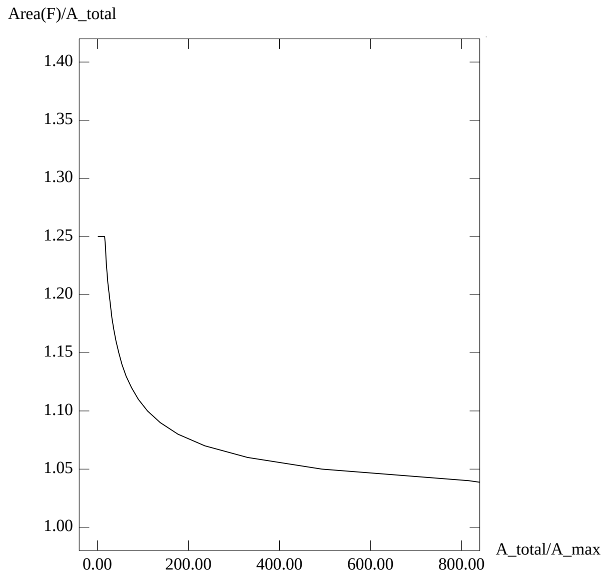

Figure 1: *Upper bound on the area of optimal slicing floorplan v.s. relative maximum area. Assume* $r = 2$.

2 Main Result

Our goal is to understand how good slicing floorplans are in packing soft modules. We have the following theorem:

Theorem 1 *Given a set of soft rectangles of total area* A_{total}*, maximum area* A_{max} *and shape flexibility* $r \geq 2$*, there exists a slicing floorplan* F *of these rectangles such that*

$$area(F) \leq \min\{(1 + \frac{1}{\lfloor\sqrt{r}\rfloor}),\ \frac{5}{4},\ (1 + \alpha)\}A_{total} \tag{2}$$

where $\alpha = \sqrt{\frac{2A_{max}}{rA_{total}}}$. *Moreover, we have*

$$1 \leq \frac{height(F)}{width(F)} \leq \begin{cases} (1 + \frac{1}{\lfloor\sqrt{r}\rfloor}) & area(F) \leq (1 + \frac{1}{\lfloor\sqrt{r}\rfloor})A_{total}, \\ \frac{5}{4} & area(F) \leq \frac{5}{4}A_{total}, \\ \frac{1+\alpha}{(1-\frac{r\alpha}{2})^2} & area(F) \leq (1 + \alpha)A_{total}. \end{cases}$$

Figure 1 shows the relationships in (2). We assume that $r = 2$, so the first term does not have any effect. The second term dominates until $\frac{A_{total}}{A_{max}} > 16$. Then the upper bound on the area of the optimal slicing floorplan drops with increasing $\frac{A_{total}}{A_{max}}$ until reaching the lower bound A_{total}, when all the areas are infinitely small comparing with A_{total}.

Theorem 1 follows directly from Lemma 1, Lemma 2 and Lemma 3. Notice that Lemma 1 applies only when the shape flexibility r is at least four, but when $2 \leq r < 4$, the term $(1 + \frac{1}{\lfloor\sqrt{r}\rfloor}) > \frac{3}{2} > \frac{5}{4}$. Therefore Theorem 1 still holds. We will prove Lemmas 1-3 in the following subsections.

2.1 A General Upper Bound

In the following, we want to show that if the shape flexibility of the soft rectangles is at least four, there exists a slicing floorplan F in which dead space is at most $\frac{1}{\lfloor\sqrt{r}\rfloor}$ of A_{total}. The shape of F resembles a square as r increases in such a way that $1 \leq \frac{height(F)}{width(F)} \leq (1 + \frac{1}{\lfloor\sqrt{r}\rfloor})$. For example, when $r = 9$, F has at most $\frac{1}{4}A_{total}$ dead space and $1 \leq \frac{height(F)}{width(F)} \leq \frac{4}{3}$

The analysis is done by constructing a simple slicing floorplan of those given soft rectangles. The areas are classified into groups such that area A is in group i when $\frac{1}{r^i} \leq A < \frac{1}{r^{i-1}}$ for $i = 1, 2, 3, \ldots$ An area A from group i will be represented by a rectangle R of width $1/r^{\frac{i-1}{2}}$ and height $r^{\frac{i-1}{2}}A$. We pack the rectangles one at a time from the largest to the smallest. When we pack a rectangle, it is always put on the lowest possible level and is pushed to the leftmost position on that level. Since the widths of the rectangles decrease by $\frac{1}{\lfloor\sqrt{r}\rfloor}$ from one group to another, there must be enough horizontal space when packing a rectangle. No dead space occur in the final floorplan, except those along the upper boundary. An example is shown in Figure 2 in which we assume that $r = 9$, so the widths of the rectangles decrease by $\frac{1}{3}$ from one group to another, and the packing is perfect except along the upper boundary.

This result gives a relationship between the size of the dead space and the shape flexibility r. It is obvious that the amount of dead space will decrease with the flexibility and it becomes infinitely small when the rectangles have very large flexibility.

