An Extended ASLD Trading System to Enhance Portfolio Management

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***Abstract*—An adaptive supervised learning decision (ASLD) trading system has been presented by Xu and Cheung to optimize the expected returns of investment without considering risks. In this paper, we propose an extension of the ASLD system (EASLD), which combines the ASLD with a portfolio optimization scheme to take a balance between the expected returns and risks. This new system not only keeps the learning adaptability of the ASLD, but also dynamically controls the risk in pursuit of great profits by diversifying the capital to a time-varying portfolio of N assets. Consequently, it is shown that 1) the EASLD system gives the investment risk much smaller than the ASLD one and 2) more returns are gained through the EASLD system in comparison with the two individual portfolio optimization schemes that statically determine the portfolio weights without adaptive learning. We have justified these two issues by the experiments.**

***Index Terms*—Extended adaptive supervised learning decision system (EASLD), expected returns, risk, portfolio optimization schemes.**

I. Introduction

IN THE literature, various theories and methods have been developed to help investors to pursue great profits in the investment. One is a computer-based trading system that produces appropriate investment decision signals (also called *trading signals* hereafter) on the basis of the available information to assist an investor make a sensible investment. In the past, one kind of widely used trading systems consists of two modules: prediction module followed by trading module. However, this type of trading system is optimized to some prediction criterion (e.g., mean square error), which is not the ultimate goal of a financial investment. Therefore, it often leads to suboptimal performance in the profit-achieved sense. To solve this problem, some efforts have been made along different directions. One is to use a prediction criterion more correlated with common trading strategies such as that proposed in [2]. Another direction is the return-based systems as proposed in the papers [1], [10], where the prediction module and the trading module are merged into one single system that optimizes the returns instead of the prediction criterion.

An alternative choice is the adaptive supervised learning decision (ASLD) network suggested by Xu and Cheung [16], where the system is built to learn the desired past investment decisions via a supervised learning neural network such as extended normalized radial basis function (ENRBF) neural network [15]. The desired past investment decision for a day is obtained just after that day is past and used as a teaching signal for the network to adaptively learn what decision should be made upon the corresponding inputs. The possibility of using this idea has also been discussed but weeded out by Moody *et al.*'s two major comments in their papers [10]. One comment is that the forming of teaching signals and the training of network by supervised learning is a two-step process still, which encounters difficulties in solving the structural and temporal credit assignment problems. The second comment is that most of the existing labeling procedures by a human expert or an automatic labeling algorithm ignore the input variables and do not consider the conditional distributions of price changes given the input variables, especially in the cases that actual transaction costs should be considered.

However, although the decision-based approach in [16] needs to obtain the past desired trading signals followed by learning it with a supervised learning network, it can actually be regarded as a single-step process that a network outputs a decision signal directly on the basis of the current input as opposed to make a decision based on the prediction of prices as in a two-step prediction-based system. Also, there is no difficulty in the so-called structural and temporal credit assignment problems because the best past investment decision for a day can be easily and surely obtained after that day is past. Moreover, although the decision-based ASLD system has not directly considered the conditional distributions of price changes given the input variables, it considers the conditional distributions of decisions given the input variables. Thus, it has actually not ignored the inputs as well as the previous-mentioned return-based systems. In addition, the ASLD system has also considered the actual transaction costs indeed. Therefore, we argue that the decision-based systems and the return-based ones are both the interesting directions for trading and portfolio management. The performance of the ASLD system has been shown in the foreign exchange market [16] and stock market [6], respectively, with considerable profits acquired. Essentially, the ASLD trading system optimizes the returns without considering the investment risk, which may result in large volatility of the returns.

To reduce the risks in pursuit of great profits, an investor usually diversifies the capital to a portfolio. In the investment community, Markowitz in 1952 [7] proposed a fundamental portfolio optimization scheme named standard Markowian portfolio optimization (SMPO), which determines the portfolio weights by maximizing the difference between the *expected returns* and

Manuscript received January 22, 2000; revised October 12, 2001. This work was supported by the Research Grant Council of the Hong Kong SAR under Project RGC Earmarked CUHK 4297/98E.

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Digital Object Identifier 10.1109/TNN.2003.809423

risk. Later, Sharpe [12] defined the Sharpe ratio as the ratio of expected returns to the risk of one asset, which basically evaluates an individual asset but not a portfolio as a whole. Consequently, an investor may empirically decide the portfolio weights of N assets on the basis of their Sharpe ratios. To our best knowledge, such a weight assignment has not been well studied yet. In both the SMPO and Sharpe ratio, a risk is defined as the return variance, which however has been realized that the variance is not an appropriate measure because it counts the positive fluctuation above the expected returns (also called *upside volatility*) as the part of a risk. Hence, Markowitz [8] and Fishburn [4] later defined the *downside risk*, which counts the volatility below the expected returns only. Recently, the downside risk has been further studied, and a new Sharpe ratio related portfolio optimization scheme called improved portfolio Sharpe ratio maximization with diversification (IPSRM-D) has been proposed [5]. By maximizing the expected returns and the upside volatility while minimizing the downside risk, this IPSRM-D takes a balance between the expected returns and risk to obtain the portfolio weights in batch way and then statically diversifies the capital to N assets based on the weights during the whole investment period.

To combine the ASLD and a portfolio optimization scheme to get the advantages of both, this paper proposes an extended ASLD (EASLD) system that is associated with a portfolio optimization scheme such as SMPO or IPSRM-D to provide a better tradeoff between the expected returns and risk. Compared to the ASLD system [16], this new one has two attractive features:

1) *Diversification of Investment*. In pursuit of great profits, the EASLD system allows more than one asset to be invested simultaneously instead of investing one asset only, whereby the risk is considerably reduced.
2) *Time-varying Portfolio Management*. The EASLD trading system selects a time-varying portfolio, in which not only the portfolio weights are changed over the time, but also the number of assets in the portfolio is time-varied. The time-varying portfolio management adaptively adjusts the balance point between the expected returns and risk such that the portfolio can follow the change of the assets' prices on line with more profits obtained.

As a result, the EASLD system not only remains the learning adaptability of the ASLD but also dynamically adjusts the portfolio weights such that the risk is considerably reduced while great profits are gained. In the experiments, we use the SMPO and improved portfolio Sharpe ratio maximization with diversification (IPSRM-D) as two examples to build the EASLD system, respectively. Compared with the ASLD trading system, the EASLD system can considerably reduce the risk, while providing reasonable long-term returns as well. Furthermore, the empirical results show that the EASLD trading system brings more profits than a human investment by using individual SMPO or IPSRM-D portfolio scheme.

This paper is organized as follows. Section II gives a brief review on SMPO and IPSRM-D portfolio schemes as well as the ASLD system. Section III presents the extended ASLD system in detail including its training and operation procedures. In Section IV, we will conduct some experiments to compare the performance of this new system with the individual SMPO and IPSRM-D, respectively, as well as the ASLD system. Last, we draw a conclusion in Section V.

II. Overviews on SMPO, IPSRM-D Portfolio Schemes, and ASLD Trading System

Let $r_i(t) = (z_i(t) - z_i(t-1))/z_i(t-1)$ be the return from asset i (the term "asset" means bonds, stock shares etc.) at time t, where $z_i(t)$, $i = 1, 2, \ldots, N$ denotes the market-value (also called *price*) of asset i at time t. Given an N-asset portfolio with a past data set $\{\mathbf{z}(t)\}_{t=0}^{M}$ with $\mathbf{z}(t) = [z_1(t), z_2(t), \ldots, z_N(t)]^T$, we calculate the portfolio returns at time t and its expected value, denoted as $r^p(t)$ and $\overline{r^p}$, respectively, by

$$r^p(t) = \sum_{i=1}^{N} w_i r_i(t), \qquad \overline{r^p} = \sum_{i=1}^{N} w_i \overline{r_i} = \mathbf{w}^T \bar{\mathbf{r}} \tag{1}$$

with

$$\sum_{i=1}^{N} w_i = 1, \qquad w_i \geq 0, \qquad i = 1, \ldots, N \tag{2}$$

where $\mathbf{w} = [w_1, w_2, \ldots, w_N]^T$ with w_i being the weight for asset i in portfolio, and $\bar{\mathbf{r}} = [\overline{r_1}, \overline{r_2}, \ldots, \overline{r_N}]^T$. $\overline{r_i}$ is the expected returns of asset i, which can be empirically calculated by

$$\overline{r_i} = \frac{1}{M} \sum_{t=1}^{M} r_i(t). \tag{3}$$

A. SMPO

For a single asset i, Markowitz [7] suggested to use its return variance

$$\text{var}(r_i) = \frac{1}{M-1} \sum_{t=1}^{M} [r_i(t) - \overline{r_i}]^2 \tag{4}$$

as a measure of the investment risk. Also, the portfolio risk of N assets is measured by

$$\text{var}(r^p) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j}^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \tag{5}$$

where the (i, j)th element of matrix $\mathbf{\Sigma}$

$$\sigma_{i,j}^2 = \frac{1}{M-1} \sum_{t=1}^{M} [r_i(t) - \overline{r_i}] [r_j(t) - \overline{r_j}] \tag{6}$$

is the covariance of those returns from asset i and j.

