

## CSCI 5020 External Memory Data Structures: Exercise List 3

In the following problems,  $B$  is the block size, and  $M$  is the memory capacity.

**Problem 1 (Interval Stabbing).** Let  $S$  be a set of  $n$  intervals in  $\mathbb{R}$ . Given a real value  $q$ , a *stabbing query* finds all the intervals in  $S$  that contain  $q$ . Describe a data structure of  $O(n/B)$  space that answers such a query in  $O(\log_B n + k/B)$  I/Os where  $k$  is the number of intervals reported. For example, if  $I = \{[2, 9], [3, 12], [5, 7], [8, 18]\}$ , then a query with  $q = 6$  returns the first 4 intervals in  $I$ .

**Problem 2 (Segment Intersection).** Let  $S$  be a set of  $n$  horizontal segments in  $\mathbb{R}^2$  (namely, each segment is in the form  $[x_1, x_2] \times y$ ). Given a vertical segment  $q = x \times [y_1, y_2]$ , a query reports all the segments in  $S$  intersecting  $q$ . Describe a data structure of  $O(n/B)$  space that answers such a query in  $O(\log_B n + k/B)$  I/Os where  $k$  is the number of segments reported.

**Problem 3 (1D Range Sum).** Let  $P$  be a set of  $n$  points in  $\mathbb{R}$ . Each point in  $P$  is associated with a real-valued *weight*. Given an interval  $q = [x, y]$  in  $\mathbb{R}$ , a *range sum* query returns the sum of all the weights in  $P \cap q$ . Design a data structure of  $O(n/B)$  space that answers a query in  $O(\log_B n)$  I/Os, and supports an insertion and a deletion in  $O(\log_B n)$  I/Os.

**Problem\* 4 (Segment Intersection Sum).** Let  $S$  be a set of  $n$  horizontal segments in  $\mathbb{R}^2$ , each of which is associated with a real-valued *weight*. Given a vertical segment  $q = x \times [y_1, y_2]$ , a query reports the sum of all the segments in  $S$  intersecting  $q$ . Describe a data structure of  $O(\frac{n}{B} \log_B n)$  space that answers such a query in  $O(\log_B n)$  I/Os.