Lemma 1 *Given a set of soft rectangles of total area A_{total} and shape flexibility $r \geq 4$, there exists a slicing floorplan F of these rectangles such that*

$$area(F) \leq (1 + \frac{1}{\lfloor\sqrt{r}\rfloor})A_{total}$$

and

$$1 \leq \frac{height(F)}{width(F)} \leq (1 + \frac{1}{\lfloor\sqrt{r}\rfloor})$$

Proof In the following, we assume that the given shape flexibility r is a perfect square. If this is not the case, we will take r as the largest perfect square smaller than the given shape flexibility. W.l.o.g. we assume that $A_{total} = 1$. The areas are classified into groups according to their sizes such that area A is in group i if and only if $\frac{1}{r^i} \leq A < \frac{1}{r^{i-1}}$ for $i = 1, 2, 3, \ldots$ We will construct a slicing floorplan F by packing the areas one at a time from the largest to the smallest. F has a width one. (Note that the areas are scaled to have $A_{total} = 1$.) An area A from group i will be represented by a rectangle R of width $1/r^{\frac{i-1}{2}}$ and height $r^{\frac{i-1}{2}}A$. Notice that $\frac{height(R)}{width(R)} = r^{i-1}A$, so $\frac{1}{r} \leq \frac{height(R)}{width(R)} < 1$ and the aspect ratio constraint is not violated. During packing, a rectangle is always put on the lowest possible level and is pushed to the leftmost position on that level. Since the widths of the rectangles decrease by $\frac{1}{\sqrt{r}}$ from one group to another, there must be enough horizontal space when packing a rectangle on the lowest possible level. The packing is perfect except some dead space occurs along the irregular upper boundary. Consider the highest rectangle R' in F, its lower boundary must be at a level below one, because $A_{total} > 1$ otherwise. Thus the maximum height of the rectangles gives an upper bound on the size of the dead space. Table 1 tabulates the areas, the heights and the widths of different groups. Since group 1 will not create any dead space, the dead space size is upper bounded by $\frac{1}{\lfloor\sqrt{r}\rfloor}$. It is not difficult to see that the final packing gives a slicing floorplan. An example is shown in Figure 2.

$\square$

	Area A	Width w	Height h
1	$\frac{1}{r} \leq A \leq 1$	$w = 1$	$\frac{1}{r} \leq h \leq 1$
2	$\frac{1}{r^2} \leq A < \frac{1}{r}$	$w = \frac{1}{\sqrt{r}}$	$\frac{1}{r\sqrt{r}} \leq h < \frac{1}{\sqrt{r}}$
3	$\frac{1}{r^3} \leq A < \frac{1}{r^2}$	$w = \frac{1}{r}$	$\frac{1}{r^2} \leq h < \frac{1}{r}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$
$2i$	$\frac{1}{r^{2i}} \leq A < \frac{1}{r^{2i-1}}$	$w = \frac{1}{r^{\frac{2i-1}{2}}}$	$\frac{1}{r^{\frac{2i+1}{2}}} \leq h < \frac{1}{2^{\frac{2i-1}{2}}}$
$2i+1$	$\frac{1}{r^{2i+1}} \leq A < \frac{1}{r^{2i}}$	$w = \frac{1}{r^i}$	$\frac{1}{r^{i+1}} \leq h < \frac{1}{r^i}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$

Table 1: *Classification of Areas in Lemma 1*

2.2 A Better Bound for Small Shape Flexibility

The result of Lemma 1 gives a good upper bound when the shape flexibility r is large. For small r, we can obtain a better bound by modifying the packing strategy and post-processing the constructed floorplan. The areas are also classified into groups and areas in different groups are represented by rectangles of different widths. Again we pack the rectangles one at a time from the largest to the smallest, and we always put a rectangle on the lowest possible level and push it to the leftmost position on that level. One big difference from the proof of Lemma 1 is that the widths of the rectangles now decrease by half from one group to another. In Lemma 1, the widths of the rectangles are dependent on r, but

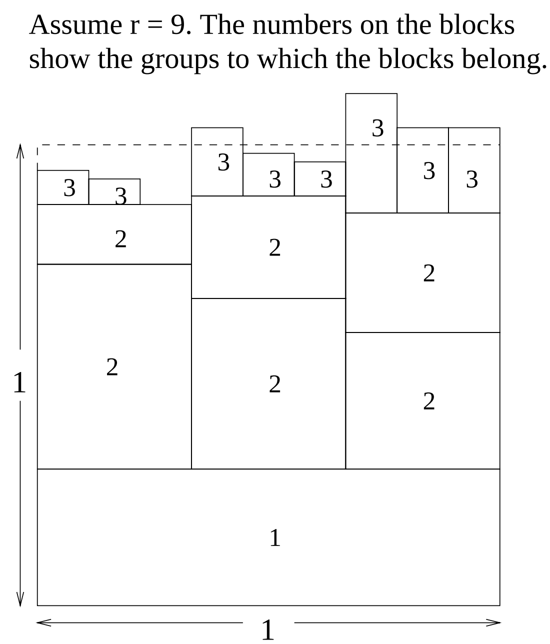

Figure 2: *A simple example on the slicing floorplans constructed by Lemma 1*

this is not the case here. Another difference is that after packing all the rectangles, we need to do some post-processing steps to rearrange some rectangles in order to obtain the desired bound. Again no dead space occurs in the interior, except those along the upper boundary. An example is shown in Figure 3.