To find out an appropriate $\mathbf{w}$, Markowitz [7] proposed an objective function in terms of the expected returns and risk as follows:

Maximize

$$S(\mathbf{w}) = u\mathbf{w}^T \bar{\mathbf{r}} - \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} \qquad \text{with respect to } \mathbf{w}$$

subject to

$$\sum_{i=1}^{N} w_i = 1, \quad \text{and} \quad w_i \geq 0, \qquad i = 1, \ldots, N \tag{7}$$

where $u \geq 0$ indicates the acceptable level of risk that an investor prefers. The larger the value of u is, the greater the importance of the expected returns is in the objective function. This is well-known SMPO. The optimum solution for (7) has been given by Ostermark [11].

B. IPSRM-D

1) Sharpe Ratio: For ranking the goodness of an asset i, Sharpe proposed the idea of Sharpe ratio [12], [13] based on Markowitz's mean-variance paradigm. The definition of Sharpe ratio S_i for asset i is as follows:

$$S_i \equiv \frac{\overline{r_i}}{\sqrt{\text{var}(r_i)}}. \tag{8}$$

Sharpe suggested calculating the Sharpe ratio for each asset and then select the asset with the greatest value to invest. Although both of SMPO and Sharpe ratio consider the balance between expected return and risk, the latter measures the goodness of an asset generally better than the former.

2) Downside Risk and Upside Volatility: According to Markowitz [7], the risk is taken as the variance of the return. However, more and more academics and practitioners claim that the variance is not an appropriate measure of risks in many practical investments. Thus, Fishburn [4] proposed a more sophisticated measure of risk associated with below-target return, through which the idea of downside risk was proposed by Sortino and Meer [14]. Basically, the downside risk is the volatility of return below the minimal acceptable return (also called *target return*) G.

For an single asset, the definition of downside risk, $\text{down}V_\alpha(G)$, is given as follows:

$$\text{down}V_\alpha(G) = \int_{-\infty}^{G} (G-r)^\alpha \, dF(r) \qquad \text{with } \alpha > 0 \tag{9}$$

where $F(r)$ is the probability distribution function of an asset's return r. The α is supposed to reflect a decision maker' feelings about the relative consequences (personal, corporate, etc.) of falling short of the minimal acceptable return G by various amounts. The detailed selection rules for α can be referred in Fishburn [4].

Since the downside risk in (9) is for a single asset only, our recent paper [5] has extended it to the case for N-asset portfolio as follows:

$$\text{down}V_\alpha(G) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \text{ down } V_\alpha(G, i, j) = \mathbf{w}^T\mathbf{D}\mathbf{w}$$

where

$$\begin{cases} \text{down}V_\alpha(G, i, j) = \displaystyle\int_{-\infty}^{G}\int_{-\infty}^{G} (G-r_i)^\alpha (G-r_j)^\alpha \\ \qquad\qquad \cdot dF(r_i)\, dF(r_j), \\ \mathbf{D} = \begin{bmatrix} \text{down}V_\alpha(G, 1, 1) & \cdots & \text{down}V_\alpha(G, 1, N) \\ \vdots & & \vdots \\ \text{down}V_\alpha(G, N, 1) & \cdots & \text{down}V_\alpha(G, N, N) \end{bmatrix}. \end{cases} \tag{10}$$

Apart from considering the variance below the target return, the paper [5] also considers to measure the variance of return above the target return by introducing a new term called "upside volatility." Similar to the calculation of downside risk, the upside volatility for an N-asset portfolio is defined as

$$\text{up}V_\alpha(G) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \text{ up } V_\alpha(G, i, j) = \mathbf{w}^T\mathbf{U}\mathbf{w}$$

where

$$\begin{cases} \text{up}V_\alpha(G, i, j) = \displaystyle\int_{G}^{\infty}\int_{G}^{\infty} (r_i - G)^\alpha (r_j - G)^\alpha dF(r_i) dF(r_j) \\ \mathbf{U} = \begin{bmatrix} \text{up}V_\alpha(G, 1, 1) & \cdots & \text{up}V_\alpha(G, 1, N) \\ \vdots & & \vdots \\ \text{up}V_\alpha(G, N, 1) & \cdots & upV_\alpha(G, N, N) \end{bmatrix}. \end{cases} \tag{11}$$

3) IPSRM-D: Since the Sharpe ratio defined in (8) is exclusively for evaluating a single asset without a scheme to decide the portfolio weights, it is actually not suitable for portfolio management. Hence, the paper [5] has extended it to the portfolio case, named improved portfolio sharpe ratio (IPSR), as follows:

$$\text{IPSR} = \frac{\mathbf{w}^T\overline{\mathbf{r}} + \mathbf{w}^T\mathbf{U}\mathbf{w}}{\mathbf{w}^T\mathbf{D}\mathbf{w}} \tag{12}$$

which not only includes the individual Sharpe ratio value of each asset, but also counts both of the downside risk and upside volatility. Based on IPSR, the paper [5] presents a portfolio optimization scheme called IPSRM-D as follows:

Maximize

$$S(\mathbf{w}) = \frac{\mathbf{w}^T\overline{\mathbf{r}} + H\mathbf{w}^T\mathbf{U}\mathbf{w}}{\mathbf{w}^T\mathbf{D}\mathbf{w}} + B\mathbf{w}^T([\mathbf{1}] - \mathbf{w})$$

with respect to $\mathbf{w}$ subject to

$$\sum_{i=1}^{N} w_i = 1, \quad \text{and} \quad w_i \geq 0 \qquad \text{for } \forall\, i = 1, \ldots, N \tag{13}$$

where $[\mathbf{1}]$ is the constant vector of 1, the parameter H represents the degree of importance of maximizing upside volatility in the optimization, and B is the degree of importance of regularization in the optimization. The optimization of (13) can be implemented by an augmented Lagrangian algorithm as listed in Appendix-I. It has been shown [5] that the IPSRM-D can effectively reduce the risk while obtaining great returns.

C. ASLD System

Suppose the nonlinear relationship between trading signals and the assets' prices can be described by a nonlinear function f. The ASLD system [16] trains a supervised learning neural network to approximate this f before its operation. The training and operation of the ASLD system are summarized as follows:

- *Training Stage of the ASLD System*. In this stage, we use a set of past data $\{\mathbf{z}(t)\}_{t=0}^{M}$ to train the system. At each time $t \in \{0, 1, 2, \ldots, M-1\}$, the desired trading signal is first extracted based on a return-optimized trading strategy because the "future" price $z_i(t+1)$, $i = 1, 2, \ldots, N$ can be look-ahead. Then, this available trading signal as a teaching signal is used to train a supervised learning neural network, e.g., extended normalized radial basis function

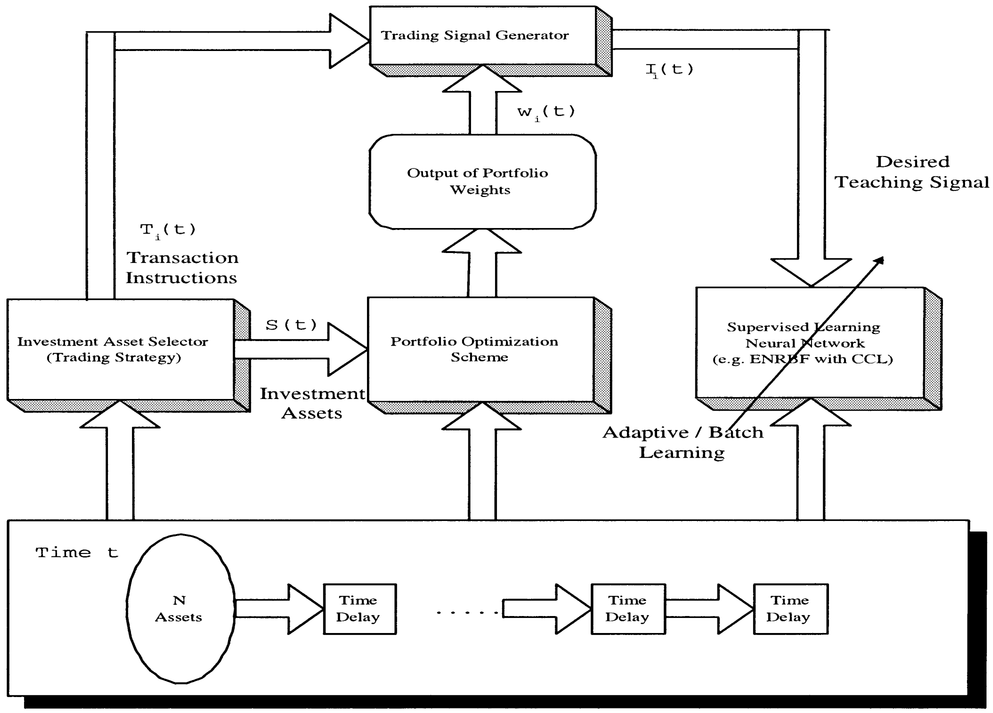


Fig. 1. Structure of the extended ASLD (EASLD) system in the training stage.