In the following, we assume that the shape flexibility r is at least two, and we can construct a slicing floorplan in which dead space is at most $\frac{1}{4}$ of the total area A_{total}.

Lemma 2 *Given a set of soft rectangles of total area A_{total} and shape flexibility $r \geq 2$, there exists a slicing floorplan F of these rectangles such that*

$$area(F) \leq \frac{5}{4} A_{total}$$

and

$$1 \leq \frac{height(F)}{width(F)} \leq \frac{5}{4}$$

Proof W.l.o.g. we assume that $A_{total} = 1$. Again we classify the areas into groups and an area A is in group i if and only if $\frac{1}{2^{2i-1}} \leq A < \frac{1}{2^{2i-3}}$ for $i = 1, 2, 3, \ldots$ The widths of the rectangles are halved from one group to another. Table 2 tabulates the areas, the widths and the heights of different groups. Here we cannot obtain an upper bound of $\frac{1}{4}$ directly

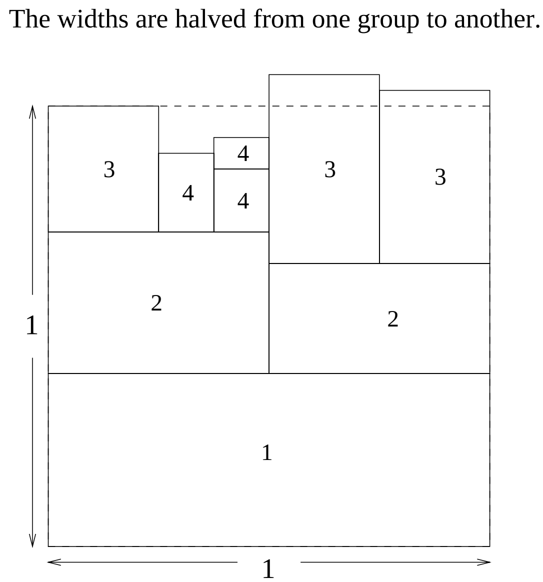

Figure 3: *A simple example on the slicing floorplans constructed by Lemma 2*

from the height. Consider the highest rectangle R in the constructed floorplan F. Let the height of R be h and the width be w. Suppose $(1-\alpha)wh$ of R, where $\alpha < 1$, is above the unit level. It is easy to see from Figure 4 that:

$$\begin{aligned} (1-\alpha)wh &\leq \alpha(1-w)h & (3) \\ \alpha &\geq w & (4) \end{aligned}$$

Therefore we can use $(1-w)h$ to upper bound the size of the dead space. However $(1-w)h$ may exceed $\frac{1}{4}$ in group 2 (when $\frac{1}{4} < A < \frac{1}{2}$) and in group 3 (when $\frac{1}{12} < A < \frac{1}{8}$). we will post-process the packing in F to obtain the desired bound. Lets consider all the cases in which the highest rectangle R have height h and width w such that $h(1-w) > \frac{1}{4}$:

Case 1 The highest rectangle comes from group 2. There are only two possibilities in which the highest rectangle has an area between $\frac{1}{4}$ and $\frac{1}{2}$ exclusively:

Subcase (i) There are one rectangle of area $\frac{1}{2} \leq A_1 \leq 1$ and one rectangle of area $\frac{1}{4} < A_2 < \frac{1}{2}$:
Let $A_1 = \frac{1}{2} + x$ where $0 \leq x < \frac{1}{4}$. ($x < \frac{1}{4}$ since $A_1 + A_2 > \frac{3}{4} + x$) Consider three separate cases:

$A_2 \leq \frac{3}{8} - \frac{x}{2}$. Then $height(A_1) + height(A_2) \leq (\frac{1}{2}+x) + (\frac{3}{8}-\frac{x}{2})/\frac{1}{2} = \frac{1}{2}+x+\frac{3}{4}-x = \frac{5}{4}$. The bound is not exceeded.

$A_2 > \frac{3}{8} - \frac{x}{2}$ and $\sqrt{2A_2} \geq \frac{3}{4}$. (That means A_2 can be a rectangle of width $\frac{3}{4}$.) Then $1 - A_1 - A_2 < 1 - (\frac{1}{2} + x) - (\frac{3}{8} - \frac{x}{2}) = \frac{1}{8} - \frac{x}{2} \leq \frac{1}{8}$. Therefore all the remaining rectangles have width $w \leq \frac{1}{4}$. We can pack A_2 as a rectangle of width $\frac{3}{4}$ (Figure 5). Then $height(A_1) + height(A_2) \leq (\frac{1}{2} + x) + (\frac{1}{2} - x)/\frac{3}{4} \leq \frac{7}{6}$. The bound is not exceeded.