(ENRBF) network [15], such that it is learned what decision should be made upon the corresponding inputs.

- *Operation Stage of the ASLD System*. In this stage, the assets' daily prices are not available until that day. That is, $\mathbf{z}(t)$ is available at time t or after. At current time t, the ASLD system not only outputs an estimate of the corresponding desired trading signal but also adaptively adjusts the parameters of the supervised learning neural network after the desired trading signal at time $t-1$ is extracted presently.

The details of the ASLD system can be found in [16].

It should be noted that the trading strategy used in [16] determines the desired trading signals by optimizing the returns one time step ahead only. Reference [6] has provided an improved trading strategy that considers q-time step ahead when extracting a desired trading signal. Since the ASLD system is proposed to optimize the returns only without considering the risks, it may often result in large volatility of the returns. In the next section, an extended ASLD system will be introduced, therefore, to take a balance between the expected returns and risks.

III. EASLD Trading System

The EASLD trading system is a combination of the ASLD system and a portfolio optimization scheme. At each time t, the system input is the observed prices of N assets and the output is a trading signal that not only shows the portfolio investment on a subset of N assets, but also indicates the transaction instructions on the assets in hand. Since the EASLD uses a supervised learning neural network to describe the nonlinear relationship between the inputs and outputs, we need to train it such that the net's parameters are tuned appropriately before operation. In the following, we will elaborate the system's training and operation stages, respectively.

A. Training of the EASLD System

As shown in Fig. 1, the EASLD system consists of four major components: Investment asset selector (IAS), portfolio optimization scheme (POS), trading signal generator (TSG), and supervised learning neural network (SNN). Its training procedure is as follows: At each time step t, the IAS component selects a subset, written as $\mathbf{S}(t)$, from N assets. We call these selected assets *investment assets*, which are not held at time $t-1$ and should be invested at time t on the basis of a pre-specified trading strategy. Also, the IAS will give the transaction instruction $T_i(t)$ for asset i to decide whether to make a transaction or not. Upon determining the investment assets, the POS component uses a portfolio optimization scheme, such as existing SMPO or IPSRM-D, to assign each asset i a portfolio weight, written as $w_i(t)$, which is determined based on the available past assets' prices up to t. Subsequently, a trading signal $I_i(t)$ on asset i, $i=1, 2, \ldots, N$, is generated by the TSG on the basis of the transaction instruction $T_i(t)$ and the portfolio weight w_i from the POS. We then use the trading signal, as a desired teaching signal at time t, to adjust the SNN system parameters with a little small step. With the adaptive training, the SNN can gradually model the nonlinear relationship between the desired trading signals and the assets' prices. In the following, we will show the detailed implementations of IAS, TSG, and SNN, respectively. As for the POS, it can be generally accomplished by using any existing portfolio optimization scheme. In this paper, we just concentrate on the SMPO and IPSRM-D, whose details as well as the relevant literatures have been introduced in Section I. We therefore omit its details hereafter.

1) Investment Asset Selector: The EASLD system defines the transaction instruction signal $T_i(t)$ in the form

$$T_i(t) = I^a{}_i(t) \cdot I_i^P(t) \tag{14}$$

with

$$T_i(t) = \begin{cases} 1, & \text{means to buy asset } i \\ -1, & \text{means to sell asset } i \\ 0, & \text{otherwise} \end{cases} \tag{15}$$

where $I^a{}_i(t)$ and $I_i^P(t)$ are called *allocating signal* and *positional signal*, respectively, with

$$I_i^P(t) = \begin{cases} 1, & \text{means to hold asset } i \\ -1, & \text{means to sell asset } i \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

and

$$I^a{}_i(t) = \begin{cases} 1, & \text{means to perform a transaction on asset } i \text{ at time } t \\ 0, & \text{otherwise.} \end{cases} \tag{17}$$

The trading strategy used in the EASLD system needs to look ahead the "future" prices such that an appropriate trading decision is made. Its rules can be stated as follows:

1) Suppose asset i is held at time $t-1$. If asset i keeps decreasing in the next q time steps with the amount of decrement larger than β times of the transaction costs, we determine to sell it out at time t to get cash back.
2) Suppose asset i is not held at time $t-1$. If its prices keep increasing in the next q time steps and the amount of the increment is larger than β times of the transaction costs, we will invest on this asset at time t.

Specifically, $I^a{}_i(t)$ and $I_i^P(t)$ are determined by the following procedure:

Step A Let $E = \{k | I_k^P(t-1) = 1 \text{ and } 1 \le k \le N\}$.

Step B For i, $1 \le i \le N$, if $i \in E$, let

$$I_i^P(t) = \begin{cases} -1, & \text{if } z_i(t+q) < z_i(t+q-1) < \cdots < z_i(t+1) < z_i(t) \\ & \text{and } [z_i(t) - z_i(t+q)] > z_i(t) * \beta * \gamma \\ 1, & \text{otherwise} \end{cases} \tag{18}$$

where γ denotes a transaction cost, and β is a constant.

Step C If i is not in E, let

$$I_i^P(t) = \begin{cases} 1, & \text{if } z_i(t+q) > z_i(t+q-1) > \cdots > z_i(t+1) > z_i(t) \\ & \text{and } [z_i(t+q) - z_i(t)] > z_i(t) * \beta * \gamma \\ 0, & \text{otherwise.} \end{cases} \tag{19}$$

Step D For $1 \le r \le N$, let

$$I_r^a(t) = \begin{cases} 1, & \text{if } (r \text{ is not in } E), \text{ and } (I_r^P(t) = 1 \text{ or } I_r^P(t) = -1) \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

Since an investment asset is the one that is not held at time $t-1$ should be invested at time t according to the above trading strategy, the IAS, therefore, chooses the investment assets at time t by

$$\text{Asset } i \text{ is an investment asset if } I_i^P(t) = 1 \text{ and } I_i^P(t-1) = 0, \qquad i = 1, 2, \ldots, N.$$

It should be noted that the above trading strategy is actually made without considering the activities of "sell short." If it is allowed in the portfolio investment, the situation then becomes somewhat complex. One possible way is to replace the symbol ">" in (19) by "<" when considering "sell short" transaction on asset i. Consequently, the signal definition of $I_i^P(t)$ in (16) and the signal assignment in (18) both need to be modified correspondingly. In our paper [16], a trading strategy that allows the "sell short" activities has been suggested. See [16] for more details. For simplicity, we will no longer consider "sell short" hereafter.

2) Trading Signal Generator (TSG): Given the transaction instruction $T_i(t)$, the TSG generates the trading signal $I_i(t)$ in the form

$$I_i(t) = \begin{cases} w_i(t), & \text{if } T_i(t) = 1 \\ T_i(t), & \text{otherwise,} \end{cases} \qquad i = 1, 2, \ldots, N \tag{21}$$

with

$$I_i(t) = \begin{cases} w_i(t), & \text{means to buy asset } i \text{ in proportion to } w_i(t) \\ -1, & \text{means to sell asset } i \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

and $\sum_{i \in \psi} w_i(t) = 1$, $w_i(t) > 0$, $\psi = \{i | T_i(t) = 1\}$.