$A_2 > \frac{3}{8} - \frac{x}{2}$ and $\sqrt{2A_2} < \frac{3}{4}$. (That means the longest side of A_2 cannot be $\frac{3}{4}$.) Then $A_2 < \frac{9}{32}$. Since $A_2 > \frac{3}{8} - \frac{x}{2}$, $x > \frac{3}{16}$. $1 - A_1 - A_2 < 1 - (\frac{1}{2} + x) - (\frac{3}{8} - \frac{x}{2}) = \frac{1}{8} - \frac{x}{2} < \frac{1}{32}$. Therefore all the remaining rectangles have width $w \leq \frac{1}{8}$. We can pack A_2 as a rectangle of width $\frac{5}{8}$ (Figure 6). (Notice that $\sqrt{\frac{A_2}{2}} < \frac{5}{8} < \sqrt{2A_2}$.) Then $height(A_1) + height(A_2) < (\frac{1}{2} + x) + \frac{9}{32}/\frac{5}{8} = \frac{19}{20} + x < \frac{5}{4}$. The bound is not exceeded.

Subcase (ii) There are three rectangles of area $\frac{1}{4} < A < \frac{1}{2}$:

Let $A_1 = \frac{1}{4} + x$, $A_2 = \frac{1}{4} + y$ and $A_3 = \frac{1}{4} + z$ where $x + y + z \leq \frac{1}{4}$. W.l.o.g. let $x \leq y \leq z$. Consider two separate cases:

$x + y \leq \frac{1}{8}$. Then $height(A_1) + height(A_2) \leq (\frac{1}{4} + x)/\frac{1}{2} + (\frac{1}{4} + y)/\frac{1}{2} \leq \frac{5}{4}$. So the bound is not exceeded.

$x + y > \frac{1}{8}$. Since $x + y > \frac{1}{8}$, $z < \frac{1}{8}$. That means all x, y and z are less than $\frac{1}{8}$. Besides $x + y > \frac{1}{8}$, thus $y > \frac{1}{16}$ and so as z. Consider $x + y + z > \frac{1}{8} + z > \frac{3}{16}$. Therefore $1 - A_1 - A_2 - A_3 = 1 - (\frac{1}{4} + x) - (\frac{1}{4} + y) - (\frac{1}{4} + z) < \frac{1}{16}$, which means the total area of the remaining rectangles is less than $\frac{1}{16}$. We shuffle the positions of A_1 (the smallest one) and A_3 (the largest one), and pack A_3 as a rectangle of width $\frac{3}{4}$ as in Figure 7. (Notice that $\frac{1}{16} < z < \frac{1}{8}$, so $\frac{5}{16} < A_1 < \frac{3}{8}$ and $\sqrt{\frac{A_1}{2}} < \frac{3}{4} < \sqrt{2A_1}$.) Then $height(A_2) + height(A_3) = (\frac{1}{4} + y)/\frac{1}{2} + (\frac{1}{4} + z)/\frac{3}{4} = \frac{1}{2} + 2y + \frac{1}{3} + \frac{4z}{3} < \frac{5}{4}$. The bound is not exceeded. For the remaining rectangles, we can pack them in the empty space sitting beside A_3, which has width $\frac{1}{4}$ and height at least $\frac{5}{12}$ (because the height of A_3 is $(\frac{1}{4} + z)/\frac{3}{4} = \frac{1}{3} + \frac{4z}{3} > \frac{1}{3} + \frac{1}{12} = \frac{5}{12}$). Since the total area of the remaining rectangles is less than $\frac{1}{16}$, we only need a space of $\frac{1}{4} \times (\frac{1}{4} \times \frac{5}{4})$ by arguing inductively on the number of rectangles, where the base case is the trivial condition that there is only one rectangle.

Case 2 The highest rectangle comes from group 3:

Let $\frac{1}{12} < A_1 < \frac{1}{8}$ be the area of the highest rectangle.

Subcase (i) Besides A_1, a width of at least $\frac{1}{2}$ is above the unit level in the final packing (Figure 8(a)). Let $(1 - \alpha)$ of A_1 is above the unit level where $\alpha < 1$. Then $(1 - \alpha)h \times \frac{1}{4} \leq \frac{\alpha h}{4}$, so $\alpha \geq \frac{1}{2}$, and $(1 - \alpha)h \leq \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The bound is not exceeded.

Subcase (ii) Besides A_1, a width of less than $\frac{1}{2}$ is above the unit level in the final packing (Figure 8(b)). Consider two separate cases:

Except A_1, no $\frac{1}{12} < A_2 < \frac{1}{8}$ in the region above the unit level (shaded in Figure 9(a)). Since $\sqrt{2A_1} < \frac{1}{2}$, we can pack A_1 as a rectangle of width $\sqrt{2A_1}$ (Figure 9). Besides $\sqrt{\frac{A_1}{2}} < \frac{1}{4}$, so the bound is not exceeded.