3) SNN Learning: Suppose there exists a nonlinear function f among $\mathbf{I}(t)$, $\mathbf{I}(t-1)$, and the past assets' prices $\{\mathbf{z}(t-m)\}_{m=0}^{d-1}$ with

$$I_i(t) = f_i[\mathbf{I}(t-1), \mathbf{z}(t), \mathbf{z}(t-1), \ldots, \mathbf{z}(t-d+1)] \qquad 1 \le i \le N \tag{23}$$

where $\mathbf{I}(t) = [I_1(t), I_2(t), \ldots, I_N(t)]^T$, $f = [f_1, f_2, \ldots, f_N]^T$, and d is the time lag. In principle, the EASLD system can use any trained supervised learning neural network to approximate this function f. Here, we prefer to use ENRBF networks because of its simple architecture and learning procedures. Further, the experiments in [6] and [16] have shown their successful performance in modeling foreign exchange and stock markets. In this paper, we use an array of N ENRBF networks trained by the CCL learning algorithm [15] as the SNN component to approximate the nonlinear function f in (23). As shown in Fig. 2, the output of network i is $\hat{I}_i(t)$ when $\mathbf{x}(t) = [\mathbf{I}(t-1), \mathbf{z}(t), \mathbf{z}(t-1), \ldots, \mathbf{z}(t-d+1)]^T$ is the input at time t. The parameter set of network i is

$$\boldsymbol{\theta}^i = \{\mathbf{W}_j^i, c_j^i, \mathbf{m}_j^i, \boldsymbol{\Sigma}_j^i | j = 1, \ldots, K\} \tag{24}$$

where K is the number of hidden units in the hidden layer, $\mathbf{W}_j^i$, c_j^i are the hidden parameters in the output layer, and $\mathbf{m}_j^i$, $\boldsymbol{\Sigma}_j^i$ are the centers and covariance matrices of the RBF hidden units. In the literature, such a network is expected to be trained by using a least-square learning technique. However, due to the difficulty

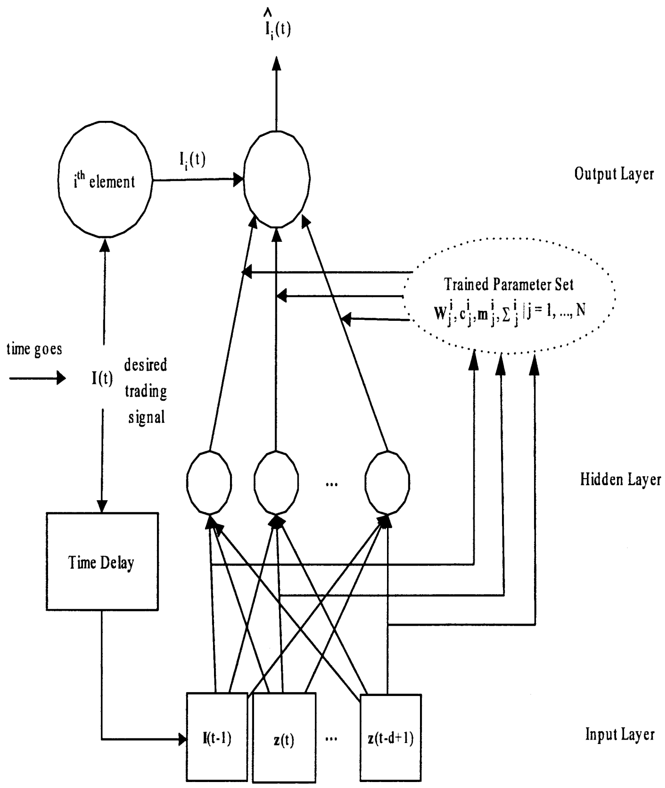


Fig. 2. Structure of ENRBF-network i.

of determining the parameters $\mathbf{m}_j^i$ and $\mathbf{\Sigma}_j^i$ in practice, learning is usually made by an approximation method with two separate steps. First, we use some kind of clustering analysis (e.g., the k-means algorithm) to group the net's input set into K clusters with their centers characterized as $\mathbf{m}_j^i$, $j = 1, 2, \ldots, K$. Second, the parameters $\mathbf{W}_j^i$, c_j^i are determined by a least-square learning technique. Using this two-step training approach, the centers are obtained directly from the input data without the need to make an improvement to the regression relation between $\hat{I}_i(t)$ and $\mathbf{x}_t$.

By combining the ENRBF network with an alternative to the mixture-of-experts model [17], Xu [15] has proposed an expectation-maximization (EM) algorithm to learn the ENRBF's parameters with a better estimate. In Appendix-III, we give out an adaptive algorithm as well as a batch one to train the parameters in each network i. For other details, the interesting readers can refer to [15] and [16].

B. Operation of the EASLD Trading System

At each time step t, determining the desired teaching signal needs to look ahead q-step the "future" assets' prices. Hence, in the operation stage, the desired trading signal of current time t is not available until time $t+q$. As shown in Fig. 3, at each time t, the EASLD trading system uses the trained SNN to give an estimate of the corresponding desired teaching signal by

$$\hat{I}_i(t) = \begin{cases} \dfrac{O^i(t)}{\sum_{r\in\Omega} O^r(t)}, & \text{if } O^i(t) > \delta \\ 0, & \text{if } |O^i(t)| < \delta \\ -1, & \text{otherwise} \end{cases} \tag{25}$$

with

$$O^i(t) = \hat{f}[\mathbf{I}(t-1), \mathbf{z}(t), \mathbf{z}(t-1), \ldots, \mathbf{z}(t-d+1)] \tag{26}$$

where $\Omega = \{i | O^i(t) > \delta\}$, and δ is a small positive threshold value, which can be generally determined via a cross-validation technique. In particular, when the SNN is implemented by the ENRBF network described in Section III-A3), (26) then becomes

$$O^i(t) = \sum_{j=1}^{K} \left[\left(\mathbf{W}_j^i\right)^T \mathbf{x}(t) + c_j^i \right] \cdot \frac{e^{-0.5[\mathbf{x}(t)-\mathbf{m}_j^i]^T (\mathbf{\Sigma}_j^i)^{-1} [\mathbf{x}(t)-\mathbf{m}_j^i]}}{\sum_{j=1}^{K} e^{-0.5[\mathbf{x}(t)-\mathbf{m}_j^i]^T (\mathbf{\Sigma}_j^i)^{-1} [\mathbf{x}(t)-\mathbf{m}_j^i]}}. \tag{27}$$

With the help of $\hat{I}_i(t)$s in (25), an investor can, therefore, know how to deal with the assets in hand, and to diversify the capital to a portfolio investment such that the risk is considerably reduced meanwhile acquiring great profits.

When the desired teaching signal of time t is determined at time $t+q$, we continually tune the SNN in the same way as the training stage to maintain the EASLD system adaptive-learning capability.

IV. Experimental Demonstration

A. EASLD in the Stock Market

We performed the experiments by investing a portfolio of six different stock indexes:

- S&P 500 Composite Price Index (USA);
- Hang Seng Index (Hong Kong);
- Shanghai SE Composite - Price Index (P.R. China);
- NIKKEI 255 Stock Average (Japan);
- CAC 40 - Price Index (France);
- Australia SE All Ordinary Price Index (Australia).

The experimental data consisted of 1365 data points from May 11, 1992 to August 1, 1997. The first 1000 points were used as the past data set, whereas the remaining 365 ones were used in the operation stage of the EASLD trading system for performance test. Furthermore, in all of the following experiments, we let the rate of transaction cost γ be 3% of the amount in each transaction. Also, we chose SMPO and IPSRM-D as two examples to build the EASLD system, respectively, by setting $H = B = \alpha = 1$ and $G = 0$ in the IPSRM-D, and setting $u = 1$ in SMPO. In making IAS trading strategy, we simply fixed $q = 3$ and $\beta = 2.5$ heuristically determined by the paper [6] in the same experimental data set, although a cross-validation technique is helpful to improve the parameter value selection.

1) Experiment 1: Comparison Between EASLD System and ASLD One: To show that EASLD system has smaller investment risk in comparison with the ASLD one, we let SMPO be the POS component of the EASLD system as an example. Hereafter, we denote it as EASLD-SMPO. Fig. 4 shows that the profit gain obtained by the EASLD system is more stable than the ASLD. The major reason is that the target of the ASLD system is to maximize the expected returns without considering the risk. As indicated in Table I, the ASLD system invested on at most

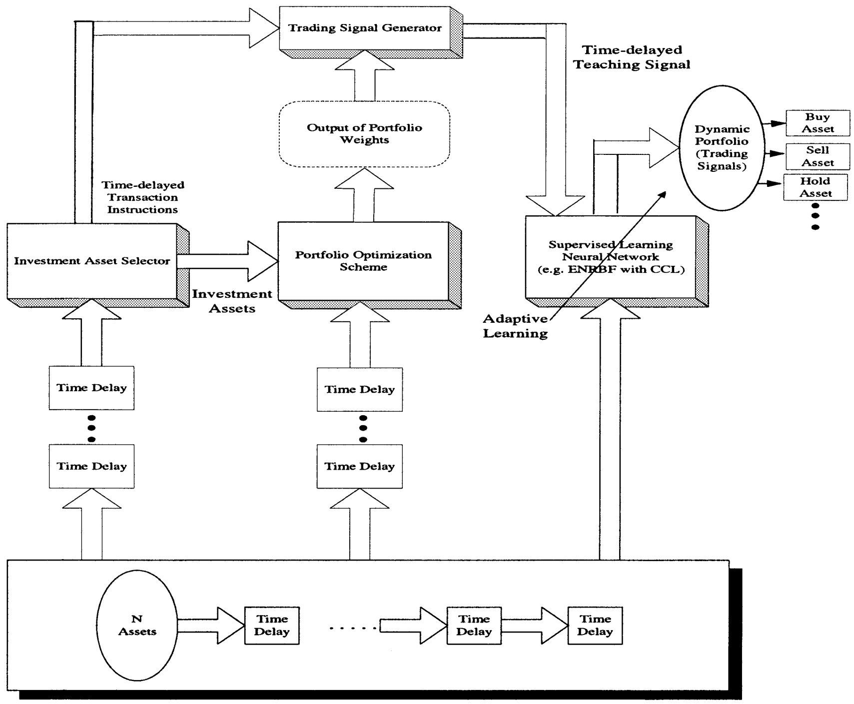


Fig. 3. Structure of the EASLD system in the operation stage.

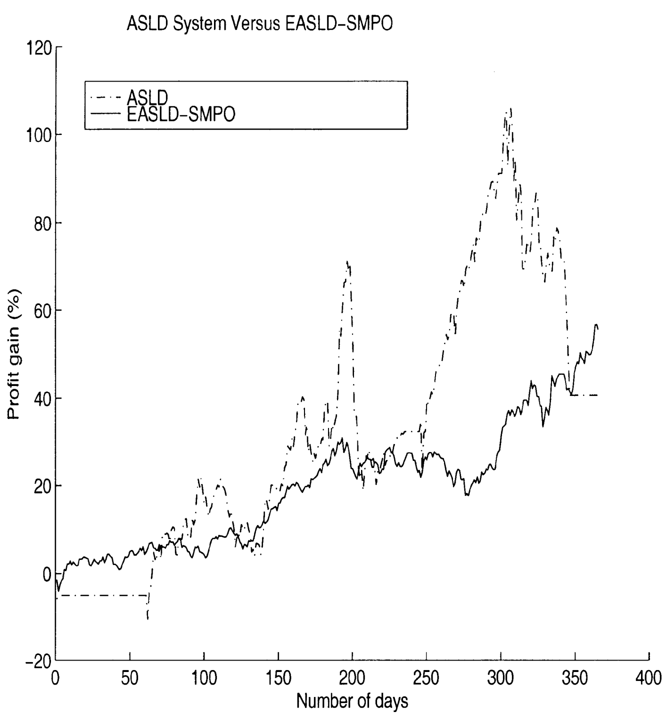


Fig. 4. Profit gain obtained by the EASLD-SMPO system and the ASLD one, respectively.

one index each day, which made the volatility of the profit gain large. On the other hand, instead of only maximizing the expected returns, the EASLD system diversified the capital to take an appropriate balance between the expected returns and the risk. We measured the degree of diversification (dod) by

$$\text{dod} = \frac{1}{\xi}\sum_{t=1}^{\xi}\mathbf{w}(t)^T[[\mathbf{1}] - \mathbf{w}(t)] \tag{28}$$

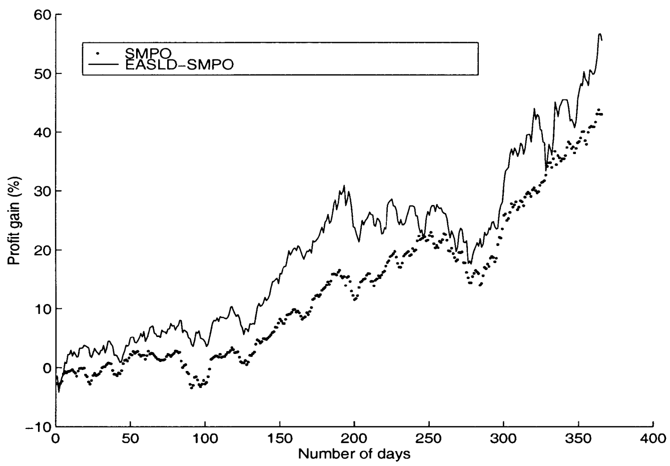
where $\mathbf{w}(t)$ is the vector of portfolio at time t, and ξ is the length of the operation time period. Table I shows that the EASLD system invested on at most three indexes in the operation period with dod $= 0.27$ in comparison with dod $= 0$ in the ASLD. Hence, the EASLD system can considerably reduce risk in contrast to the ASLD, while bringing reasonable long-term profits as well. We also compared the EASLD-IPSRM-D (i.e., we build the EASLD system with the POS component being IPSRM-D) with the ASLD, resulting in the same conclusion as shown in Table I.

2) Experiment 2: Comparison Between EASLD and Individual Portfolio Schemes: This experiment is to show that the EASLD system can bring more profits than an individual portfolio scheme. When individually using SMPO and IPSRM-D, we calculated the portfolio weights $\mathbf{w}$ on the past data set as shown in Section II. We then fixed the $\mathbf{w}$ and diversified the capital to six assets in proportion to $\mathbf{w}$ and held them during the whole operation period.

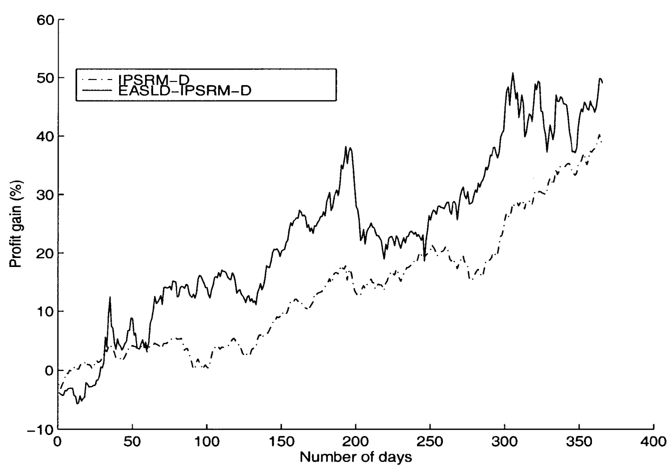
Fig. 5(a) shows the performance of the SMPO and the EASLD-SMPO. It can be seen that the return curve of the latter is always above the former's one. That is, the latter can consistently bring more profit. The major reason is that the adaptive trading signal provided by the EASLD system can keep track of the dynamic financial market indexes but the individual SMPO cannot. To justify the significance of the EASLD in profit gains, we further conducted the statistical hypothesis test. We calculated the individual average monthly returns of EASLD-SMPO and SMPO, written as $R^m_{\text{EASLD–SMPO}}$ and R^m_{SMPO}, respectively, where $m = 1, 2, \ldots$, is the monthly indexes. We then further calculated their mean, denoted as $\overline{R}_{\text{EASLD–SMPO}}$ and $\overline{R}_{\text{SMPO}}$, respectively. We set the null hypothesis H_0 be $\overline{R}_{\text{EASLD–SMPO}} = \overline{R}_{\text{SMPO}}$, while the alternative hypothesis H_1 is $\overline{R}_{\text{EASLD–SMPO}} > \overline{R}_{\text{SMPO}}$. We performed

TABLE I
PERFORMANCE COMPARISON AMONG THE ASLD, EASLD-SMPO, AND EASLD-IPSRM-D SYSTEMS

	The ASLD	EASLD-SMPO	EASLD-IPSRM-D
Maximum Number of Indices Invested at a Time	1	3	3
Degree of Diversification	0	0.27	0.35
Mean Daily Returns	0.137	0.130	0.192
IPSR	1.424	1.882	2.122
Risk	5.393	0.905	1.356
Upside Volatility	0.786	0.424	0.535
Downside Risk	0.648	0.295	0.343



(a)



(b)

Fig. 5. Performance comparison. (a) Between EASLD-SMPO and SMPO. (b) Between EASLD-IPSRM-D and IPSRM-D.

the t-test at the 0.05% level of significance. We obtained the t value to be 10.0542, which is much larger than the critical value $t_{0.0005} = 4.14$. It is, therefore, the experimental results reject H_0 at the 0.05% level of significance. That is, we have the strong confidence that the EASLD-SMPO brings the profit more than the SMPO. In Fig. 5(b), we also compared the IPSRM-D and EASLD-IPSRM-D. We found again that the EASLD can obtain more profit gain than the corresponding individual portfolio scheme. We further performed the t test in the same way as the previous one. We obtained the t value to be 6.1004, which is still larger than the critical value 4.14. The experimental results, therefore, support again that EASLD-IPSRM-D outperforms IPSRM-D in the sense of profit gains.

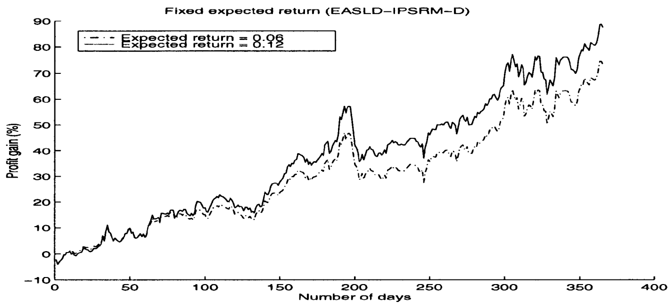
Furthermore, from Fig. 5(a) and (b), we also found that the return volatility of EASLD-SMPO and EASLD-IPSRM-D systems were larger a little than the individual SMPO and IPSRM-D, respectively. Even so, we found that the returns from EASLD-SMPO are always much more than SMPO during the whole operation period. Similarly, EASLD-IPSRM-D gave far greater returns than IPSRM-D in the most time. This implies that the EASLD has made an appropriate balance between the returns and the investment risk.