Another $\frac{1}{12} < A_2 < \frac{1}{8}$ in the region above the unit level (shaded in Figure 10(a)). Let the height of A_2 be h' and $(1-\alpha)$ of A_2 is above the unit level where $\alpha < 1$. Then $(1-\alpha)h' \times \frac{1}{4} \times 2 \leq \frac{\alpha h'}{2}$, so $\alpha \geq \frac{1}{2}$ and $(1-\alpha)h' \leq \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Thus the height of A_2 does not exceed the bound. Similarly, we can pack A_1 as a rectangle of width $\sqrt{2A_1} < \frac{1}{2}$ (Figure 10). Since $\sqrt{\frac{A_1}{2}} < \frac{1}{4}$, the bound is not exceeded by A_1 neither.

$\square$

	Area A	Width w	Height h	$B = h(1-w)$
1	$\frac{1}{2} \leq A \leq 1$	$w = 1$	$\frac{1}{2} \leq h \leq 1$	0
2	$\frac{1}{8} \leq A < \frac{1}{2}$	$w = \frac{1}{2}$	$\frac{1}{4} \leq h < 1$	$\frac{1}{8} \leq B < \frac{1}{2}$
3	$\frac{1}{32} \leq A < \frac{1}{8}$	$w = \frac{1}{4}$	$\frac{1}{8} \leq h < \frac{1}{2}$	$\frac{3}{32} \leq B < \frac{3}{8}$
4	$\frac{1}{128} \leq A < \frac{1}{32}$	$w = \frac{1}{8}$	$\frac{1}{16} \leq h < \frac{1}{4}$	$\frac{7}{128} \leq B < \frac{7}{32}$
5	$\frac{1}{512} \leq A < \frac{1}{128}$	$w = \frac{1}{16}$	$\frac{1}{32} \leq h < \frac{1}{8}$	$\frac{15}{512} \leq B < \frac{15}{128}$
...	...	...	...	...

Table 2: *Classification of Areas in Lemma 2*

2.3 Another Bound Considering the Relative Sizes of the Areas

In the above analyses, we did not take into account the relative sizes of the rectangles. It should be reasonable to predict a better packing if all the rectangles are small comparing with A_{total}. We will consider this factor in the following.

The floorplan is divided into columns of equal width W initially where the value W depends on A_{max}. We classify the areas into groups such that area A is in group i when $\frac{W^2}{4^{i-1}r} \leq A < \frac{W^2}{4^{i-2}r}$ for $i = 1, 2, 3, \ldots$ An area A from group i is represented as a rectangle R of width $\frac{W}{2^{i-1}}$ and height $\frac{2^{i-1}A}{W}$. Note that the widths of the rectangles decrease by half from one group to another. Then we pack the areas one at a time from the largest to the smallest, using the same strategy, i.e. pack the rectangle on the lowest possible level (among all the columns) and push it to the rightmost position "within that column". An example is shown