3) Cautious Investment: From the practical viewpoint, we noticed that some conservative investors are more concerned about risk than return. Therefore, a strategy that may satisfy them is to minimize the risk while controlling the expected return. In the IPSRM-D, we fixed the expected return to be 0.06 and 0.12, respectively, while minimizing the downside risk and maximizing the upside risk. The algorithm of IPSRM-D with control of expected returns is listed in Appendix-II. Fig. 6(a) is the experimental results of EASLD-IPSRM-D with controlling expected return at different levels. The result statistics are shown in Table II, through which we can see the following.

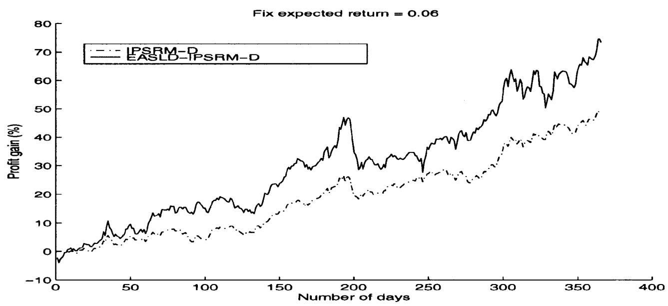
1) The higher the expected return that is specified, the higher the resulting return is obtained and vice versa.
2) The smaller the expected return that is specified, the smaller the resulting risk is met, as expected.

Under the different return control level, we again compared the profit gain performance of the EASLD-IPSRM-D system with the individual IPSRM-D. Fig. 6(b) and (c) showed the comparison results, where we can see that the EASLD-IPSRM-D can bring much more profit gain than the IPSRM-D, which is also consistent with the previous findings.

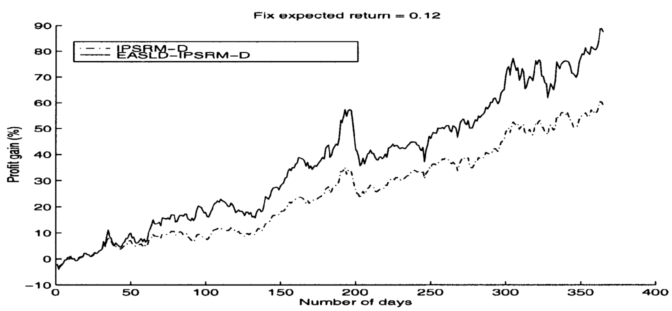
In contrast, we also noticed that some aggressive investors are more concerned about return than risk. Therefore, a strategy that may satisfy them is to maximize the expected return while controlling the expected downside risk. Consequently, we fixed the expected downside risk, which is specified by the investor, and the optimization essentially becomes a maximization of re-



(a)

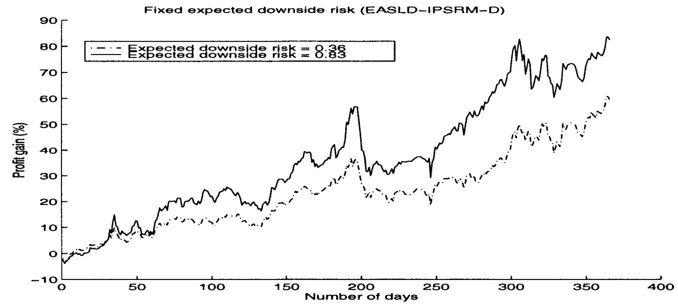


(b)

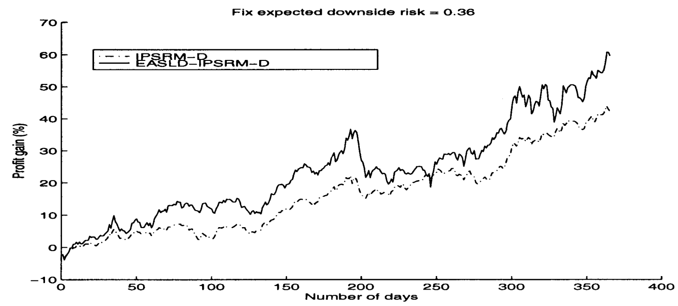


(c)

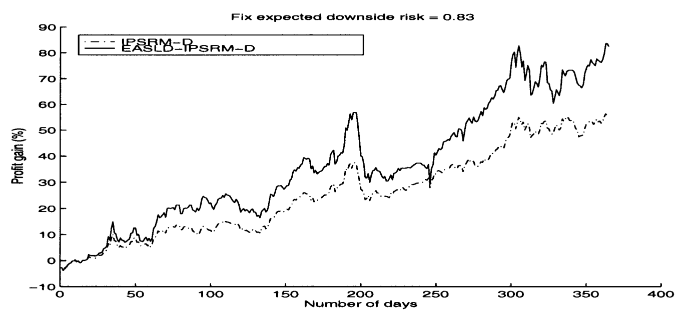
Fig. 6. (a) Profit gains by EASLD-IPSRM-D under different controlled expected returns. (b) Comparison of EASLD-IPSRM-D with controlling expected returns at 0.06 and individual IPSRM-D. (c) Comparison of EASLD-IPSRM-D with controlling expected returns at 0.12 and individual IPSRM-D.



(a)



(b)



(c)

Fig. 7. (a) Profit gain by EASLD-IPSRM-D under different controlled expected downside risk. (b) Comparison of EASLD-IPSRM-D with controlling expected downside risk at 0.36 and individual IPSRM-D. (c) Comparison of EASLD-IPSRM-D with controlling expected downside risk at 0.83 and individual IPSRM-D.

TABLE II
PROFIT GAIN OF THE EASLD-IPSRM-D SYSTEM WITH DIFFERENT EXPECTED RETURNS

	Expected return = 0.06	Expected return = 0.12
Mean return	0.164	0.192
Risk	1.098	1.356
IPSR	2.043	2.122
Upside Volatility	0.480	0.535
Downside Risk	0.315	0.343

TABLE III
PROFIT GAIN OF EASLD-ITSRM-D WITH DIFFERENT EXPECTED DOWNSIDE

	Expected downside risk = 0.36	Expected downside risk = 0.83
Mean return	0.140	0.183
Risk	1.089	2.026
IPSR	1.864	1.883
Upside Volatility	0.464	0.597
Downside Risk	0.324	0.414

turn and upside risk. We give the algorithm of IPSRM-D with control of the expected downside risk in Appendix-II as well.

Fig. 7(a) is the experimental result of EASLD-IPSRM-D with the different expected downside risks. The statistics are shown in Table III, through which we can see the following.

1) The smaller the expected downside risk that is specified, the smaller the resulting downside risk is met and vice versa.
2) The larger expected downside risk is that specified, the larger the resulting return is obtained, as expected.

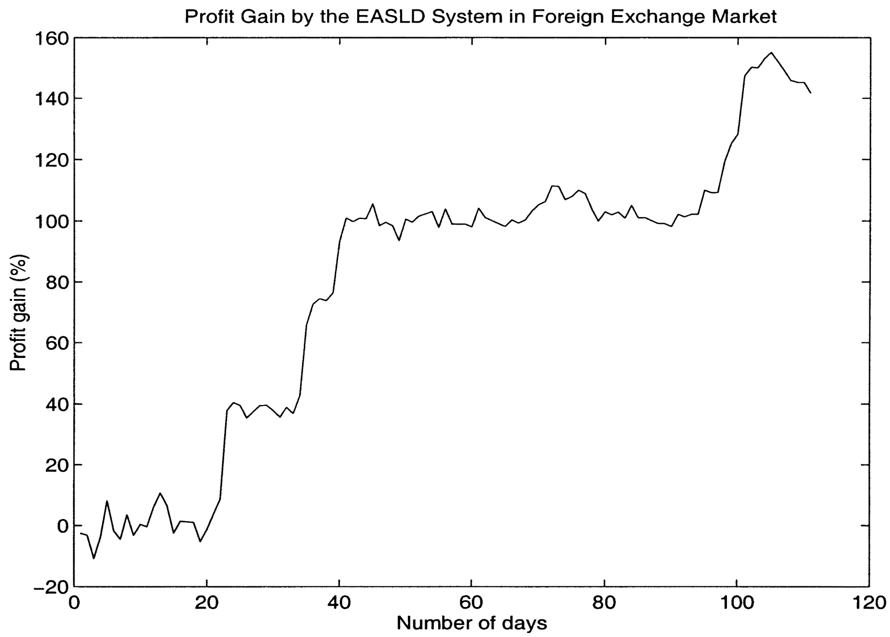


Fig. 8. Profit gains from the investment in foreign exchange market by the EASLD system.

Again, under the different controls of expected downside risk, we compared the performance of EASLD-IPSRM-D with the IPSRM-D, as shown in Fig. 7(b) and (c). We found that the return volatility of the former is similar with the latter because of the risk control, but the former gives much more profits.