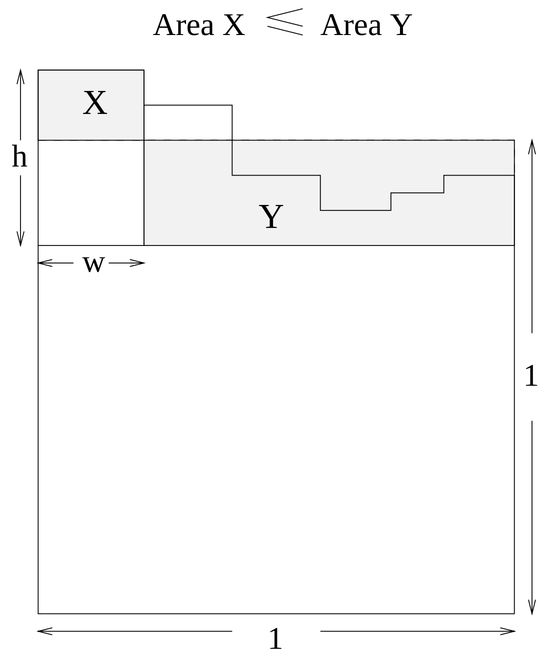

Figure 4: *An example showing the relationship in equation (3)*

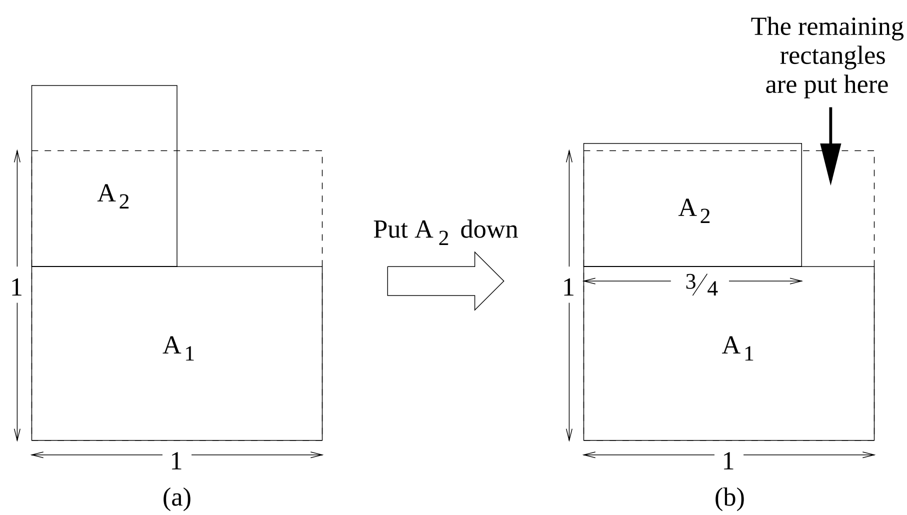

Figure 5: *A post-processing step in case 1(i) of Lemma 2*

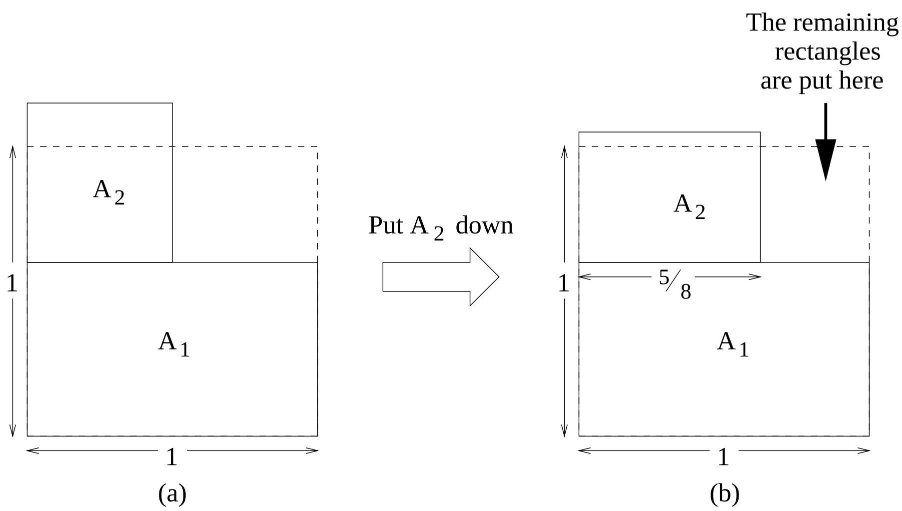

Figure 6: *A post-processing step in case 1(i) of Lemma 2*

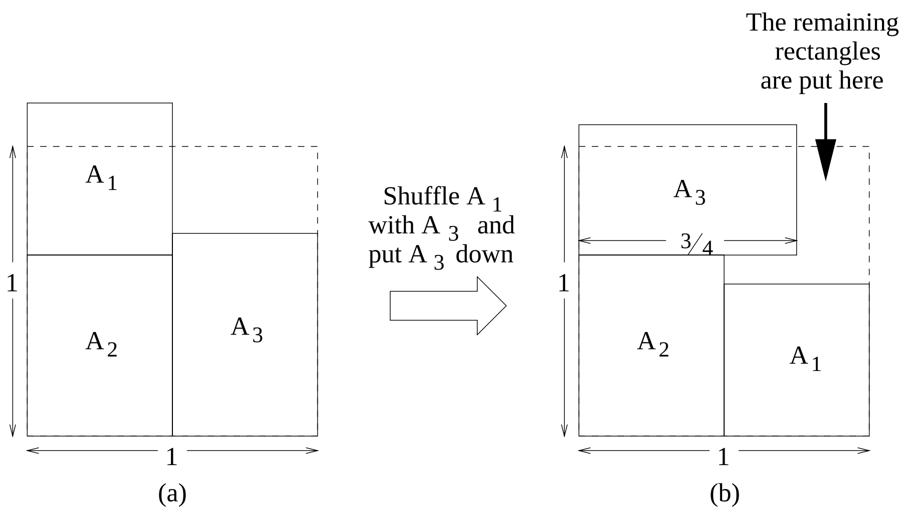

Figure 7: *A post-processing step in case 1(ii) of Lemma 2*

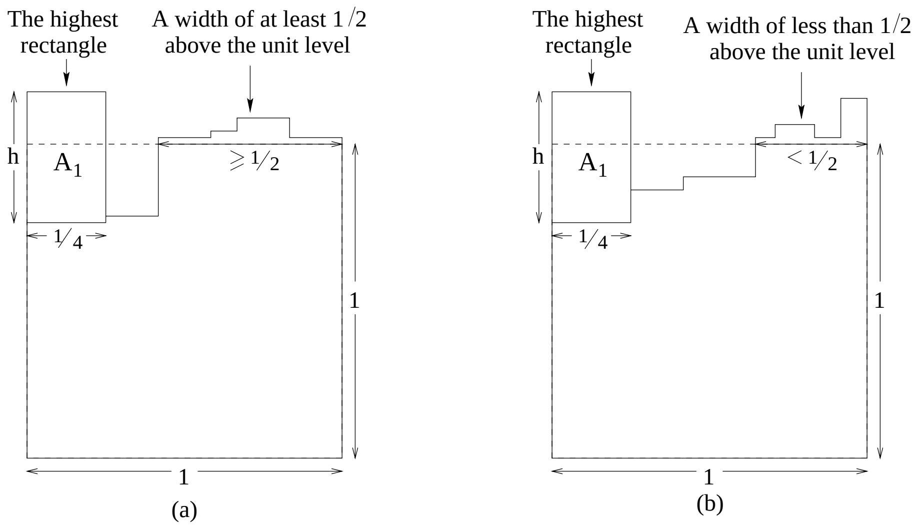

Figure 8: *An example showing the situation in case 2(i) and case 2(ii) of Lemma 2*

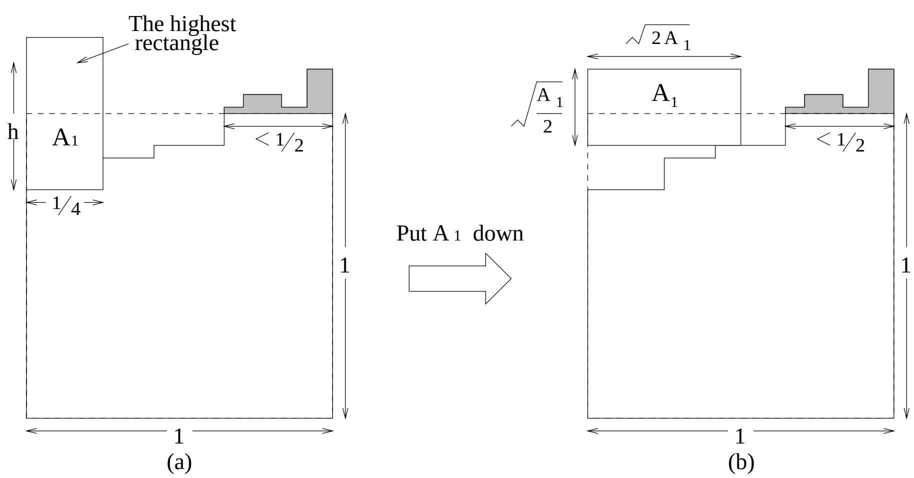

Figure 9: *A post-processing step in case 2(ii) of Lemma 2*

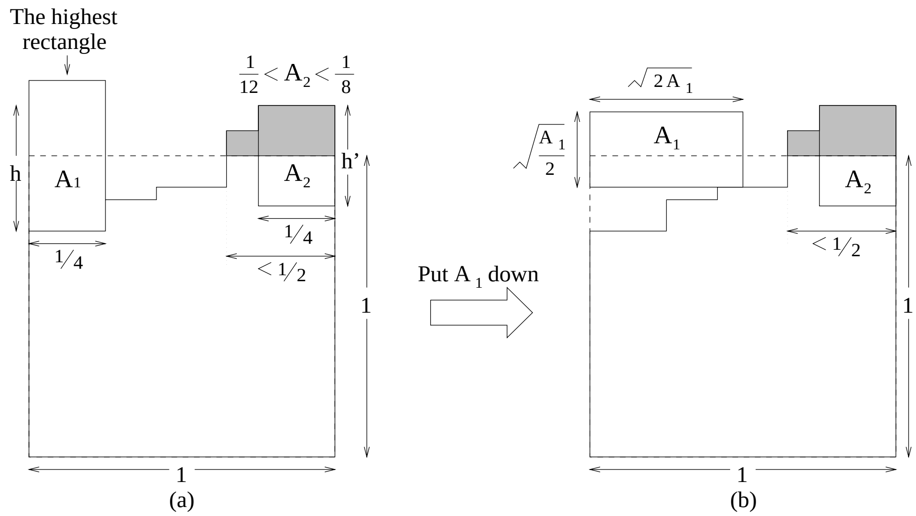

Figure 10: *A post-processing step in case 2(ii) of Lemma 2*

in Figure 11 in which we assume that $\lfloor \frac{\sqrt{A_{total}}}{W} \rfloor = 3$, so there are totally three columns. Again no dead space occurs in the interior, except those along the upper boundary.

We can show that the dead space in the resulting floorplan is at most $\sqrt{\frac{2A_{max}}{rA_{total}}}$ of A_{total}. For example, when $r = 2$, $A_{max} = \frac{A_{total}}{100}$, the percentage of dead space is at most 9%. Therefore, the smaller the maximum area comparing with the total, the better can be the packing. This result gives a good bound when all the areas are small in comparison with the total area.