B. EASLD in Foreign Exchange Market

In the previous experiments, we have shown the outstanding performance of EASLD in the stock market in comparison with the ASLD and the two individual portfolio management schemes. This section will further demonstrate the performance of EASLD in foreign exchange market. We considered a portfolio of the following six foreign exchange rates:

1) Australian Dollar (AUD);
2) Canadian Dollar (CAD);
3) German Deutschmark (DEM);
4) French Franc (FRN);
5) Japanese Yen (JAP);
6) Swiss Franc (CHF).

The experimental data were from November 26, 1991 to August 30, 1995 with 1112 samples in total. In our experiment, the first 1000 data were used in the training stage, while the other 112 ones were used in the operation stage.

Fig. 8 shows the curve of profit gains from the EASLD system. It can be seen that the investment returns nearly 158% profit gain following the trading signal generated by the EASLD system but with a small downside risk. It further again demonstrates the capability of the EASLD in making a tradeoff between high returns and the investment risk.

V. CONCLUSION

In this paper, we have extended the ASLD trading system by combining it with a portfolio optimization scheme. Since this new system has the advantages of both of them, it not only keeps the learning adaptability of the ASLD but also controls the risk in pursuit of considerable profits by diversifying the capital to a time-varying portfolio of N assets. The experimental results have shown that the EASLD system can considerably reduce the risk in comparison with the ASLD one while keeping the reasonable long-term returns. Furthermore, the EASLD brings far more profits than the two individual portfolio optimization schemes with a little sacrifice of the risk, which can actually be further improved by controlling of the downside risk. The proposed system has therefore provided a new way to enhance the portfolio management by making a better balance between the expected returns and the investment risk.

APPENDIX I
AUGMENTED LAGRANGIAN ALGORITHM FOR IPSRM-D IMPLEMENTATION

We show the derivation of the optimization algorithms for IPSRM-D as follows.

Let $g^-(w_j,\mu_j,c) = \min\{w_j,-(\mu_j/c)\}$ and $g^-(\mathbf{w},\boldsymbol{\mu},c)$ be a column vector containing $g^-(w_j,\mu_j,c)$, $j = 1,2,\ldots,N$. Then, we have

$$\frac{\partial g^-(w_j,\, \mu_j,\, c)}{\partial w_j} = \begin{cases} 1, & \text{if } w_j < -\dfrac{\mu_j}{c} \\ 0, & \text{otherwise} \end{cases}$$

and

$$\frac{\partial g^-(w_j,\, \mu_j,\, c)}{\partial w_k} = 0, \qquad \text{if } k \neq j. \tag{29}$$

The augmented Lagrangian is given by

$$\begin{aligned} L = {} & \frac{\mathbf{w}^T\bar{\mathbf{r}} + H\mathbf{w}^T\mathbf{U}\mathbf{w}}{\mathbf{w}^T\mathbf{D}\mathbf{w}} + B\mathbf{w}^T([\mathbf{1}] - \mathbf{w}) - \lambda(\mathbf{w}^T[\mathbf{1}] - 1) \\ & - \tfrac{1}{2}\, c(\mathbf{w}^T[\mathbf{1}] - 1)^2 - \boldsymbol{\mu}^T g^-(\mathbf{w},\, \boldsymbol{\mu},\, c) \\ & - \tfrac{1}{2}\, c[g^-(\mathbf{w},\, \boldsymbol{\mu},\, c)]^T g^-(\mathbf{w},\, \boldsymbol{\mu},\, c) \end{aligned} \tag{30}$$

and its derivative with respect to w_j is

$$\frac{\partial L}{\partial w_j} = \frac{1}{(\mathbf{w}^T\mathbf{D}\mathbf{w})^2} \cdot \left\{(\mathbf{w}^T\mathbf{D}\mathbf{w})[\bar{\mathbf{r}}+2H\mathbf{U}\mathbf{w}]_j - 2(\mathbf{w}^T\bar{\mathbf{r}}+H\mathbf{w}^T\mathbf{U}\mathbf{w})[\mathbf{D}\mathbf{w}]_j\right\} + B(1-2w_j) - \lambda - c(\mathbf{w}^T[1]-1) - \frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}\mu_j - cg^-(w_j,\mu_j,c)\frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}. \tag{31}$$

The first-order multiplier iteration method is used. The recursive algorithm for implementing IPSRM-D by the augmented Lagrangian method is to update $\{\mathbf{w}, \lambda, \boldsymbol{\mu}, c\}$ at every iteration t by the following equations until $\mathbf{w}$ converges:

$$\begin{aligned} w_j(t+1) &= w_j(t) + \eta\frac{\partial L}{\partial w_j}, \qquad j = 1,\ldots,N \\ \lambda(t+1) &= \lambda(t) + c(t)(\mathbf{w}^T[\mathbf{1}]-1) \\ \mu_j(t+1) &= \mu_j(t)\times g^-(w_j,\mu_j(t),c(t)) \\ c(t+1) &= \beta\times c(t) \end{aligned} \tag{32}$$

where β is a constant greater than one, η is a small positive learning rate, and $c(t)$ is a penalty parameter which should be kept increasing during the optimization process.

APPENDIX II
ALGORITHM FOR IPSRM-D WITH CONTROL OF EXPECTED RETURN OR DOWNSIDE RISK

For IPSRM-D with control of expected return, the augmented Lagrangian in (30) then becomes

$$L_{\text{fixr}} = \frac{r_{\text{spce}}+H\mathbf{w}^T\mathbf{U}\mathbf{w}}{\mathbf{w}^T\mathbf{D}\mathbf{w}} + B\mathbf{w}^T([\mathbf{1}]-\mathbf{w}) - \lambda_1\left(\mathbf{w}^T\bar{\mathbf{r}}-r_{\text{spce}}\right) \tag{33}$$

$$-\lambda_2(\mathbf{w}^T[\mathbf{1}]-1) - \tfrac{1}{2}c\left(\mathbf{w}^T\bar{\mathbf{r}}-r_{\text{spce}}\right)^2 - \tfrac{1}{2}c(\mathbf{w}^T[\mathbf{1}]-1)^2 \tag{34}$$

$$-\boldsymbol{\mu}^T g^-(\mathbf{w},\boldsymbol{\mu},c) - \tfrac{1}{2}c[g^-(\mathbf{w},\boldsymbol{\mu},c)]^T g^-(\mathbf{w},\boldsymbol{\mu},c) \tag{35}$$

and its derivative with respect to w_j is

$$\frac{\partial L_{\text{fixr}}}{\partial w_j} = \frac{1}{(\mathbf{w}^T\mathbf{D}\mathbf{w})^2} \cdot \left\{(\mathbf{w}^T\mathbf{D}\mathbf{w})[\bar{\mathbf{r}}+2H\mathbf{U}\mathbf{w}]_j - 2(\mathbf{w}^T\bar{\mathbf{r}}+H\mathbf{w}^T\mathbf{U}\mathbf{w})[\mathbf{D}\mathbf{w}]_j\right\} + B(1-2w_j) - \lambda_1\bar{r}_j - \lambda_2 - c\left(\mathbf{w}^T\bar{\mathbf{r}}-r_{\text{spec}}\right)\bar{r}_j - c(\mathbf{w}^T[1]-1) - \frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}\mu_j - cg^-(w_j,\mu_j,c)\frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}. \tag{36}$$

The recursive algorithm for implementing IPSRM-D with control of expected return by the augmented Lagrangian method is to update $\{\mathbf{w}, \lambda_1, \lambda_2, \boldsymbol{\mu}, c\}$ at every iteration t by the following equations until $\mathbf{w}$ converges

$$\begin{aligned} w_j(t+1) &= w_j(t) + \eta\frac{\partial L_{\text{fixr}}}{\partial w_j}, \qquad j = 1,\ldots,N \\ \lambda_1(t+1) &= \lambda_1(t) + c(t)\left(\mathbf{w}^T\bar{\mathbf{r}} - r_{\text{spce}}\right) \\ \lambda_2(t+1) &= \lambda_2(t) + c(t)(\mathbf{w}^T[\mathbf{1}]-1) \\ \mu_j(t+1) &= \mu_j(t)\times g^-(w_j,\mu_j(t),c(t)) \\ c(t+1) &= \beta\times c(t). \end{aligned} \tag{37}$$

For IPSRM-D with control of downside risk, the augmented Lagrangian is given by

$$L_{\text{fixv}} = \frac{\mathbf{w}\bar{\mathbf{r}}+H\mathbf{w}^T\mathbf{U}\mathbf{w}}{v_{\text{spec}}} + B\mathbf{w}^T([\mathbf{1}]-\mathbf{w}) - \lambda_1\left(\mathbf{w}^T\mathbf{D}\mathbf{w}-v_{\text{spce}}\right) \tag{38}$$

$$-\lambda_2(\mathbf{w}^T[\mathbf{1}]-1) - \tfrac{1}{2}c\left(\mathbf{w}^T\mathbf{D}\mathbf{W}-v_{\text{spce}}\right)^2 - \tfrac{1}{2}c(\mathbf{w}^T[\mathbf{1}]-1)^2 \tag{39}$$

$$-\boldsymbol{\mu}^T g^-(\mathbf{w},\boldsymbol{\mu},c) - \tfrac{1}{2}c[g^-(\mathbf{w},\boldsymbol{\mu},c)]^T g^-(\mathbf{w},\boldsymbol{\mu},c) \tag{40}$$

and its derivative with respect to w_j is

$$\frac{\partial L_{\text{fixv}}}{\partial w_j} = \frac{\bar{r}_j}{v_{\text{spec}}} + \frac{2H[\mathbf{U}\mathbf{w}]_j}{v_{\text{spec}}} + B(1-2w_j) - 2\lambda_1[\mathbf{D}\mathbf{w}]_j - \lambda_2 - c\left(\mathbf{w}^T\mathbf{D}\mathbf{w}-v_{\text{spec}}\right)[\mathbf{D}\mathbf{w}]_j - c(\mathbf{w}^T[\mathbf{1}]-1) - \frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}\mu_j - cg^-(w_j,\mu_j,c)\frac{\partial g^-(w_j,\mu_j,c)}{\partial w_j}. \tag{41}$$

The recursive algorithm for implementing IPSRM-D with control of downside risk by the augmented Lagrangian method is to update $\{\mathbf{w}, \lambda_1, \lambda_2, \boldsymbol{\mu}, c\}$ at every iteration t by the following equations until $\mathbf{w}$ converges

$$\begin{aligned} w_j(t+1) &= w_j(t) + \eta\frac{\partial L_{\text{fixv}}}{\partial w_j}, \qquad j = 1,\ldots,N \\ \lambda_1(t+1) &= \lambda_1(t) + c(t)\left(\mathbf{w}^T\mathbf{D}\mathbf{w} - v_{\text{spce}}\right) \\ \lambda_2(t+1) &= \lambda_2(t) + c(t)(\mathbf{w}^T[\mathbf{1}]-1) \\ \mu_j(t+1) &= \mu_j(t)\times g^-(w_j,\mu_j(t),c(t)) \\ c(t+1) &= \beta\times c(t). \end{aligned} \tag{42}$$

APPENDIX III
CCL LEARNING ALGORITHM FOR ENRBF

In the following, we will give out two CCL algorithms for ENRBF learning. One is for batch-way learning, and the other is for adaptive learning.

a) Batch CCL-ENRBF Algorithm. Give the training set $\{\mathbf{x}(t), I^i(t)\}_{t=1}^N$, we have the following iterative procedure:

Step 1) Fix $\boldsymbol{\theta}^i$, get $s(j|\mathbf{x}(t))$ by

$$s(j|\mathbf{x}(t)) = \begin{cases} 1, & \text{if } j = \arg\min_r \{d_{i,r}[\mathbf{x}(t) + \log|\Pi_r^i| + e_{i,r}^2(\mathbf{x}(t))]\} \\ 0, & \text{otherwise} \end{cases} \tag{43}$$

with

$$\begin{aligned} d_{i,j} &= [\mathbf{x}(t) - \mathbf{m}_j^i]^T (\boldsymbol{\Sigma}_j^i)^{-1} [\mathbf{x}(t) - \mathbf{m}_j^i] \\ e_{i,j}^2 &= \left[I^i(t) - (\mathbf{W}_j^i)^T \mathbf{x}(t) - c_j^i\right]^2 \Big/ \Pi_j^i \end{aligned} \tag{44}$$

where Π_j^i is the variance of the regression error $I^i(t) - (\mathbf{W}_j^i)^T\mathbf{x}(t) - c_j^i$.

Step 2) Update $\boldsymbol{\theta}^i$ by

$$\begin{aligned} \alpha_j^{\text{new}} &= \frac{1}{N} \sum_{t=1}^N s(j|\mathbf{x}(t)) \\ \mathbf{m}_j^{i\,\text{new}} &= \frac{1}{\alpha_j^{\text{new}} N} \sum_{t=1}^N s(j|\mathbf{x}(t))\mathbf{x}(t) \\ \boldsymbol{\Sigma}_j^{i\,\text{new}} &= \frac{1}{\alpha_j^{\text{new}} N} \sum_{t=1}^N s(j|\mathbf{x}(t)) \left[\mathbf{x}(t) - \mathbf{m}_j^{i\,\text{new}}\right] \cdot \left[\mathbf{x}(t) - \mathbf{m}_j^{i\,\text{new}}\right]^T \\ \mathbf{W}_j^{i\,\text{new}} &= (\boldsymbol{\Sigma}_j^i)^{-1} \mathbf{R}^i \\ c_j^{i\,\text{new}} &= E_j^i - \left(\mathbf{W}_j^{i\,\text{new}}\right)^T \mathbf{m}_j^{i\,\text{new}} \\ \Pi_j^{i\,\text{new}} &= \frac{1}{\sum_{t=1}^N s(j|\mathbf{x}(t))} \left[I^i(t) - \left(\mathbf{W}_j^{i\,\text{new}}\right)^T \mathbf{x}(t) - c_j^{i\,\text{new}}\right]^2 \end{aligned} \tag{45}$$

with

$$\begin{aligned} E_j^i &= \frac{1}{\sum_{t=1}^N s(j|\mathbf{x}(t))} \sum_{t=1}^N s(j|\mathbf{x}(t)) I^i(t) \\ \mathbf{R}^i &= \frac{1}{\sum_{t=1}^N s(j|\mathbf{x}(t))} \sum_{t=1}^N s(j|\mathbf{x}(t)) \left[I^i(t) - E_j^i\right] \left[\mathbf{x}(t) - \mathbf{m}_j^{i\,\text{new}}\right]^T \end{aligned} \tag{46}$$

where E^i refers to the expectation of I^i, and $\mathbf{R}^i$ is the cross correlation of $\mathbf{x}$ and I^i.

b) Adaptive CCL-ENRBF Algorithm. Given each pair $\{\mathbf{x}(t), I^i(t)\}$, go through the following steps once:

Step 1) Fix $\boldsymbol{\theta}^i$, get $s(j|\mathbf{x}(t))$ by (43) and let $j^* = \arg\max_j s(j|\mathbf{x}(t))$.

Step 2) Update $\boldsymbol{\theta}^i$ by

$$\begin{aligned} \alpha_{j^*}^{\text{new}} &= \frac{n_{j^*}}{\sum_{j=1}^K n_j} \\ \mathbf{m}_{j^*}^{i\,\text{new}} &= \mathbf{m}_j^{i\,\text{old}} + \eta \left[\mathbf{x}(t) - \mathbf{m}_{j^*}^{i\,\text{old}}\right] \\ \boldsymbol{\Sigma}_{j^*}^{i\,\text{new}} &= (1-\eta)\boldsymbol{\Sigma}_{j^*}^{i\,\text{old}} + \eta \left[\mathbf{x}(t) - \mathbf{m}_{j^*}^{i\,\text{old}}\right]\left[\mathbf{x}(t) - \mathbf{m}_{j^*}^{i\,\text{old}}\right]^T \\ E_{j^*}^{i\,\text{new}} &= E_{j^*}^{i\,\text{old}} + \eta \left[I^i(t) - E_{j^*}^{i\,\text{old}}\right] \\ c_{j^*}^{i\,\text{new}} &= E_{j^*}^{i\,\text{new}} - \left(\mathbf{W}_{j^*}^{i\,\text{old}}\right)^T \mathbf{m}_{j^*}^{i\,\text{old}} \\ \mathbf{W}_{j^*}^{i\,\text{new}} &= \mathbf{W}_{j^*}^{i\,\text{old}} + \eta \left[\mathbf{x}(t) - \left(\mathbf{W}_{j^*}^{i\,\text{new}}\right)^T \mathbf{x}(t) - c_{j^*}^{i\,\text{new}}\right] \mathbf{x}(t)^T \\ \Pi_{j^*}^{i\,\text{new}} &= (1-\eta)\Pi_{j^*}^{i\,\text{old}} + \eta \left[I^i(t) - \left(\mathbf{W}_{j^*}^{i\,\text{old}}\right)^T \mathbf{x}(t) - c_{j^*}^{i\,\text{new}}\right]^2 \end{aligned} \tag{47}$$

where η is a small positive learning rate, and n_j is the number that $j = j^*$ in the past.

ACKNOWLEDGMENT

The authors would like to give many thanks to anonymous reviewers for their valuable comments and suggestions.

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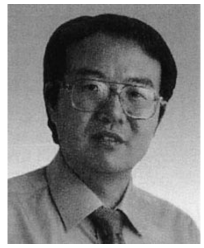

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