Lemma 3 *Given a set of soft rectangles of total area A_{total}, maximum area A_{max} and shape flexibility $r \geq 2$, there exists a slicing floorplan F of these rectangles such that*

$$area(F) \leq (1+\alpha)A_{total}$$

where $\alpha = \sqrt{\frac{2A_{max}}{rA_{total}}}$. Moreover, we have

$$1 \leq \frac{height(F)}{width(F)} \leq \frac{1+\alpha}{(1-\frac{r\alpha}{2})^2}$$

Proof We construct a slicing floorplan F by dividing it into columns of fixed width W and packing the rectangles into these columns simultaneously. The areas are again classified into groups. The areas, the widths and the heights of different groups are shown in Table 3. We use a similar packing technique as before. Given a rectangle, we always put it on the lowest possible level (among all the columns) and push it to the leftmost position on that level "within the same column" (Figure 11).

If we set $W = \sqrt{\frac{rA_{max}}{2}}$, the height is at most $\sqrt{\frac{2A_{max}}{r}}$ in every group. Therefore we can upper bound the size of the dead space by hX, where $X = \lfloor \frac{\sqrt{A_{total}}}{W} \rfloor \times W$ is the width of the floorplan F:

$$\begin{aligned} \Delta(F) &\leq h\lfloor \frac{\sqrt{A_{total}}}{W} \rfloor \times W \\ &\leq \sqrt{\frac{2A_{max}}{r}}\sqrt{A_{total}} \\ &= \sqrt{\frac{2A_{max}}{rA_{total}}}A_{total} \\ &= \alpha A_{total} \end{aligned}$$

where $\alpha = \sqrt{\frac{2A_{max}}{rA_{total}}}$. Consider the aspect ratio of the final floorplan, it must be at least one since the width of F is at most $\sqrt{A_{total}}$. Also,

$$\begin{aligned} \frac{height(F)}{width(F)} &\leq \frac{\frac{(1+\alpha)A_{total}}{\sqrt{A_{total}}-W}}{\sqrt{A_{total}} - W} \\ &= \frac{1+\alpha}{(1-\frac{W}{\sqrt{A_{total}}})^2} \\ &= \frac{1+\alpha}{(1-\frac{r\alpha}{2})^2} \end{aligned}$$

$\square$

	Area A	Width w	Height h
1	$\frac{W^2}{r} \leq A \leq A_{max}$	$w = W$	$h \leq \frac{A_{max}}{W}$
2	$\frac{W^2}{4r} \leq A < \frac{W^2}{r}$	$w = \frac{W}{2}$	$h < \frac{2W}{r}$
3	$\frac{W^2}{16r} \leq A < \frac{W^2}{4r}$	$w = \frac{W}{4}$	$h < \frac{W}{r}$
4	$\frac{W^2}{64r} \leq A < \frac{W^2}{16r}$	$w = \frac{W}{8}$	$h < \frac{W}{2r}$
...	...	...	...

Table 3: *Classification of Areas in Lemma 3*

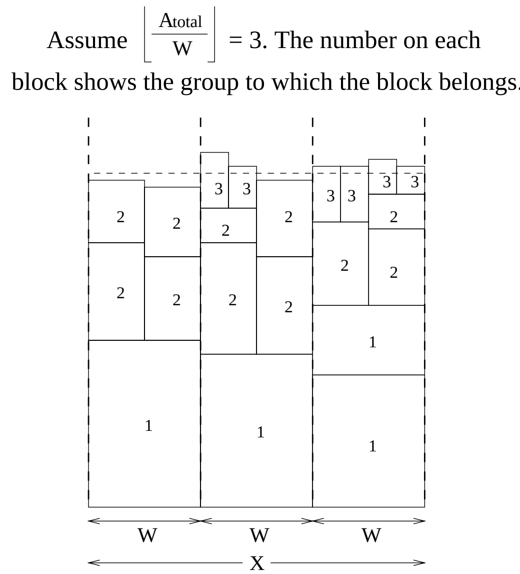

Figure 11: *An example on the slicing floorplans constructed by Lemma 3*

3 Concluding Remarks

Experimental results show that slicing floorplans can actually do better than what we have proved mathematically. We applied the system in [6] to 25 test problems, each with 100 soft rectangles of shape flexibility two. On the average, 2.2% of dead space was obtained. We have also applied the system to the same 25 test problems using a cost function which takes into consideration both the area and the wiring. On the average, we obtained 4.9% dead space, which is higher than before but still quite reasonable. These show that slicing floorplans are good. We hope to be able to incorporate wiring into our analyses in the future.

Finally, note that our problem is quite different from 2-D bin packing [2, 1]. In 2-D bin packing, one considers packing hard rectangles (no flexibility in shape) into a long strip of a constant width and the aim is to minimize the total height. Since the width of the strip is fixed and is independent of the areas of the rectangles, so the resulted packing is usually a long narrow piece with very large aspect ratio. However, we want the resulting shape to be close to a square in floorplan design and the width is thus dependent on the total area of the rectangles. Another difference is that 2-D bin packing considers packing hard rectangles, so their analyses do not take into account the shape flexibility which is, on the contrary, an important issue in our case.

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